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SPECIFIC FEATURES OF NATURAL CONVECTION HEAT TRANSFER IN MAGNETIC FLUIDS

B.M.Berkovsky, V.E.Fertman,
V.K.Polevikov, S.V.Isaev

The Luikov Heat and Mass Transfer Institute,
Minsk,USSR

ABSTRACT

The results are presented of the numerical and experimental study of laminar heat convection in a vertical circular magnetic fluid layer on a cylindrical energized cable. Specific features of heat transfer in the presence of simultaneous gravitational and magnetic forces are investigated. A simple criterial equation is obtained for heat transfer across the fluid layer when only thermomagnetic convection takes place. Validity of the prediction technique is confirmed experimentally.

NOMENCLATURE

- b = characteristic magnetic field gradient, A/m
- c = bulk magnetic particle concentration per unit liquid volume, m^3/m^3
- $c_{p,\mathcal{H}}$ = specific heat at constant pressure and constant magnetic field, J/kgK
- d = characteristic temperature gradient, K/m
- Gr = Grashof number
- Gr_m = magnetic Grashof number
- g_n = acceleration of gravity, 9.80665 m/s
- H = height, m
- $\mathcal{H}$ = intensity of magnetic field, A/m
- I = electric current, A
- $\mathcal{K}$ = pyromagnetic coefficient, A/mK
- l = characteristic length, m
- M = fluid magnetization, A/m
- M_s = saturation magnetization of a solid magnetic, A/m
- m = magnetic moment per particle, A/m
- Nu = Nusselt number
- n = number of particles per unit volume
- P = pressure, N/m^2
- Pr = Prandtl number
- R = radius, m
- Ra = Rayleigh number
- Ra_m = magnetic Rayleigh number
- T = absolute temperature, K
- t = time, s
- V = magnetic particle volume, m^3
- v = velocity, m/s
- β_{th} = volume expansion coefficient, K^{-1}
- η = dynamic viscosity, kg/m s
- κ = thermal diffusivity, m /s
- λ = thermal conductivity, W/mK
- μ_o = magnetic permeability of vacuum, $1.26.10^{-6}$ H/m
- ν = kinematic viscosity, m^2/s
- ρ = mass density, kg/m^3
- ψ = stream function
- ω = vorticity

Superscripts

- $-$ = averaged quantity
- $\hat{}$ = non-dimensional quantity

INTRODUCTION

Heat transfer processes in media of finite extents under the conditions of thermal gravitational convection have been widely studied in connection with the design of many industrial devices as, for example, high-voltage transmission lines where it is essential that cables and transformers be effectively cooled (1) . The idea of employing magnetic fluids for the purpose (2 - 4) has motivated the interest to natural convection in magnetic fluids in the presence of magnetic fields. The popular model of magnetic fluid suggested by Rosensweig and Neuringer (5) assumes the magnetic colloidal suspension to be a continuous and isotropic medium in the presence of gravitational and non-uniform magnetic fields and the viscous stress tensor to be symmetric and proportional to the shear rate tensor. Magnetization of such a fluid is assumed to be instantaneous and its extent independent of the fluid motion while the fluid motion depends on magnetization (noninductive approximation). Physically it means that the reverse effect of motion on the magnetic field structure is not taken into account,i.e. magnetic perturbations are being ignored. By its magnetostatic properties the considered magnetic fluid is similar to a paramagnetic gas in which the carrier of magnetization is not individual molecules but ferromagnetic particles of submicron size ($\sim$100A). With the size as such, each particle is in a uniform magnetized subdomain state (6) and its magnetic moment far from the Curie point is expressed as $m=M_S V$, where M_S is the saturation magnetization of a solid ferromagnetic and V is the particle volume. Then the macroscopic magnetization per unit volume of a ferromagnetic fluid is $\vec{M}=\Sigma\vec{m}/V$, where summation is carried out over all n particles contained in unit volume. With bulk concentration of a solid phase amounting to 10 per cent, magnetization of a magnetic fluid at saturation is only by one order less than magnetization of a solid ferromagnetic. The considered model of a magnetic fluid is thus in complete conformity with definition of the "superparamagnetic" medium suggested in (7) to describe the magnetic properties of a system of subdomain particles the magnetic moment of which by some orders exceeds the moment of an individual atom. The set of equations for convective heat transfer of Rosensweig-Neuringer's magnetic fluid is

