SOME NOTES ABOUT NONPARAMETRIC MODELING OF NONLINEAR DYNAMICAL PROCESSES

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The problem of nonlinear dynamical systems of Wiener type identification is considered. The parametric structure of linear dynamical part of the system is unknown. The common type of nonlinearity is assumed to be known with the set of parameters. In this theses are presented the algorithm of nonparametric model of such class of nonlinear systems, and some results of numerical experiments.

Keywords: nonparametric model, nonlinear dynamical system, Wiener model.

FORMULATION OF THE IDENTIFICATION TASK

The problem of nonlinear dynamical system identification is one of the most important one in the theory of control. In spite of the existing a lot of methods for dynamical systems identification, there is no universal theory that allows to design the models of such systems. In this paper the dynamic systems identification "in the broad sense” is considered. In this case the parameterization of the investigated object model is not available or one can partially parameterized the model on the base of available a priori information [1–3].

A lot of nonlinear systems can be considered in the form of a sequence connected linear dynamic and nonlinear static blocks (Wiener model) [3]. The main difficulty in such model design is measures unavailability of intermediate signals $w(t)$ (it is the value of the first block output). The total scheme of nonlinear dynamical system identification is shown in Fig. 1.

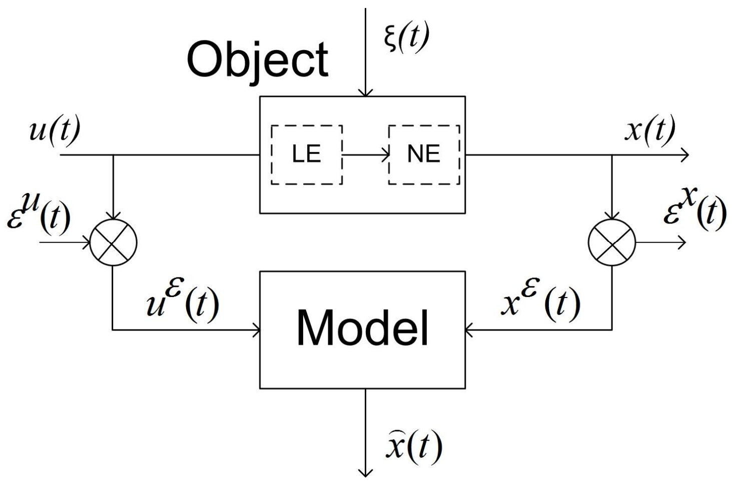

Fig. 1. The general scheme of the identification problem

Where «Object» – nonlinear dynamical object, which is consist of a sequence connected linear dynamic (LE) and nonlinear static (NE) blocks, $u(t)$ and $x(t)$ – input and output variables of the object, $u_t^\varepsilon, x_t^\varepsilon$ – or $\{u_i, x_i, \ t=\overline{1,s}\}$ – appropriate observation of process variables in discrete moments of time t, $\xi(t)$ – unobserved random effects, $\varepsilon^u(t), \varepsilon^x(t)$ – random noise in measure channels: $M\{\xi\}=0, \ D\{\xi\}<\infty$, $\hat{x}(t)$ – output of the object model.

Available priory information is uneven sample of input and output variables of the object's measures of s size $\{u_i, x_i, i=\overline{1,s}\}$. A structure and parameters of linear dynamic block of such system is unknown. The common type of the nonlinear function is assumed to be known with the set of parameters. It is required to design the mathematical model of the stochastic object according to the measures of process. That would describes objects behavior at arbitrary input effects and additive noise the presence on the output. Problem of nonlinear system identification can be divided into two tasks. At first, the parameters of nonlinear block and step response of the system are estimated, and then – a nonparametric model of the nonlinear dynamical system is designed.

LINEAR DYNAMICAL SYSTEM IDENTIFICATION

Nonparametric model of a nonlinear dynamical object can be designed on the base of the system step response. The step response of a system in a given initial state consists of the time evolution of its outputs $x(t) = h(t)$ when its control inputs are Heaviside step function $u(t) = 1(t)$. It is required on the base of measures $\{u_i, x_i, \ t=\overline{1,s}\}$ to construct the linear dynamical system (LDS) model, if the order of the system is unknown. The reaction of linear dynamical system $x(t)$ to the input signal $u(t)$ is described with the Duhamel integral [2]. And then the nonparametric model is:

$$w(t) = \int_0^t h'(t-\tau)u(\tau)d\tau = \int_0^t k(t-\tau)u(\tau)d\tau, \tag{1}$$

where $h(t)$ – step response of this system, $k(t)$ – impulse response of the system (weight function).

