Different Types of Bistability upon Multiwave Mixing in a Nonlinear Interferometer

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Interaction of light beams in nonlinear Fabry-Perot interferometer has been theoretically studied in conditions of rescattering from the diffraction dynamic structures formed in the medium volume. Consideration has been given to the scheme for the symmetric propagation of two light beams in the interferometer and the scheme for the normal incidence of a reference beam upon oblique incidence of the signal one. It has been demonstrated that one can realize optical bistability of the S-, N-, butterfly-type and, owing to the pitchfork bifurcation, asymmetric modes of bistability.

Key words: dynamic structures, optical bistability

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1 Introduction

Recently quite a number of studies have been concerned with self-organization phenomena and cooperative processes in nonlinear optical systems.The combining of the bistable properties of a nonlinear interferometer with energy-exchange processes between the light beams on parametric interaction looks very promising for the improvement of optical data processing, control of bistable modes of four-wave mixing, symmetry breaking bifurcations and the creation of optical logical devices [1]- [6].

This work presents the results obtained during a theoretical study of light beam interaction in a nonlinear interferometer in conditions of rescattering of the waves from the dynamic diffraction structures formed in the medium volume. The consideration has been given to the scheme for symmetric incidence of two light beams at the Fabry-Perot interferometer and the scheme for normal incidence of a pump beam in case of oblique incidence of the signal one. In the first scheme two light beams, being reflected from the output mirror of the interferometer, create counter-propagating waves for each other in such a way that degenerate four-wave mixing is realized within the resonator. In the second scheme scattering of the waves from the dynamic gratings in the medium volume and reflection from the cavity mirrors lead to the formation of new light beams in the interferometer, and double four-wave mixing takes place. At the interferometer output two light beams duplicate the phase of a signal wave, and two beams represent their phase conjugates.

A theoretical analysis and numerical simulation have been carried out with thedevelopment of a theory of four-wave mixing in resonant media with regard to nonlinear absorption and diffraction of the propagating light beams from the dynamic gratings recorded by these beams. A bistable behaviour of such a system may be due to the internal (wave interaction in a nonlinear medium) as well as an external (reflection from the cavity mirrors) feedback.

2 Scheme for symmetric propagation of two light beams in the nonlinear interferometer Fabry-Perot

Fig.1a. demonstrates the scheme for the interaction of light beams in a symmetric geometry. As seen, such a system reveals four-wave mixing (FWM). A theoretical description of FWM is based on the nonlinear susceptibility $\chi(I)$ series expansion in terms of the harmonics of two dynamic gratings with linearly independent vectors. We use resonant type of nonlinearity, also nonlinear absorption and energy exchange between all interacting waves taken into account. In conditions of Bragg reflection we can write a system of reduced wave equations fields as follows [7]:

$$\frac{\partial E_{1F,1B}}{\partial z} = \pm\frac{i2\pi\omega}{cn_0}(\chi_{0,0}E_{1F,1B} + \chi_{\pm1,\pm1}E_{1B,1F} + \\ +\chi_{0,\pm1}E_{2F,2B} + \chi_{\pm1,0}E_{2B,2F}),$$
$$\frac{\partial E_{2F,2B}}{\partial z} = \pm\frac{i2\pi\omega}{cn_0}[\chi_{0,0}E_{2F,2B} + \chi_{\pm1,\mp1}E_{2B,2F} + \\ +\chi_{0,\mp1}E_{1F,1B} + \chi_{\pm1,0}E_{1B,1F}] \quad (1)$$

where n_o is a refractive index of the unexcited medium, ω is an optical frequency, c is a light velocity.