$$\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\nabla)\vec{v}\right]=-\nabla P+\eta\nabla^2\vec{v}+\rho\vec{g}_n+\mu_0 M\nabla\mathcal{H}, \qquad (1)$$

$$\rho C_{p,\mathcal{H}}\left[\frac{\partial T}{\partial t}+\vec{v}\nabla T\right]=\lambda\nabla^2 T, \qquad (2)$$

$$\operatorname{div}\vec{v}=0, \qquad (3)$$

$$\rho=\rho(T), \quad M=M_S(\rho,T), \qquad (4)$$

$$\operatorname{div}(\vec{\mathcal{H}}+M)=0, \quad \operatorname{rot}\vec{\mathcal{H}}=0, \ [\vec{M}\times\vec{\mathcal{H}}]=0. \qquad (5)$$

The magnetic fluid is assumed to be incompressible nonconducting and saturated with a strong magnetic field. All the transfer coefficients are considered to be constant. Viscous dissipation is ignored. Since in the fluid constant non-uniformity of temperature is maintained by means of an external heat source or sink, equation (3) lacks the magnetocaloric term accounting for a change in the fluid temperature with time-variation of the external magnetic field or fluid displacement into the region of different magnetic intensity. The intervals of temperature variation being small, equation(4) can be linearized to

$$\left.\begin{aligned} \rho&=\bar{\rho}\left[1-\beta_{th}(T-\bar{T})\right]\\ M&=\bar{M}-(\mathcal{K}+\beta_{th}\bar{M})(T-\bar{T}) \end{aligned}\right\} \qquad (6)$$

in which $\beta_{th}=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)$ is the volume expansion coefficient, $\mathcal{K}=-\left(\frac{\partial M}{\partial T}\right)_{\bar{P},\bar{\mathcal{H}}}$ the pyromagnetic coefficient. The linear theory of convective stability of a horizontal magnetic fluid layer in the Boussinesq approximation provides expression for the Rayleigh number (8)

$$Ra^* = \frac{[\beta_{th}g_n + \mu_0/\bar{\rho}\,(\mathcal{K}+\beta_{th}\bar{M})\,6]\,dt^4}{\kappa\,\nu} \qquad (7)$$

Equation (7) implies that when the gravity force g and the constant magnetic intensity gradient $\nabla\mathcal{H}$ are collinear and allowance is made of the similarity between equations of state (6), the gravitational mechanism of convective instability is fully identical to the magnetic one. When coinciding in direction, the magnetic field gradient and the forces of gravity reinforce each other. Oppositely directed magnetic forces may cancel the gravitational induction of convective motion ($\nabla\mathcal{H}\uparrow\downarrow\nabla\vec{T}$). In the case $g_n=0$ and $\nabla\mathcal{H}\uparrow\downarrow\nabla\vec{T}$ they are responsible for instability of mechanically equilibrium state of a non-isothermal magnetic fluid. In each specific case the critical Rayleigh number, Ra^*_{cr}, is equal to a certain critical Ra_{cr} of an ordinary fluid (9-11). Thus, for an infinite horizontal plane-parallel layer of magnetic fluid with rigid boundaries Ra^*_{cr} = 1708. Thus, convective heat transfer in cavities filled with magnetic fluids and exposed to constant gradient magnetic fields is governed by the criterial relations obtained for ordinary fluids but with a different parametric group for the Rayleigh number. The form of these relations, $Nu = f(Ra^*)$, is dictated by the cavity geometry and boundary conditions at its surface. In case of combined effect of the gravitational and magnetic forces and $\vec{g}_n \parallel \nabla\mathcal{H}$, the terms in the numerator of (7) are either summed up or subtracted depending on relative direction of the gravitational and magnetic forces. In intermediate position of the body forces, $\nabla\mathcal{H}$ = const and $\vec{g}_n$ the known relationships for convective heat transfer across a laterally heated inclined flat layer of a heavy fluid will hold for a magnetic fluid on respective renormalization of the terms of the body force (12).