In this case we can calculate the output value of object $x(t)$ only if its weight function $h(t)$ is known. But in practice, as a rule, it is impossible to get the weight function of the object, and therefore is used the method which is shown hereinafter. The main idea of LDS identification in the nonparametric uncertainty conditions [4] is based on the nonparametric estimation of the system impulse response. If we applies to the system input the Heaviside function effect, we obtain the values of its step response function at discrete moments of time t_i, $i=\overline{1,s}$. Therefore we can write the estimation of step response of the system as a stochastic approximation of regression as follows [4]:

$$\hat{h}(t) = \frac{1}{sc_s} \cdot \sum_{i=1}^{s} h_i H\left(\frac{t-t_i}{c_s}\right), \tag{2}$$

where $\{h_i = x_i, \ i=\overline{1,s}\}$ $u(t) = 1(t)$ – sample values of step response of LDS, that is, reaction of a system with zero initial conditions to the step input action $u(t) = 1(t)$. Wherein in equation (2) $H(\cdot)$ – Kernel function and c_s – bandwidth parameter are satisfied the conditions of convergence[2]:

$$c_s > 0; \lim_{s\to\infty} c_s = 0; \lim_{s\to\infty} sc_s = \infty, \tag{3}$$

$$\int_{\Omega(u)} H'(u)du = 0, \quad c_s \int_{\Omega(u)} H'(u)udu = -1, \quad u = \frac{\tau - t}{c_s}, \tag{4}$$

$$\lim_{s\to\infty} c_s^{-1} H\left(\frac{\tau - t}{c_s}\right) = \delta(\tau - t).$$

It is known, that the system impulse response $k(t)$ is calculated as $k(t) = dh(t)/dt$. Then nonparametric estimation of the system impulse response assumes the form:

$$k_s(t) = h_s'(t) = \frac{1}{sc_s} \cdot \sum_{i=1}^{s} h_i H'\left(\frac{t - t_i}{c_s}\right). \tag{5}$$

By substituting the impulse response estimation in the Duhamel integral, we obtain a nonparametric model of LDS. Linear dynamic systems can be described with the following mathematical formula [3]:

$$\hat{w}(t) = \frac{1}{sc_s} \cdot \sum_{i=1}^{s} \sum_{j=1}^{t/\Delta\tau} \hat{h}_i H'\left(\frac{t - \tau_j - t_i}{c_s}\right) u(\tau_j)\Delta\tau, \tag{6}$$

where τ – integration variable, $\Delta\tau$ – sampling step. Then let's consider the algorithm of nonlinear system identification.

WIENER SYSTEM IDENTIFICATION

Let's consider the system that can be represented as a model of Wiener type (Fig. 2) [3]

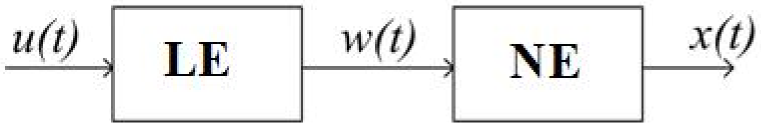

Fig. 2. Wiener model, LE – linear dynamical and NE – nonlinear parts of the system, $u(t)$ – input action, $w(t)$ – the output of intermediate part of object (is not measured), $x(t)$ – object output

The nonlinear element output $w(t)$ is not available to measure. We assume that the parameterized structure of the linear element in Wiener model is unknown, but the type of nonlinearity is known with a set of parameters. According to Figure 2 the relationship between input $u(t)$ and output $x(t)$ signals of the object with zero initial conditions can be described by the following system of equations [5]:

$$x(t) = f(w(t)),$$

$$w(t) = \int_0^t h'(t-\tau)u(\tau)d\tau, \tag{7}$$

where $u(t)$ and $x(t)$ – input and output variable of the system; $w(t)$ – the output of the linear part of the system (is not measured); $f\{w(t), \alpha\}$ – nonlinear function, the structure of which is known with the set of parameters.

In this case according to (7) the object output is calculated as the following equation:

$$\hat{x}(t) = f(\hat{w}(t)) = \hat{f}\left(\int_0^t \hat{k}'(t-\tau)u(\tau)d\tau, \hat{a}\right) = \hat{f}\left(\int_0^t h(t-\tau)u(\tau)d\tau, a\right). \tag{8}$$

However, due to the fact that $w(t)$ – is not measured value, in the model the response function estimation $\hat{h}(t)$ of system LE and parameters a of the NE function $f(w(t), a)$.

