ABOUT THE IDENTIFICATION OF DYNAMIC PROCESSES

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The task of nonparametric identification of dynamic objects with discrete-continuous nature of the process is conceded. The method of dynamic processes modeling, based on the nonparametric algorithms is offered. The complexity of dynamic process modeling under condition of incomplete information is discussed. The results of computing experiment explicitly are presented which show efficiency of this method, in case of simulation tasks solution.

Keywords: dynamic processes, nonparametric identification, adaptive systems.

INTRODUCTION

At the present moment the parametric theory are widely spread. The problem of parametrical identification is investigated by different authors in particular Cypkin Ja. in his theory of adaptive systems [1]. The parametrical theory based on the statistical solution is analyzed by Feldbaum A. in his publication [2]. In these works the stage of posing the identification task of parametric structure of the dynamic process model being selected by means of different methods is defined with precisions of parameters.

However the issue of identification should be analyzed from the point nonparametric theory. The problem of identification under condition of incomplete information is very topical because of many of the dynamic processes are not deeply studied. The factor of unknown distribution random noises causes the complexity of solving the identification task. In the case of insufficiency a priori information for selecting the structure of a parametric model of the dynamic process the theory of nonparametric systems is applied [2, 3]. In comparison with the parametric theory the nonparametric theory is applied for identification tasks if only the qualitative characteristics of the system are known.

The purpose of the given work is developing and researching the algorithms of identification of dynamic processes by both case nonparametric and partially parametric classes of the model.

The tasks of the work are to develop the extended algorithms for modeling dynamic objects and to carry out experimental research of the real objects and their comparison with the presented objects of the model;

The main idea of this research is to reduce the problem of identification to a mathematical modeling by using nonparametric model of a function regression.

1. THE LEVER OF PRIORY INFORMATION

Different levels of prior information are considered by A. Feldbaum [4]. In this paper the following levels of prior information is analyzed [2].

The level of parametric uncertainty is the first levels of prior information, which is conceded below. The parametric level of prior information means that the parametric structure of a model and some characteristics of random noises with zero mathematical expectation and limited dispersion are known. The iterative probable procedures are used for estimating various parameters. Under these conditions the problem of identification is solved in “narrow sense”.
The following level of prior information is the level of nonparametric uncertainty. Nonparametric level of prior information doesn't imply knowledge about this parametric model, but applies that some information of qualitative character of dynamic process is known, for example linearity for dynamic processes or the nature of its nonlinearity is required. The methods of nonparametric statistics are applied to the solution of identification tasks (identification in "all-inclusive sense" [1]).

The level of parametric and nonparametric uncertainty is the level under conditions of the amount of information, which does not correspond to any of the above described types. In this respect solving the task of identification is formulated in conditions of both case parametric, and nonparametric prior information. The models represent interdependent system of parametric and nonparametric ratios. The solution of identification problems in this level is important from the point of practical problem solving.

2. NONPARAMETRIC IDENTIFICATION

Let’s consider the dynamic object from the point of different levels of a priori information. The first level implies the determination of linearity of dynamic object, but the structure of the parametric model is unknown. The order of the equation can’t be determined from the priori information.

In the second case, the dynamic process is described by the equation:

$$x_t = f(x_{t-1}, x_{t-2}, \dots, x_{t-k}, u_t) \tag{1}$$

where $f(\cdot)$ is unknown functional, x_t is the output variable of the process, u_t is control actions, k is known "depth" of memory [4], which found on the basis of a priori information. The form of the function is not defined to within parameters.

The block diagram of a simulation of the process is show in Figure 1.

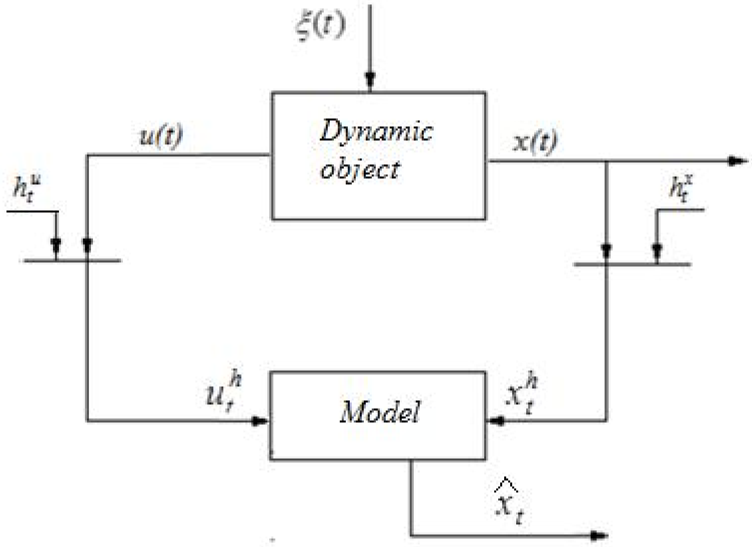

Fig. 1. Block - scheme of modeling the dynamic object

The notation is accepted in Figure 1: (t) is continuous time; index t is discrete time, $\hat{x}_t$ is output model of the object, random noise measurement h_t^x, h_t^u corresponding to the process variables, $\xi(t)$ is vector random interference.