In the case when $\nabla\mathcal{H}\neq$ const the magnetic force in equation (1) is a function of the coordinate, and in equations describing thermal convection in a magnetic fluid there appear variable coefficients. This case is similar to a simultaneous effect of the constant gravitational force and of the radius-variable buoyancy force, being due to the centrifugal field, in a non-isothermal rotating fluid (13). Due to complexity of our problem, the density of the magnetic force was first adopted to be dependent on one coordinate only. Let us consider convection heat transfer of a magnetic fluid in a vertical annular gap in the magnetic field of a cylindrical cable with current, which as is known is described by $\mathcal{H}=I/2\pi R$.

NUMERICAL INVESTIGATION

Let R_1 be the inner radius of the vertical annular gap, R_2, the outer radius, H, the height of the layer and I be the current passing through the conductor. It is assumed that the temperature of the conductor, T_1, exceeds the temperature of the outer surface of the annular gap, T_2. The stated problem is symmetric about the vertical z-axis and its solution is fully described in the cylindrical coordinates r and z not involving dependence on the angular coordinate ϕ. Performing the rot operation on equation of motion (1), we introduce the vorticity ω and then the stream function ψ

$$\omega = -\frac{1}{r}\frac{\partial v_r}{\partial z} - \frac{1}{r}\frac{\partial (r v_z)}{\partial r}; \quad v_r = \frac{1}{r}\frac{\partial \psi}{\partial z}, \quad v_z = -\frac{\partial \psi}{\partial r} \quad (8)$$

Rearranging (1) and (2) yields

$$\frac{1}{r}\frac{\partial (r v_r T)}{\partial r} + \frac{\partial (v_z T)}{\partial z} = \kappa \nabla^2 T, \quad (9)$$

$$\frac{1}{r}\frac{\partial (r v_r \omega)}{\partial r} + \frac{\partial (v_z \omega)}{\partial z} = \nu \left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (r^2 \omega)}{\partial r}\right] + \frac{\partial^2 \omega}{\partial z^2} \right\} +$$

$$+ \beta_{1h} g_n \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\mu_0 (\mathcal{K} + \beta_{1h}\bar{M})}{\rho}\,\frac{1}{r}\left(\frac{\partial \mathcal{H}}{\partial r}\frac{\partial T}{\partial z} - \frac{\partial \mathcal{H}}{\partial z}\frac{\partial T}{\partial r}\right). \quad (10)$$

Then we introduce the dimensionless variables

$$\hat{r} = (r - R_1)/l; \quad \hat{v} = \frac{v \cdot l}{\nu};$$

$$\hat{T} = (T - T_2)/\Delta T; \quad \hat{\mathcal{H}} = \mathcal{H}/\Delta\mathcal{H},$$

where $l = R_2 - R_1$; $\Delta T = T_1 - T_2$; $\Delta\mathcal{H} = \mathcal{H}_1 - \mathcal{H}_2 = \mathcal{H}(R_1) - \mathcal{H}(R_2) =$

$$= \frac{I}{2\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

The system of equations (8)-(10) can be written in a dimensionless form as

$$\frac{1}{R}\frac{\partial (R\hat{v}_r\hat{T})}{\partial \hat{r}} + \frac{\partial (\hat{v}_z \hat{T})}{\partial \hat{z}} = \frac{1}{Pr}\left[\frac{1}{R}\frac{\partial}{\partial \hat{r}}\left(R\frac{\partial \hat{T}}{\partial \hat{r}}\right) + \frac{\partial^2 \hat{T}}{\partial \hat{z}^2}\right], \quad (11)$$

$$\frac{1}{R}\frac{\partial (R\hat{v}_r\hat{\omega})}{\partial \hat{r}} + \frac{\partial (\hat{v}_z\hat{\omega})}{\partial \hat{z}} = \frac{1}{R}\frac{\partial}{\partial \hat{r}}\left[\frac{1}{R}\frac{\partial (R^2\hat{\omega})}{\partial \hat{r}}\right] +$$

$$+ \frac{\partial^2 \hat{\omega}}{\partial \hat{z}^2} + \frac{1}{R} Gr \frac{\partial \hat{T}}{\partial \hat{r}} - Gr_m \frac{\xi(\xi+1)}{R^3}\frac{\partial \hat{T}}{\partial \hat{z}}, \quad (12)$$

$$\frac{\partial}{\partial \hat{r}}\left(\frac{1}{R}\frac{\partial \hat{\psi}}{\partial \hat{r}}\right) + \frac{1}{R}\frac{\partial^2 \hat{\psi}}{\partial \hat{z}^2} + \hat{\omega} R = 0, \quad (13)$$

$$\left(\hat{v}_r = \frac{1}{R}\frac{\partial \hat{\psi}}{\partial \hat{z}}, \ \hat{v}_z = -\frac{1}{R}\frac{\partial \hat{\psi}}{\partial \hat{r}}, \ R = \hat{r} + \xi, \ \xi = \frac{1}{\frac{R_2}{R_1} - 1}\right).$$

