APPLICATION OF THE ESTIMATIONS OF THE MIXED CUMULANT OF THE FOURTH ORDER FOR RESEARCH OF RATES OF EXCHANGE ON THE FINANCIAL MARKET

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In this article the statistics of the higher order – the estimations of the mixed cumulant of the 4th order for rates of exchange on the financial market are investigated. For construction of estimations the initial data are the prices of opening, closing, minimum, maximum for currency pair EUR/USD are used. Under this data the plot of the estimations of the mixed cumulant of the 4th order and the density plot are constructed. The properties inherent in estimations of the mixed cumulant are revealed and some laws are traced. Researches are spent in package *Mathematica*.

Keywords: cumulant, estimation, stochastic process, financial market.

The present stage of development of probability theory and the mathematical statistics is characterized by considerable expansion of theoretical researches under the statistical spectral analysis (to the analysis in private area) time series and their practical application in many areas of human activity, such, as economy, spectroscopy, medicine, biology, insurance, the finance, sociology, radio electronics, the electrical engineer, geophysics, geology and many other things. Also time series are applicable and in the financial market Forex.

As a time series named the sequence of supervision usually ordered in time though streamlining and to any other parameter within the limits of the world financial market it is possible to name a time series quotations of currency pairs.

The purposes of studying of time series can be various. It is possible, for example, to aspire to predict the future on the basis of knowledge of the past, to operate the process generating a number, to find out the mechanism generating a number, or is simply compressed to describe prominent features of a series. Therefore, as the statistical spectral analysis of time series understand the statistical spectral analysis of stationary stochastic processes.

One of the main tasks of the spectral analysis of time series is construction and research of estimations of spectral density of stationary stochastic processes as they give the important information on process structure. It is very important at studying of financial market Forex. Knowing more detailed behavior of any currency pair, it is possible on the basis of last historical given to foresee the further behavior of the considered currency pair. This knowledge will allow to make profitable transactions further.

In the given work the analysis only discrete stationary stochastic processes that allows to calculate statisticans with computer use is carried out. The purpose of given article is construction of the estimations of mixed cumulant of the 4th order for one currency pair EUR/USD, and also studying and revealing of the latent properties of the constructed estimations.

Let's consider four-dimensional stationary stochastic process $X^4(t)=\{X_a(t), a=\overline{1,4}\}$, $t \in Z$. Let are available T consecutive supervision through equal intervals $X_a(0), X_a(1),\ldots,$ $X_a(T-1)$ behind process $X_a(t), a=\overline{1,4}, t\in Z$. As the estimation of the mixed cumulant of 4th order we will consider statistics of a following kind:

$$\hat{c}_4(t_1,t_2,t_3)=\hat{m}_4(t_1,t_2,t_3)-\hat{m}_2(t_1)\hat{m}_2(t_3-t_2)-\hat{m}_2(t_2)\hat{m}_2(t_3-t_1)-\hat{m}_2(t_3)\hat{m}_2(t_2-t_1), \tag{1}$$

$$\hat{m}_2(t_1)=\hat{c}_2(t_1)=\frac{1}{T}\sum_{t=0}^{T-1}x(t_1+t)x(t), \tag{2}$$

$$\hat{m}_4(t_1,t_2,t_3)=\frac{1}{T}\sum_{t=0}^{T-1}x(t_1+t)x(t_2+t)x(t_3+t)x(t), t, t_j\in Z,\ j=\overline{1,3}. \tag{3}$$

The estimation of the mixed cumulant of the 4th order $\hat{c}_4(t_1,t_2,t_3)$ possesses following properties: is asymptotically not displaced, well-founded in mean square sense and has asymptotic normal distribution with a population mean $c_4(t_1,t_2,t_3)$ [1].