The Fourier series- expansion components for the medium susceptibility $\chi_{\pm1,\pm1}, \chi_{\pm1,\mp1}$ are respectively associated with self-diffraction of $E_{1B,1F}$ and $E_{2F,2B}$ fields, the components $\chi_{0,\pm1}, \chi_{\pm1,0}$ take into consideration both the processes of self-action and parametric interaction, and $\chi_{0,0}$ describes the amplitude wave absorption. This system of differential equations requires specification of the following boundary conditions taking into account reflection from the mirrors of Fabry-Perot interferometer with the reflection coefficient R:

$$\begin{aligned} E_{1F}(0) &= E_{10}\sqrt{1-R_1} + E_{2B}(0)\sqrt{R_1} \\ E_{2F}(0) &= E_{20}\sqrt{1-R_1} + E_{1B}(0)\sqrt{R_1} \\ E_{1B}(l) &= E_{2F}(l)\sqrt{R_2} \\ E_{2B}(l) &= E_{1F}(l)\sqrt{R_2} \end{aligned} \quad (2)$$

The system of equations (1) describes the process of degenerate four-wave mixing in Fabry-Perot interferometer for any intensity ratio of the waves E_{10} and E_{20}. Let us consider a case of the interacting light beams with equal intensities. As demonstrated in Fig.la, this system is symmetric relatively to the bisector of the angle between the incident waves of

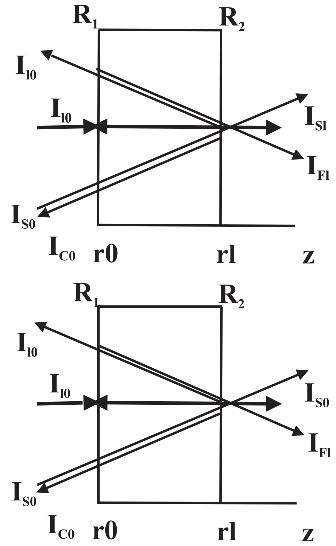

FIG. 1. Schematic diagram of interacting light beams in nonlinear interferometer: (a) symmetric geometry, (b) normally incident pump beam and oblique signal beam.

equal input intensities. Consequently, the intensities of light beams following passage through the interferometer should remain equal to each other. In the process one could realize the conditions for realization of optical bistabilities of the S-type as in case of a single-beam Fabry-Perot interferometer. Such a mode of interaction is shown in Fig. 2a. The calculations have been performed in the Gaussian approximation of mirror-symmetric absorption and emission bands for two-level model with the following parameters of medium and resonator: frequency detuning from the absorption band center $\xi = (\omega - \omega_{12})/\Delta = 1.5$, where Δ is a halfwidth of the absorption and emission contours; the Stokes shift $\eta = (\omega_{12} - \omega_{21})/\Delta = 1.6$; $k(\omega)l = 0.05, R = 0.9$, phase detuning of interferometer out of resonance

Let us consider the effect of the medium optical density *kl* on the conditions required for the optical bistability realization in the system. As seen from Fig. 2b, an increase in *kl* leads to the qualitatively new dependences for the transfer functions of the interferometer. At low intensities of the interacting waves one observes the only symmetric solution ($I_1(l) = I_S(l)$), described by the S-shaped transmission curves. However, some parts of the upper branch of the S-shaped function be-

