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Flow and energy dissipation in a magnetic fluid drop around a permanent magnet

V. Bashtovoi[a], O. Lavrova[b], T. Mitkova[b,*], V. Polevikov[c], L. Tobiska[b]

[a] *UNESCO Chair, Belarusian National Technical University, Minsk, Belarus*
[b] *Institute for Analysis and Numerical Mathematics, Otto-von-Guericke University, Magdeburg, Germany*
[c] *Department of Computational Mathematics, Belarusian State University, Minsk, Belarus*

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Abstract

This paper is devoted to the numerical modeling and simulation of hydrodynamic and dissipation processes in damper systems, where the working element is a magnetic fluid drop around a permanent magnet. Flow patterns and dimensionless dissipation coefficient depending on the Reynolds number and magnet position are established.

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Keywords: Damper system; Magnetic fluid; Finite element method

1. Introduction

The application of magnetic fluids as a working media in damper systems [1–3] requires the study of hydrodynamic and dissipative properties of these systems.

From the hydromechanical point of view these systems are presented by a double connected fluid domain which boundary comprises external free surface sections and solid walls. We consider a damper system which consists of a magnetic fluid drop around a permanent magnet placed between two plates with a plane–parallel motion [3]. The magnet is magnetized with a constant magnetization in the direction parallel to the plates. The geometry of the damper system is presented in Fig. 1.

*Corresponding author. Tel.: +49 391 67 18211; fax: +49 391 67 18073.
E-mail address: teodora.mitkova@mathematik.uni-magdeburg.de (T. Mitkova).

2. Statement of the problem

The behavior of a damper system is described mathematically by a coupled system of nonlinear partial differential equations with free boundaries.

A reduction of the model results in three subproblems, the calculation of the magnetic field intensity in a fixed domain, the calculation of new boundaries for the given flow and magnetic data and, finally, the computation of the flow in a fixed domain. The magnetic field around a permanent magnet of a rectangular shape can be calculated analytically [4]. If we neglect the capillary forces then the free surface coincides with a line of a constant magnetic field intensity value. This constant value is determined by the volume conservation condition for the fluid. We consider a finite element solving strategy for the flow part, which builds the main effort within the overall algorithm.

The mathematical flow model is governed by the following Navier–Stokes equations for two-dimensional stationary incompressible flow under the action of

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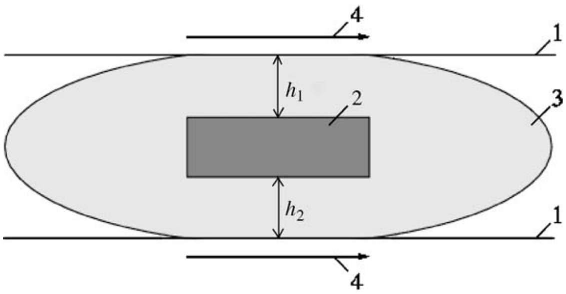

Fig. 1. Geometry of the damper system: 1—moving plates, 2—nonmoving magnet, 3—magnetic fluid, 4—direction of the moving

magnetic forces in the non-gravity case [1]

$$-\frac{1}{\mathrm{Re}}\Delta\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \nabla p = 0 \ \text{ in } \Omega$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega.$$

Here $\mathbf{u}$, p denote the velocity and pressure, respectively, Re the Reynolds number, Ω the fluid domain. The boundary conditions on the free surfaces Γ_F and solid surfaces Γ_M (the boundary of the magnet) and Γ_P (the boundary of the plates) complete the model

$$\mathbf{u}\cdot\mathbf{n} = 0,\ \mathbf{n}\cdot\sigma(\mathbf{u},p)\cdot\tau = 0 \ \text{ on } \Gamma_F;$$

$$\mathbf{u} = \mathbf{0} \ \text{ on } \Gamma_\mathrm{M}; \quad \mathbf{u} = (1,0) \ \text{ on } \Gamma_\mathrm{P},$$

where $\sigma(\mathbf{u},p)$ is the hydrodynamic part of the stress tensor, $\mathbf{n},\tau$ unit normal and tangential vectors.

The quality of the damper system can be evaluated by computing the energy dissipation [5]

$$\mathrm{E} = -\eta u_0^2 lk,\ k = \int_\Omega \left(\frac{\delta u_i}{\delta x_j} + \frac{\delta u_j}{\delta x_i}\right) \mathrm{d}\Omega,$$

where η is the dynamic viscosity, u_0 the velocity of the plates, ℓ the length in the coordinate direction perpendicular to the cross section area, κ the dimensionless dissipation coefficient.