Here $Pr = \frac{\nu}{\kappa}$ is the Prandtl number; $Gr = \frac{g_n \beta_{1h} l^3 \Delta T}{\nu^2}$ is the Grashof number;

$Gr_m = \mu_0(\mathcal{K} + \beta_{1h}\bar{M}) l^2 \Delta T \Delta\mathcal{H}/\rho\nu^2$ is the dimensionless parameter characterizing the magnetic mechanism of convective motion (the magnetic Grashof number). Besides Gr, Gr_m, Pr, R_2/R_1, solution of the problem will be governed by the ratio H/l on account of the finite height of the vertical layer. For the sake of definiteness we shall further assume that the system of coordinates r and z has the origin at the point where the plane of the circular layer base intersects the axis of the cylindrical cable. Both the butt ends and side walls of the cylindrical layer are assumed to be rigid, stationary and impermeable, so that the following boundary conditions are valid

$$\hat{\psi}(0,\hat{z}) = \hat{\psi}(1,\hat{z}) = \frac{\partial \hat{\psi}(0,\hat{z})}{\partial \hat{r}} = \frac{\partial \hat{\psi}(1,\hat{z})}{\partial \hat{r}} = 0, \quad (14)$$

$$\hat{\psi}(\hat{r},0) = \hat{\psi}(\hat{r},H/l) = \frac{\partial \hat{\psi}(\hat{r},0)}{\partial \hat{z}} = \frac{\partial \hat{\psi}(\hat{r},H/l)}{\partial \hat{z}} = 0. \quad (15)$$

The temperature at the surfaces of the cylinders is supposed to be constant and different

$$\hat{T}(0,\hat{z}) = 1, \qquad \hat{T}(1,\hat{z}) = 0. \quad (16)$$

At the butt ends the condition of thermal insulation is assumed

$$\frac{\partial \hat{T}(\hat{r},0)}{\partial \hat{z}} = \frac{\partial T(\hat{r},H/l)}{\partial \hat{z}} = 0 \quad (17)$$

Dimensionless equations (11)-(13) subject to boundary conditions (14)-(17) have been solved by the finite difference method using a monotonous conservative scheme of the second order of accuracy (14). The details of the technique are given in (15).

EXPERIMENTAL PROCEDURE

The experiments were conducted on the setup composed of two vertical coaxial cylinders the gap between which was filled with a magnetic fluid, being a colloidal dispersion of magnetite with kerosene as a carrier liquid and oleic acid as stabilizer. The magnetic fluid had the following parameters: $M_s = 4.10$ A/m; $\rho = 1.485.10^3$ kg/m^3; $\beta = 7.10^{-4}$K^{-1}; $\mathcal{K} = 0.3.10^2$ A/m.K; $\eta = 1.0$ N.s/m^2; $\lambda = 0.23$W/m.K. The inner hollow copper cylinder 0.01m in diameter was energized by passing the constant current, I= 100-300 A, from a source of the BY-12/600 type. The outer aluminium cylinder with internal diameter of 0.028 m was cooled with a thermostated water pumped at the rate of 95 cm/sec. The height of the annular gap, H=0.3m, was bounded by two textolite lids that also centered the coaxial system. The local temperature difference between the walls was measured by copper-constantan thermocouples (with diameter of wires 0.1 mm) and by the И-37 compensating constant-current amplifier with subsequent output of the results to a self-recording H-39 millivolmeter. The M-82 indicating instrument was used for controlling purposes. The outer cylinder was encased in a sheath of varnished cloth 0.135 mm thick (a heat meter) to both sides of which two resistance thermometers of copper wire (ϕ =0.05 mm) were glued. The resistance thermometers ($\mathcal{R}_1 = \mathcal{R}_2 \simeq 200\ \Omega$) composed two arms of the bridge scheme for determining the temperature drop in the probe being proportional to the integral flux through the annular gap. To calibrate the meter the internal cylinder was replaced by a cylindrical heater of nichrome wire being fed from a universal constant current УИП-I source. The annular gap was filled with boron nitride powder to effect heat transfer by heat conduction alone. Temperature and cooling water discharge during calibration and tests were the same, with temperature corresponding to the mean temperature of the room. To further decrease the effect of the room temperature, the setup was covered with a casing of foam plastic. The criteria Ra=Gr.Pr and $Ra_m = Gr_m \cdot Pr$ were calculated using the physical properties of fluid at mean temperature of the layer and at mean intensity of the magnetic field.