The mathematical model of the nonlinear object can be represented as a set of equations (8), in that, instead of the weight function $h(t)$ and the parameters α are used their statistical estimations. To obtain this estimation it is necessary to form the sample $\{u_i, \omega_i\}, i = \overline{1,s}$. This new sample is generated [6] in the same initial conditions (that is, the same values of input actions, sampling step and the noise value) as the sample of «input-output» system values $\{u_i, x_i\}, i = \overline{1,s}$ was. Where u_i – input action measures, w_i – estimation of system linear dynamical block output [2]. The step response value $h(t)$ is the linear dynamic element reaction to the input $u(t) = 1$, that is $h(t) = w(t/u(t) = 1)$. The value of $w(t)$ is not available to measure, because we can measure only the nonlinear object output (is denoted $x^1(t)$), which is equal $x^1(t) = f(h(t))$.

In the case when for some classes of nonlinear elements, the function $f(w(t), a)$ can be solved for $w(t)$, we have [6]:

$$w(t) = \hat{f}^{-1}(x(t)). \tag{9}$$

Then the values of response function of the system linear element at discrete points in time can be estimated as follows:

$$h_i = \hat{f}^{-1}(x^1{}_i, \alpha), \tag{10}$$

where $x^1{}_i$ – measure values of experimental values of the investigated objects reaction to the step input action $u(t) = 1(t)$, h_i – estimated values of the objects linear element step response. Selection the nonlinear function $f^{-1}(x)$ and its parameters estimation algorithm depends on NE type. Further, the system step response can be estimated on the basis of discrete values sample as the nonparametric regression stochastic approximation type (2) [7]. By substituting the step response estimation in Duhamel integral in accordance with (6), we obtain a nonparametric model, which estimating the output of the linear element [5]:

$$\hat{w}(t) = \frac{1}{sc_s} \cdot \sum_{i=1}^{s} \sum_{j=1}^{t/\Delta\tau} h_i H' \left(\frac{t - \tau_j - t_i}{c_s} \right) u(\tau_j) \Delta\tau, \tag{11}$$

where τ – integration variable, $\Delta\tau$ – sampling step.

In this case, the nonparametric model of nonlinear object is the following:

$$\hat{x}(t) = \hat{f}\left(\int_0^t h'(t-\tau) u(\tau) d\tau, \hat{\alpha} \right) = \hat{f}\left(\frac{1}{sc_s} \cdot \sum_{i=1}^{s} \sum_{j=1}^{t/\Delta\tau} \hat{h}_i H' \left(\frac{t - \tau_j - t_i}{c_s} \right) u(\tau_j) \Delta\tau, \hat{\alpha} \right), \tag{12}$$

where $u(t)$ – input signal of the system; $x(t)$ – output signal of the system; $f\{w(t), a\}$ – nonlinear function estimation, $\hat{\alpha}$ – estimation of system nonlinear element parameters, $\hat{h}(t)$ – estimation of the linear element step response. Thus, we get the algorithm for modeling of Wiener type nonlinear dynamical systems.

NONPARAMETRIC MODEL OF SYSTEMS WITH A QUAD

Let's consider a system that is represented as the Wiener model. The nonlinear part of the system is a quad, that is described by a function of the form: $f(w) = aw^2$, $a = const$. The object output is calculated as follows: $x(t) = f(w, a) = aw^2$. However, the values $w(t)$ are not available for the researcher measuring. Nonparametric modeling method of the object is proposed.

If the value of input action $u(t) = 1$, then the output of nonlinear system is equal to $x^1(t) = aw(t)^2 = ah(t)^2$. That is, the step response of a linear element $h(t)$ can be represented by the output of the process as follows:

$$h(t) = \sqrt{x^1(t)/a}. \tag{13}$$

For an arbitrary input action and zero initial conditions the output of linear part of the system is described by the equation (9). Considering (13) the output of the linear element is:

$$\hat{w}(t) = \frac{1}{sc_s}\sum_{i=1}^{s}\sum_{j=1}^{t/\Delta t}\sqrt{\frac{x^1_i}{a}}\cdot H'\left(\frac{t-\tau_j-t_i}{c_s}\right)u(\tau_j)\Delta\tau, \qquad (14)$$

where x^1 – the reaction of a nonlinear system (if $u(t) = 1$), $u(t)$ – a test input action, a – unknown value of a quad parameter.