3. Finite element discretization of the model

We discretizate the presented flow model by using the isoparametric $P_2\backslash P_1$ finite element approximation, i.e., the velocity $\mathbf{u}$ is approximated by quadratic functions while the pressure p is approximated by linear functions. The curved boundaries of the fluid domain require the isoparametric approach. The isoparametric finite element pair $P_2\backslash P_1$ satisfies the Babuška–Brezzi stability condition, which guarantees a stable discretization of the model problem. We consider a discretization where the slip boundary condition $\mathbf{u}\cdot\mathbf{n} = 0$ on Γ_F is incorporated in the ansatz and the finite element space.

The discretization of the flow model corresponds to a nonlinear algebraic system. The slip boundary condition is incorporated directly in the nonlinear algebraic system. The nonlinearity within the model is resolved by using a fixed-point iteration (linearization of the convective term). Thus, it remains to solve efficiently a large linear systems in each nonlinear iteration step. The geometric multigrid method is used as an effective solver.

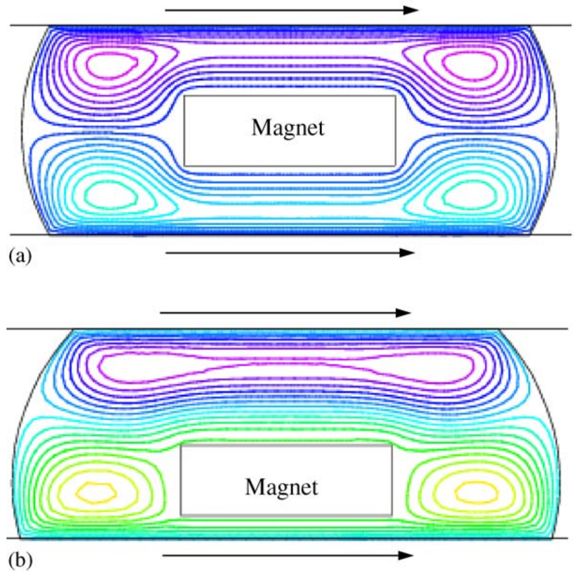

Fig. 2. Flow structures for the Reynolds number in the range $0.1 < Re < 10$ ((a) $h_2/h_1 = 1$; (b) $h_2/h_1 = 0.2$).

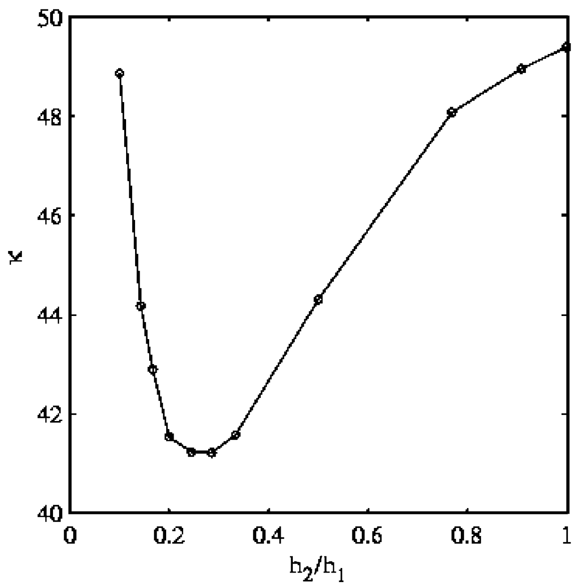

Fig. 3. The dependence of the dimensionless dissipation coefficient κ on the magnet position for $Re = 1$.

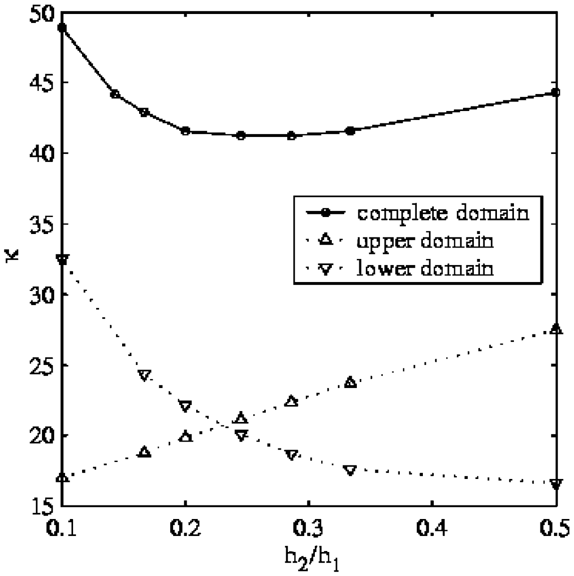

Fig. 4. The dependence of the dimensionless dissipation coefficient κ on the magnet position for $Re = 1$ in the complete, upper and lower fluid domain.