DISCUSSION OF RESULTS

Analysis of numerical and experimental results has shown that peculiar features of thermal convection in a magnetic fluid start to exhibit themselves at $Ra_m \gtrsim Ra$.

Up to this moment the convective motion in the magnetic fluid layer corresponds to gravitational convection in the layer of an ordinary fluid confined between two vertical coaxial cylinders with uniform heating of the inner cylinder. When the gravitational and magnetic forces become commensurable, there occurs superposition of convective structures, which are due to ponderomotive coupling of the magnetic fluid with a magnetic field, on the ordinary thermal convection, which however does not significantly change heat transfer through the fluid layer. Since in the problem considered $\nabla\mathcal{H} \uparrow\uparrow \nabla T$ and the magnetic mechanism of convective motion supplements the gravitational one, then, with further increase in Ra_m at constant Ra, the convective motion in the layer intensifies. Maintaining the temperature of the inner cylinder constant by decreasing the temperature of the water pumped through it, an integral heat flux through the annular gap was measured for the given temperature drop with increase in the current passed through the conductor. When $Ra_m \geq 10^2$ Ra, the magnetic mechanism prevails completely over the gravitational one with a periodic cellular convection in the form of circular rolls being established in the layer. Isotherms and structure of convective motion in a half-section of the circular layer are shown in Fig.1. The number of rolls was determined by the parameter H/l . Thus, the magnetic mechanism of the convective motion enhances heat transfer through the layer at constant temperature difference on the boundaries. This points to the possibility of using the magnetic fluid to control the process of cooling current conductors as stronger current causes increase in the intensity of the magnetic field thereby increasing the amount of heat released from the conductor due to thermomagnetic contribution to the convective motion. Similar results have been obtained in (15), where specific features of thermal convection in a horizontal annular gap of a magnetic fluid are discussed in detail.

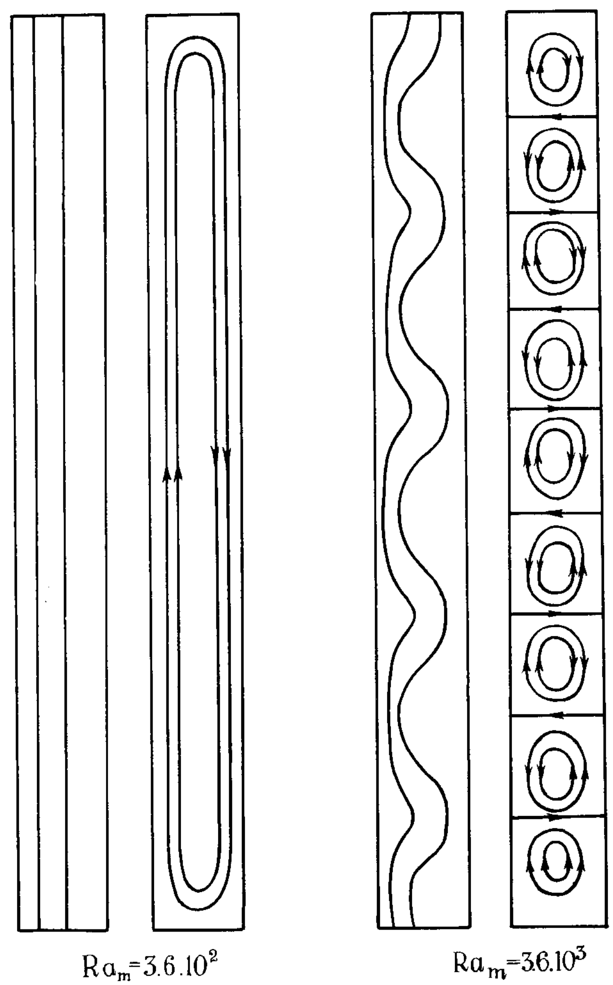

Figure 1 Typical isotherms and convection structure at $R_2/R_1=3$, $H/l=10$, Ra=36. The isolines differ by 1/3 from ψ_{max} and T_{max}.