Then the model of the nonlinear dynamic object of Wiener type is:

$$\hat{x}(t) = \left[\frac{1}{sc_s}\sum_{i=1}^{s}\sum_{j=1}^{t/\Delta t}\sqrt{x^1_i}\cdot H'\left(\frac{t-\tau_j-t_i}{c_s}\right)u(\tau_j)\Delta\tau\right]^2, \qquad (15)$$

where $u(t)$ – a test input action.

Example. Let's consider a nonlinear dynamical system (NDS) consisting of a quad (with parameter $a = 7.2$) and the differential equation (simulating object):

$$1\cdot y''(t) + 0{,}7\cdot y'(t) + 2{,}43y(t) = u(t). \qquad (16)$$

The following figures presents the results of considered object modeling when the measurement channels acts random noise 5% of the object output value.

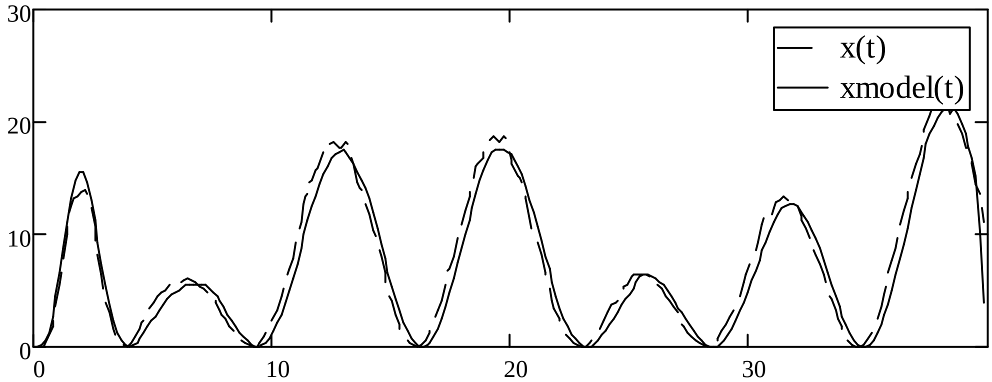

Fig. 3. xmodel (t) – a model of a nonlinear system, x_i – the system output, the sample size $s = 250$, sampling interval $\Delta t = 0.16$, noise 5%, input action: $u(t) = 3cos(0.5t) + sin(0.2t)$, the relative average error of simulation 3.11%

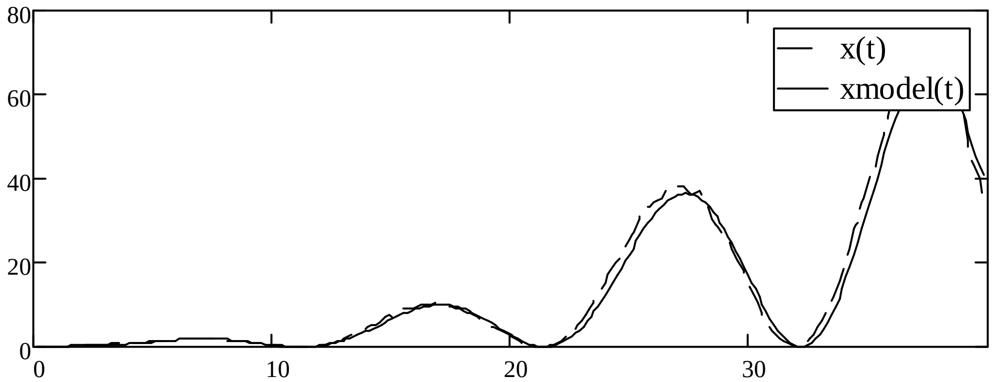

Fig. 4. xmodel (t) – a model of a nonlinear system, x_i – the system output, the sample size $s = 250$, sampling interval $\Delta t = 0.16$, noise 5%, input action: $u(t) = 0.2sin(0.3t) + 0.4cos^2(0.4t)$, the relative average error of simulation 1.9%

The presented results of the numerical experiments show that nonparametric model satisfactorily describes a nonlinear systems behavior with different input effects.

CONCLUSION

In this paper we consider the problem of nonlinear dynamical systems identification. The investigated objects are presented as a consequent combination of linear dynamic and nonlinear static blocks (Wiener model). Thus, we consider the problem of modeling of the nonlinear dynamical processes under conditions of partial parameterization of model structure. Presented methods of the nonlinear system identification are based on the combining the models of linear dynamic and nonlinear static processes in the overall model of the system. These techniques do not require the presence of full a priori information about the structure of the object. The practical part presents the results of numerical experiments, in that were designed the models nonlinear dynamical processes of Wiener type in the cases of quad nonlinearity.

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