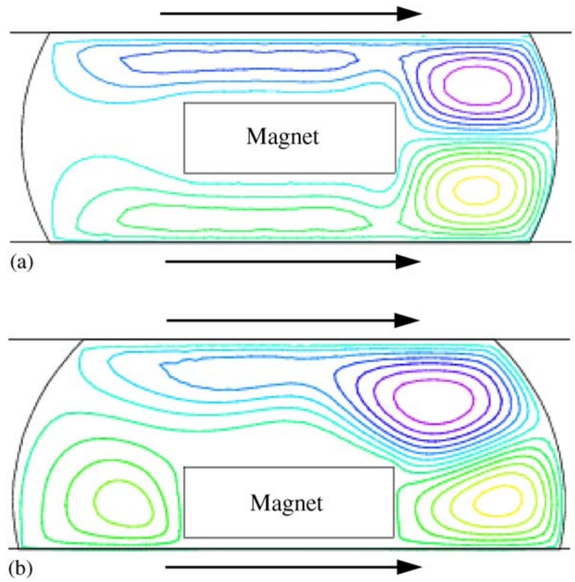

Fig. 6. Flow structures for $Re = 300$ ((a) $h_2/h_1 = 1$; (b) $h_2/h_1 = 0.1$).

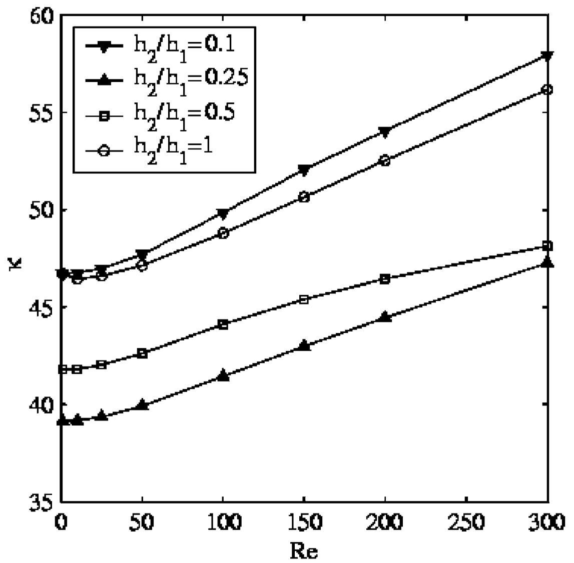

Fig. 5. The dependence of the dimensionless dissipation coefficient κ on Reynolds numbers for the different magnet positions $h_2/h_1 = \{0.1, 0.25, 0.5, 1\}$.

4. Results of numerical simulations

We have obtained numerically the values of the dimensionless dissipation coefficient and the flow structures (streamlines) in a wide range of Reynolds numbers. A position of the magnet is defined by the ratio h_2/h_1 of its distances to the lower and upper plates, respectively.

The numerical results show that the flow structure and dissipation coefficient do not depend on the Reynolds number in the range $0.1 < Re < 10$. The flow structure for two different magnet positions are presented in Fig. 2. For the symmetric position of the magnet ($h_2/h_1 = 1$) the flow structure is in a form of separate vortices. However, a magnet displacement (e.g. at $h_2/h_1 = 0.2$) cause a cicular fluid motion around the magnet.

The value of the dissipation coefficient with Reynolds number in the range $0.1 < Re < 10$ is mostly determined by the location of the magnet inside the magnetic fluid and takes the minimum value $h_2/h_1 \approx 0.25 - 0.28$ as shown in Fig. 3. By a displacement of the magnet the area of the upper fluid subdomain is increased, however, the gradients of the characteristic velocity in the same subdomian are decreased. At the same time the area of the lower fluid subdomain is decreased and gradients of the velocity are increased. Increase (decrease) of the velocity gradients results in the increasing (decreasing) dissipation energy. Fig. 4 illustrates this effect.

The results of the numerical experiments show that the flow structure and the values of the dissipation coefficient depend on the Reynolds number in the range $Re > 10$. As Fig. 5 presents, the values of the dissipation coefficient κ are increased by increasing of the Reynolds number. The increasing of the Reynolds number results in the displacement of the velocity vortices in the direction of the moving plates (see Fig. 6).

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