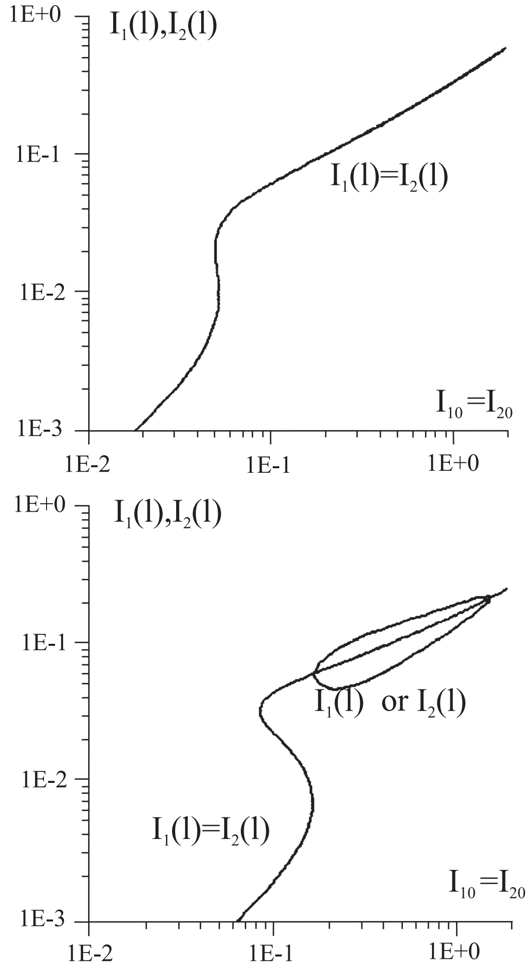

FIG. 2. Output intensities of waves I_1 and I_2 as a function of the equal input intensities ($I_{10} = I_{S0}$) in dimensionless units (all intensities are normalized to the saturation intensity of the transition $S_0 - S_1$), $\xi = 1.5$, $\eta = 1.6$, $R = 0.9$, $\Phi_0 = 0$, $k(\omega)l$= 0.05(a), 0.1(6).

came unstable through pitchfork bifurcation, and in a limited intensity range one could observe an asymmetric solution characterized by different intensities of two light beams at the interferometer output ($I_1(l) \neq I_S(l)$). It should be noted that similar bifurcations breaking down a symmetry have been considered in different interferometers using two-photon processes [8] and local Kerr nonlinearity [5]-[6] or cubic nonlinearity [9] of the resonator material. Our investigation covers the wide class of molecular and atomic media that could be characterized by resonant nonlinearity. More detailed results specified the conditions of realization of the asymmetric modes of bistability will be presented elsewhere. Here we only note that appearance of the asymmetric modes of FWM is caused by the nonlinear energy exchange between the light waves upon the diffraction on dynamic gratings within the medium volume. In the media with resonant nonlinearity the refractive index gratings (phase gratings), determining phase mismatch of the interacting waves, play a leading role in the energy exchange processes and lead to the symmetry breaking bifurcation. At the same time, the pitchfork bifurcation doesnt realize when only amplitude gratings are taken into account.

Another interesting detail, the asymmetric regimes of interaction exhibit not only for the case of equal input intensity of light beam but also for some input intensity ratio. To illustrate it let us consider the Fig.3, where presented the dependence of output intensities as a function of input intensity one of light beams (I_{20}). In the process, input intensity of the second light beam keeps constant. As seen, when I_{20} is weak, the transmission of beam I_1 is much higher than for beam I_2. With increasing of I_{20} the transmission of I_1 tends to decrease when for I_2 it grows. Than there is the range of input intensity I_{20} where transmission characteristics exhibit bistability with S-shaped curve for I_2 and with inverted S-shaped curve for I_1. When $I_{20} = I_{10}$ there are three transmission states: symmetric and two asymmetric. The symmetric state lies on unstable parts of bistable characteristics and as a result it never takes place. In dependence of previous input intensity ratio, one of passed through interferometer waves is characterized with higher transmission state and another one is less.

3 Scheme for the normally incident reference beam upon oblique incidence of the signal beam

The geometry of interaction is shown in Fig. 1b. Being multiply reflected from the cavity mirrors, the pump wave I_0 gives rise two counter-propagating waves in a nonlinear medium of the interferometer.

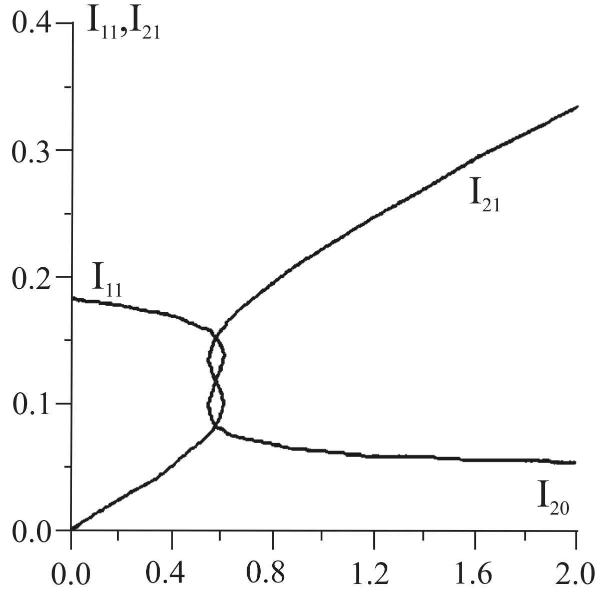

FIG. 3. Output intensities of waves I_1 and I_2 as a function of the input intensity I_{20} in dimensionless units (all intensities are normalized to the saturation intensity of the transition $S_0 - S_1$), $I_{10} = 0.6,\ \xi = 1.5,\ \eta = 1.6,\ R = 0.9,\ \Phi_0 = 0,\ k(\omega)l = 0.1$.