Let's investigate the estimation of the mixed cumulant of 4th order (1) for multidimensional stochastic processes. Let's write down it in a kind (4):

$$\hat{c}_{a_1a_2a_3a_4}(t_1,t_2,t_3)=\frac{1}{T}\sum_{t=0}^{T-1}x_{a_1}(t_1+t)x_{a_2}(t_2+t)x_{a_3}(t_3+t)x_{a_4}(t) \tag{4}$$

$$-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_1}(t_1+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_2}(t_3-t_2+t)x_{a_3}(t)-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_2}(t_2+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_1}(t_3-t_2+t)x_{a_3}(t)$$

$$-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_3}(t_3+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_1}(t_2-t_1+t)x_{a_2}(t),\ t,t_j\in Z,\ j=\overline{1,3}.$$

Research of the estimations (4) has shown that from 24 estimations 6 are unique only, other 18 turn out from the given 6 turn along an axis of values.

Further we take quotations of currency pair EUR/USD with the period of updating of the data equal one day. At the prices of opening, maximum and minimum and the closing price.

The plots of the raw datas are presented on the following drawing below (Fig. 1).

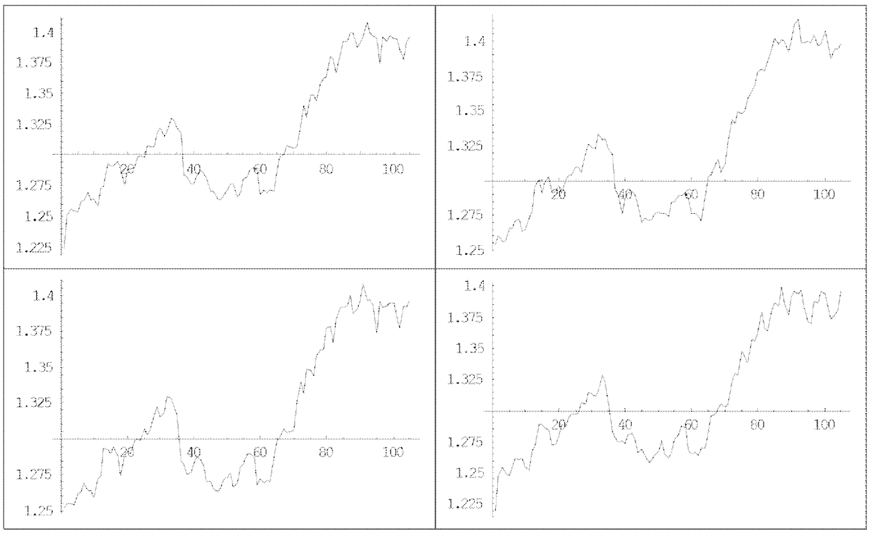

Fig. 1. The plot of the quotations of currency pair EUR/USD

The obtained data isn't stationary. We will lead them stationary after operation of a capture of differences. For research of the initial data we will use package *Mathematica*. Plots of the initial data are presented more low in following drawings (Fig. 2).

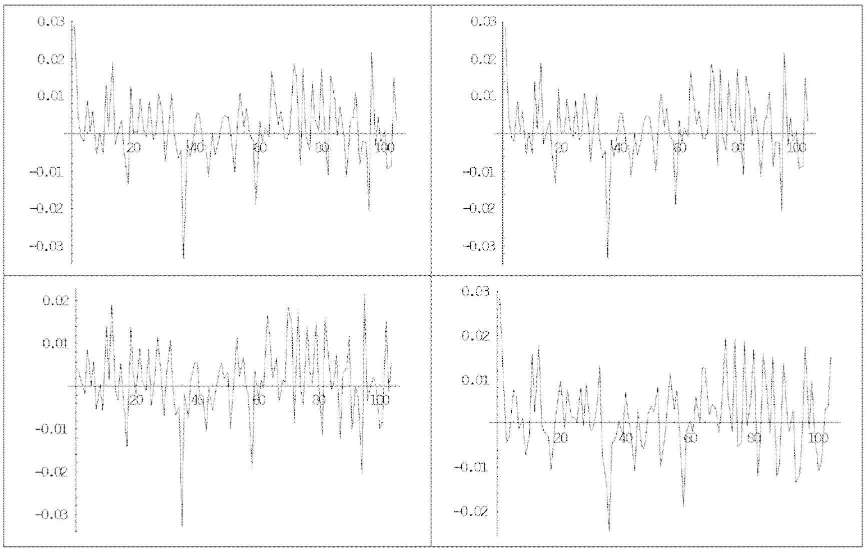

Fig. 2. The plot of the initial data $x_{a_1}(t), x_{a_2}(t), x_{a_3}(t), x_{a_4}(t)$