Control of variables is performed with a time interval, forming sampling "input-output" variables $\{x_i, u_i, i = \overline{1, s}\}$, where s is the amount of sampling.

In this case, the parametric structure of the process is partially unknown.

Let the object be described by a linear differential equation of unknown order. In this case, $x(t)$ under zero initial conditions [5] is:

$$x(t) = \int_0^t h(t - \tau) u(\tau)\, d\tau \tag{2}$$

where $h(t - \tau)$ is the weight function of the system, which is a derivative of the transfer function: $h(t) = k'(t)$. It is known that the inverse operator (2) is the operator [5]:

$$u(t) = \int_0^t \nu(t - \tau) x(\tau)\, d\tau \tag{3}$$

where $\nu(t - \tau)$ is the weight function of the object in the direction of "output - input" and $\nu(t) = \omega'(t)$, where $\omega(t)$ is a transfer function of the system in the same direction. Therefore, the problem now is to find the weight functions $h(t)$, $\nu(t)$. One way of solving this problem is to measure the transient function and evaluation of weighting function using the results of the measurements $\{x_i = k_i, t_i, i = \overline{1, s}\}$. The nonparametric model (2) has the form:

$$x_s(t) = \int_0^t h_s(t - \tau, \overline{k_s}, \overline{t_s}) u(\tau)\, d\tau \tag{4}$$

where $\overline{k_s}, \overline{t_s}$ is time vectors: $\overline{k_s} = (k_1, \ldots, k_s)$, $\overline{t_s} = (t_1, \ldots, t_s)$, and $h_s(\cdot)$ is:

$$h_s(t) = \frac{1}{sC_s} \int_0^t k_i H\left(\frac{t - t_i}{C_s}\right) dt, \tag{5}$$

where $H(\cdot)$ is bell-shaped (nuclear) function, C_s is a blur parameter satisfying, the certain conditions of convergence [4]:

$$C_s > 0; \quad H(C_s^{-1}(t - t_i)) < \infty; \tag{6}$$

$$\lim_{s \to \infty} C_s > 0; \quad C_s^{-1} \int H(C_s^{-1}(t - t_i))\, dx < 1; \tag{7}$$

$$\lim_{s \to \infty} sC_s = \infty; \quad \lim_{s \to \infty} C_s^{-1} H(C_s^{-1}(t - t_i)) = \delta(t - t_i). \tag{8}$$

It is proposed to get the transfer function $\nu(t)$ model in the direction of "output - input", i.e. "reversed". Thus:

$$x_s(t) = 1(t) = \int_0^t h_s(t - \tau, \overline{k_s}, \overline{t_s}) u(\tau)\, d\tau \tag{9}$$

can receive sample $\{u_i, t_i, i = \overline{1, s}\}$.

In the case where the structure of the dynamic process can be partially parameterized, i.e., the object is described by the equation $x(t) = f(x(t - 1), x(t - 2), \ldots, x(t - k), u(t))$, where k is determined on the basis of a priori information. The model of the process can be defined by the following nonparametric estimation of a regression function:

$$\hat{x} = \frac{\sum_{i=1}^{s} u_i \Phi\left(\frac{u_s - u_i}{C_s}\right) \cdot \Phi\left(\frac{x_{s-1} - x_i}{C_s}\right) \cdot \ldots \cdot \Phi\left(\frac{x_{s-k} - x_{i-k}}{C_s}\right)}{\sum_{i=1}^{s} \Phi\left(\frac{u_s - u_i}{C_s}\right) \cdot \Phi\left(\frac{x_{s-1} - x_i}{C_s}\right) \cdot \ldots \cdot \Phi\left(\frac{x_{s-k} - x_{i-k}}{C_s}\right)} \tag{10}$$

where Φ(·) is nuclear function of the form $H(\cdot)$.

3. THE COMPUTATION EXPERIMENT

The verifying of the nonparametric identification algorithm is carried by statistical modeling. For computational experiment object is described by equations of the form: $x_t = 0{,}4 \cdot x_{t-1} - 0{,}3 \cdot x_{t-3} + u_t$ where x_t is the output variable of the process, u_t is the input process variable. Transient response of the object is shown in Figure 2.

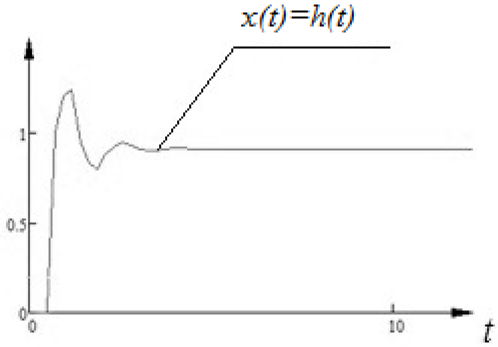

Fig. 2. The transfer function of dynamic process

The input controlled variable is defined by the equation: $u(t) = \sin(0{,}5 \cdot t)$. The model of the object is constructing by using a non-parametric model (4). The simulation results are shown in Figure 3.