Criterial interpretation of the numerical data on heat transfer for $10 \leqslant Pr \leqslant 10^4$, $0 \leqslant Ra \leqslant 10^6$; $0 \leqslant Ra_m \leqslant 10^8$; $1.5 \leqslant R_2/R_1 \leqslant 5$; $2 \leqslant H/l \leqslant 10$ and treatment of the experimental data for the fluid considered for $10^4 \leqslant Ra \leqslant 5.10^5$; $10^4 \leqslant Ra_m \leqslant 2.10^6$; $R_2/R_1 = 2$; $H/l = 30$ at $Ra_m \gg Ra$ have provided the correlation being valid at $Ra_m > 5.10^3$:

$$\bar{Nu} = 0.25\, Ra_m^{0.24} \qquad (18)$$

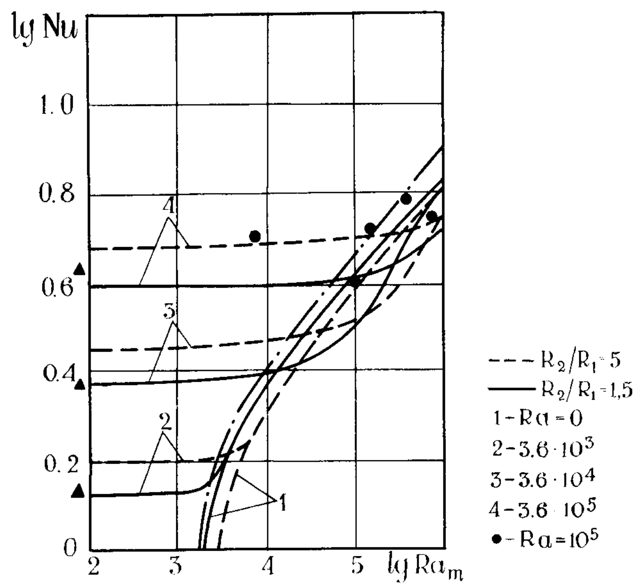

Figure 2 Effect of Ra_m on convective heat transfer at $H/l = 5$.

In Fig.2 the relationship $Nu=f(Ra_m)$ is presented for different values of Ra. The experimental points (triangles) have been taken from (16), where they were obtained under the conditions of thermal gravitational convection in circular layers of ordinary fluid at the same Ra as the curves 2,3,4. Calculated by the empirical relation lacking the parameter which takes into account the geometry of the system, these points lie interior to the boundary curves plotted for the studied range of R_2/R_1, thus being indicative of the slight effect of R_2/R_1 on heat transfer. The dash-dotted line taken from (17) correlates the data of different authors on heat transfer through a horizontal layer heated from below. It has been plotted by laying off the values of lgRa on the abscissa axis. Comparison of the dash-dotted line and the curves 1 for thermomagnetic convection alone shows that there is a qualitative agreement between the pattern of heat transfer through a magnetic fluid layer in the intensity gradient field at $\nabla\mathcal{H} \uparrow\uparrow \nabla T$ and heat transfer through a horizontal layer heated from below at Ra greater than Ra_{cr}. Dependence of the magnetic field intensity gradient on R results in decrease of the integral heat flux through the layer by about 10%, with specificity of the magnetic fluid exhibiting itself at large R_2/R_1. That the other geometrical parameter, H/l, has no effect on heat transfer under the conditions of developed thermomagnetic convection is confirmed by a good coincidence of experimental data obtained for $H/l=30$ with the predictions for $2 \leqslant H/l \leqslant 10$. An experimental verification of the computational procedure has shown that the simple model of a magnetic fluid used adequately describes the convective heat transfer at laminar natural convection. In the presence of combined effects of the gravitational and magnetic forces the summation of the both mechanisms at $\nabla\mathcal{H} \uparrow\uparrow \nabla T$ makes the magnetic fluid particularly promising for the purposes of cooling current conductors.

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