Under this data we will construct plots of the estimation of the mixed cumulant of the 4th order and we will fix the estimation concerning one of planes. Let's construct the estimations of the mixed cumulant of the 4th order of a three-dimensional kind, and also the density plot. We will consider the estimation $\hat{c}_{a_1 a_2 a_3 a_4}(t_1, t_2, t_3)$ kind (4) (Fig. 3–4).

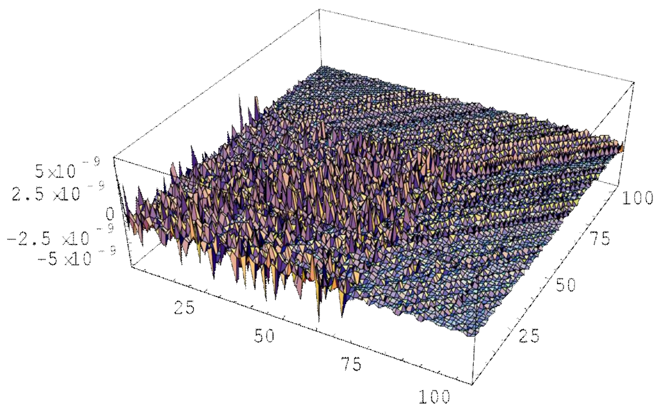

Fig. 3. The plot of the estimation (4)

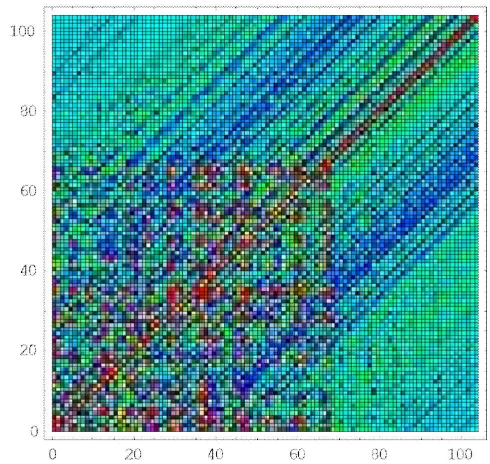

Fig. 4. The plot of density of the estimation (4)

Also we will construct table of pairs periodicity and the plot of the maximum points which are resulted more low in following drawings (Fig. 5–6).

x	y	z
28	61	3.5268×10^{-9}
28	29	3.01177×10^{-9}
35	36	2.84545×10^{-9}
28	45	2.77512×10^{-9}
35	29	2.7281×10^{-9}
36	61	2.60245×10^{-9}
18	45	2.57691×10^{-9}
57	60	2.35239×10^{-9}
18	19	2.35018×10^{-9}
28	48	2.32677×10^{-9}
35	68	2.25447×10^{-9}
28	68	2.24071×10^{-9}
28	36	2.23963×10^{-9}
12	13	2.21001×10^{-9}
18	61	2.17216×10^{-9}
65	60	2.08313×10^{-9}
44	45	2.05038×10^{-9}
4	61	2.0471×10^{-9}
35	45	2.03195×10^{-9}
5	6	2.02322×10^{-9}