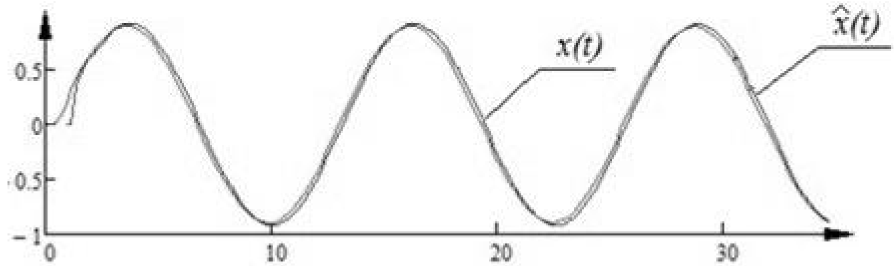

Fig. 3. Results of the identification process using the model (4)

The notation is accepted in Figure 3: $x(t)$ is output of the object, $\hat{x}(t)$ is output of the model. Square error of the simulation is 0,015. The model of the object is constructing by using a non-parametric model (10). The use of this model is acceptable, when the parametric structure of object is partially known. The simulation results are presented in Figure 4.

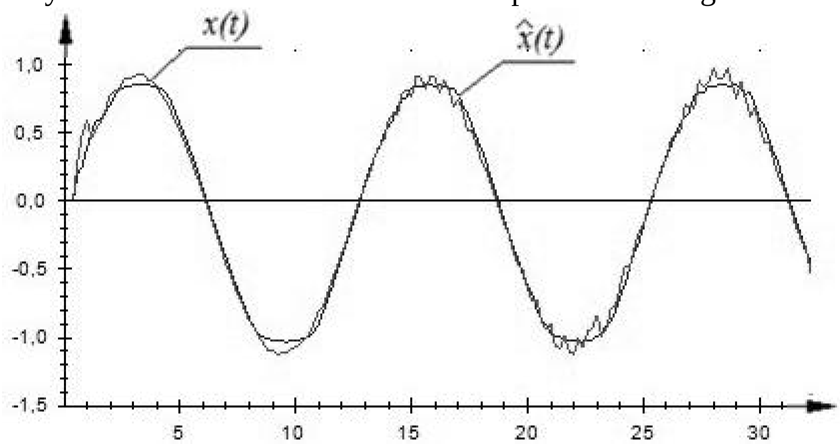

Fig. 4. Results of the identification process using the model (10)

Square error of modeling is 0,023. In the next experiment the input controlled variable is defined by the equation: $(t) = \sin(0{,}2 \cdot t)$. The model of the object is constructing by using a non-parametric model (10). The simulation results are shown in Figure 5.

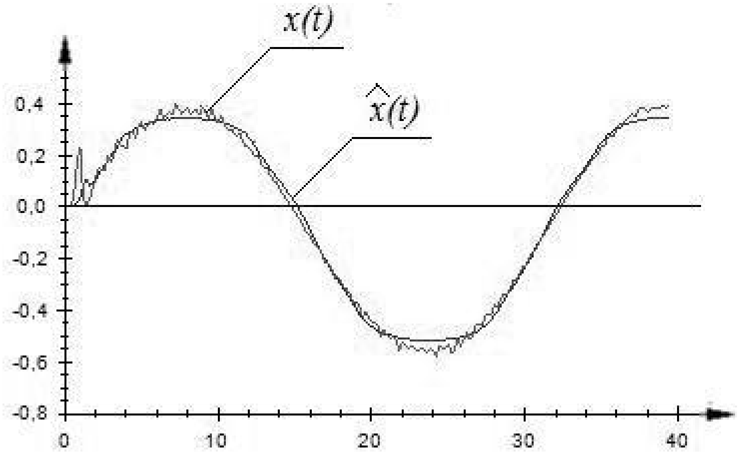

Fig. 5. Results of the identification process using the model (10)

Square error of modeling is 0,056. The use of models (4), (10) may be used for controlling a dynamic process.

CONCLUSION

In the article the analysis of models and algorithms for nonparametric identification under condition of non-parametric uncertainty is carried out, i.e. the case where a priori information about the object is small and do not allow choose a parametric model of the object. In this case, the Duhamel integral is used for describe the process. The problem reduces to the solution non-parametric estimation of the weight function of the system as a result of observations "input-output" of the object. The non-parametric algorithms under partial nonparametric uncertainty are shown in the computational experiment. The conclusion of use of non-parametric models is made.

REFERENCES

1. *Cypkin, Ja.* Adaptation and training in automated systems / Ja. Cypkin. Moscow : Nauka, 1968. 400 p.

2. *Medvedev, A.* The theory of nonparametric systems. Modeling / A. Medvedev // Vestnik SibGAU. 2010. № 4 (30). P. 4–9.

3. *Medvedev, A.* The theory of nonparametric systems. Processes / A. Medvedev // Vestnik SibGAU. 2010. № 3 (29). P. 4–9.

4. *Nadaraya, E.* Non-parametric estimation of the probability density and the regression curve / E. Nadaraya. Tbilisi : Izd. Tbil. un-ta, 1983. 194 p.