REDUCED WHITEHEAD GROUPS FOR OUTER FORMS OF ANISOTROPIC GROUPS OF TYPE A_n

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Let K be a field and D a central finite-dimensional division K-algebra.

We will be interested in the problem of describing of reduced Whitehead groups of anisotropic groups of type A_n. It is well known that there are two forms (inner and outer) of such groups. As for inner case the groups of K-rational points of anisotropic groups can be described as

$$SL(D) = \{d \in D^* | Nrd_D(d) = 1\},$$

where Nrd_D is the reduced norm homomorphism of D^* to K^*. The describing of normal structure of such groups is a very vast problem for the arbitrary field K, but at least the special case of algebraic number fields K we have the following fundamental Segev's result [1,2].

If one looks at the similar problem for outer forms in anisotropic situation, so there are no any complete results even in the case related to computation of reduced Whitehead groups of such forms. In our talk we will discuss the problem of description of reduced Whitehead groups for multiplicative groups of tame henselian division algebras. More precisely, let k be any field of characteristic different from 2, and let K be a quadratic extension. Denote the non-trivial Galois automorphism of K over k by σ. Let D be a central division algebra over K (i.e. K = center of D) such that σ extends to an involution τ of D (in this case τ is an unitary involution). One can define the unitary group of D as

$$U(D,\tau) = \{d \in D^* | d^\tau d = 1\},$$

and special unitary group

$$SU(D,\tau) = U(D,\tau) \cap SL(D).$$

The latter group is the group of k-rational points of outer form of an anisotropic k-defined algebraic group of type ${}^2A_{n-1}$. The main problem in this situation is the problem of computation of so-called reduced Whitehead group of D:

$$SUK_1^{an}(D,\tau) = SU(D,\tau)/[U(D,\tau), U(D,\tau)],$$

where $[U(D,\tau), U(D,\tau)]$ is the commutator subgroup of $U(D,\tau)$.

For some time there was a widespread opinion that the above group is trivial at least in the situation of global fields k, but Sury proved in [3] that this is not the case (see also [4] for other kind of fields k). Since groups $SUK_1^{an}(D,\tau)$ are nontrivial in general, then the problem of its computation arises. In the talk among other topics [see 5–10] we will discuss some results, which deal with tame henselian division algebras D and allows us to reduce mainly the computation of $SUK_1^{an}(D,\tau)$ to processing with objects defined over residue algebra $\overline{D}$.

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