Fig. 5. The table of pairs periodicity

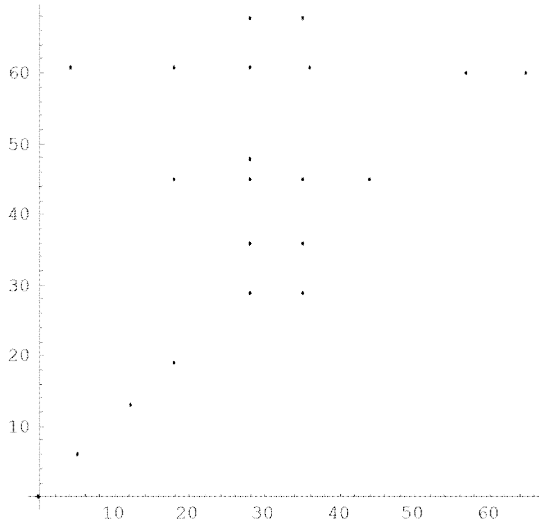

Fig. 6. The plot of the maximum points

For the estimation (4) 20 local maximum are found. For the estimation $\hat{c}_{a_1a_3a_2a_4}(t_1,t_2,t_3)$, $\hat{c}_{a_1a_4a_2a_3}(t_1,t_2,t_3)$ plots of the estimations and density, the table of pairs periodicity and a drawing of the maximum points coincide.

Now we will consider the estimation $\hat{c}_{a_2a_3a_1a_4}(t_1,t_2,t_3)$. The mixed cumulant of 4th order (1) which is presented in the form of (5):

$$\hat{c}_{a_2a_3a_1a_4}(t_1,t_2,t_3) = \frac{1}{T}\sum_{t=0}^{T-1} x_{a_2}(t_1+t)x_{a_3}(t_2+t)x_{a_1}(t_3+t)x_{a_4}(t) \tag{5}$$

$$-\frac{1}{T}\sum_{t=0}^{T-1} x_{a_2}(t_1+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1} x_{a_3}(t_3-t_2+t)x_{a_1}(t) - \frac{1}{T}\sum_{t=0}^{T-1} x_{a_3}(t_2+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1} x_{a_2}(t_3-t_2+t)x_{a_1}(t)$$

$$-\frac{1}{T}\sum_{t=0}^{T-1} x_{a_1}(t_3+t)x_{a_4}(t)\frac{1}{T}\sum_{t=0}^{T-1} x_{a_2}(t_2-t_1+t)x_{a_3}(t),\quad t,t_j \in Z,\ j=\overline{1,3}.$$

Let's construct the estimation $\hat{c}_{a_2a_3a_1a_4}(t_1,t_2,t_3)$ three-dimensional kind, and also the density plot (Fig. 7–8).

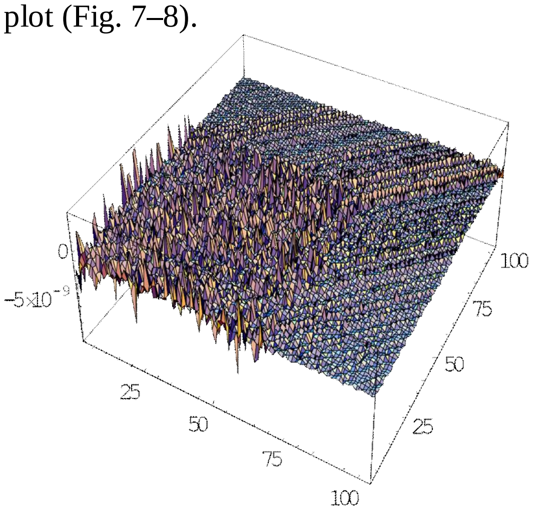

Fig. 7. The plot of estimation (5)

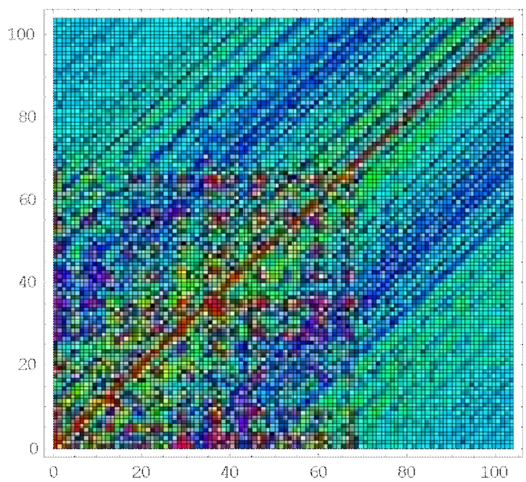

Fig. 8. The plot of density of the estimation (5)

x	y	z
35	24	3.19775×10^{-9}
57	61	3.17856×10^{-9}
35	60	2.79251×10^{-9}
29	60	2.72317×10^{-9}
28	24	2.65514×10^{-9}
57	29	2.62894×10^{-9}
57	36	2.4629×10^{-9}
9	36	2.37111×10^{-9}
55	24	2.34356×10^{-9}
57	39	2.29334×10^{-9}
57	45	2.21929×10^{-9}
24	61	2.19072×10^{-9}
18	24	2.18149×10^{-9}
66	61	2.02797×10^{-9}
9	61	2.01669×10^{-9}
34	38	1.97989×10^{-9}
13	24	1.8627×10^{-9}
9	30	1.80912×10^{-9}
35	66	1.79945×10^{-9}
57	68	1.79262×10^{-9}

Fig. 9. The table of pairs periodicity

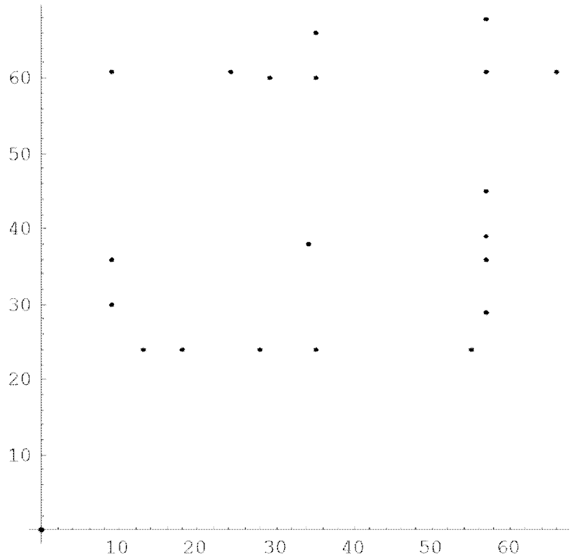

Fig. 10. The plot of the maximum points

For the estimation $\hat{c}_{a_2a_3a_1a_4}(t_1,t_2,t_3)$ 20 local maximum are found. For the estimation $\hat{c}_{a_2a_3a_1a_4}(t_1,t_2,t_3)$, $\hat{c}_{a_2a_4a_1a_3}(t_1,t_2,t_3)$ plots of the estimation and density, the table of pairs periodicity and a drawing of the maximum points coincide (Fig. 9–10).

Let's consider the estimation $\hat{c}_{a_3a_4a_1a_2}(t_1,t_2,t_3)$. The mixed cumulant of 4th order (1) which is presented in the form of (6):

$$\hat{c}_{a_3a_4a_1a_2}(t_1,t_2,t_3)=\frac{1}{T}\sum_{t=0}^{T-1}x_{a_3}(t_1+t)x_{a_4}(t_2+t)x_{a_1}(t_3+t)x_{a_2}(t) \tag{6}$$

$$-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_3}(t_1+t)x_{a_2}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_4}(t_3-t_2+t)x_{a_1}(t)-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_4}(t_2+t)x_{a_2}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_3}(t_3-t_2+t)x_{a_1}(t)$$

$$-\frac{1}{T}\sum_{t=0}^{T-1}x_{a_1}(t_3+t)x_{a_2}(t)\frac{1}{T}\sum_{t=0}^{T-1}x_{a_3}(t_2-t_1+t)x_{a_4}(t),\quad t,t_j\in Z,\; j=\overline{1,3}.$$

Let's construct the estimation $\hat{c}_{a_3a_4a_1a_2}(t_1,t_2,t_3)$ three-dimensional kind, and also the density plot (Fig. 11–12).

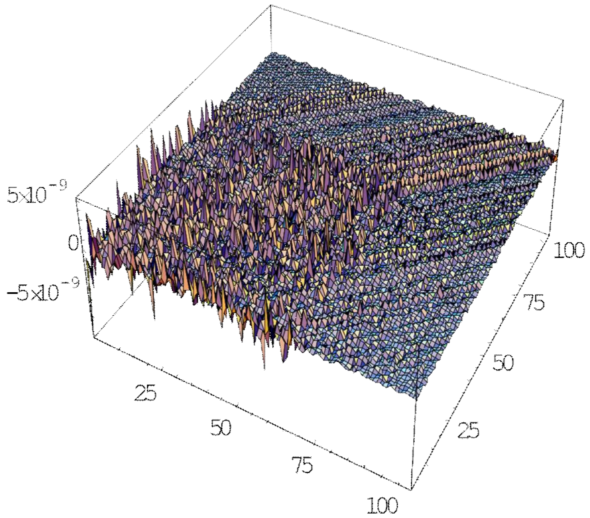

Fig. 11. The plot of the estimation (6)

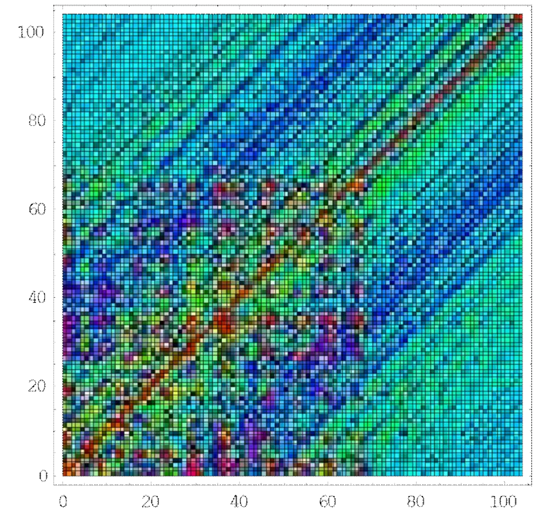

Fig. 12. The plot of density of the estimation (6)

x	y	z
35	24	3.23973×10^{-9}
57	61	3.17834×10^{-9}
28	24	2.74302×10^{-9}
35	60	2.71757×10^{-9}
29	60	2.69175×10^{-9}
57	29	2.56072×10^{-9}
57	36	2.40494×10^{-9}
57	39	2.31959×10^{-9}
9	36	2.23636×10^{-9}
57	45	2.20944×10^{-9}
18	24	2.20897×10^{-9}
24	61	2.19852×10^{-9}
55	24	2.19501×10^{-9}
66	61	2.03807×10^{-9}
9	61	1.97707×10^{-9}
34	38	1.88098×10^{-9}
35	66	1.83472×10^{-9}
57	42	1.82548×10^{-9}
57	48	1.81492×10^{-9}
24	36	1.77655×10^{-9}

Fig. 13. The table of pairs periodicity

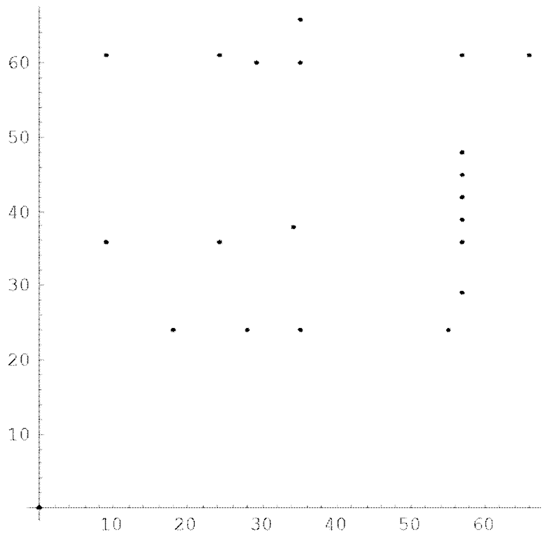

Fig. 14. The plot of the maximum points

For the estimation $\hat{c}_{a_3 a_4 a_1 a_2}(t_1, t_2, t_3)$ 20 local maximum are found (Fig. 13–14).

During researches it was revealed that among points of a local extremum of the constructed estimations there are repeating. The received estimations can be used for revealing of the periods hidden in the initial data.

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