The beam I_D is formed due to reflection of the signal beam I_S from the interferometer mirror R_2 ,whereas phase-conjugated waves I_C and I_F are formed owing to four-wave mixing of each of the waves I_S and I_D with the pumping ones.

A theoretical analysis is performed for the Fabry-Perot interferometer with cubic nonlinearity in the approximation of weak signal waves in relation to the pumping field and taking into consideration linear absorption of the waves in the medium volume. Such simplifications restrict considerably the scope of the phenomena at issue, yet enabling one to find an analytical solution for the amplitudes of interacting waves.

The relationship between the field intensity in the medium Iin and the incident pump wave intensity I0 in the approximation of a low-intensity signal beam ($I_{so} << I_0$) may be represented as [10]:

$$I_{in} = \frac{I_0(1-R)(1-\tau)(1+R\tau)}{kl((1-R\tau)^2 + 4R\tau\sin^2(\Phi))} \qquad (3)$$

Here, $\tau = e^{-kl}$ is transmission of a nonlinear layer with a thickness 1, k is the medium absorption coefficient, $\Phi = 2\pi nl/\lambda - m\pi$ is the phase detuning of the interferometer out of resonance, n is the refractive index of a nonlinear medium, λ is the radiation wavelength, m is an integer number. Introducing the initial interferometer detuning for the pump wave $\Phi_o = 2\pi n_o l/\lambda - m\pi$ one can represent the phase detuning as $\Phi = \Phi_o + gI_{in}l$, g being the proportionality coefficient that is determined by the particular nonlinearity mechanism and in the general case is of the following form for the medium with a cubic nonlinear susceptibility $X^{(3)}$:

$$g = 16\pi^3 X^{(3)}/cn_0^2\lambda \qquad (4)$$

In the approximation of the given pumping-wave intensity, the rescattering processes of weak signal and phase-conjugated waves from the recorded dynamic gratings are described by the following system of differential equations:

$$\begin{aligned} \frac{\partial E_{S,F}}{\partial z} &= i(\psi E_{S,F} + \varphi E^*_{C,D}) \\ \frac{\partial E_{C,D}}{\partial z} &= -i(\psi E_{C,D} + \varphi E^*_{S,F}) \end{aligned} \qquad (5)$$

Here we used the phase synchronism conditions $\vec{k}_1 + \vec{k}_2 = \vec{k}_S + \vec{k}_G$ and $\vec{k}_1 + \vec{k}_2 = \vec{k}_F + \vec{k}_D$ determining generation of the phase-conjugate beams E_C and E_F respectively. The coefficients ψ and φ describe the processes of self-action and parametric coupling of the waves. In a medium model with cubic nonlinearity and linear absorption they are of the form

$$\begin{aligned} \psi &= (i + 2g' I_{in}) \\ \varphi &= g' I_{in} \end{aligned} \qquad (6)$$

where $g' = 2g/k$. The boundary conditions for the coupled equations (5) are determined by the external feedback through the cavity mirrors:

$$\begin{aligned} E_S(0) &= \frac{E_{S0}\sqrt{1-R}}{1 - R\tau\exp(i\Phi_S)}, \\ E_D(l) &= E_S(0)\sqrt{\tau R}\exp(i\Phi_S) \\ E_C(l) &= E_F(l)\sqrt{R}, \\ E_F(0) &= E_C(0)\sqrt{R} \end{aligned} \qquad (7)$$

where $\Phi_S = \Phi_{S0} + gI_{in}l,\ \Phi_{S0}$ is the initial detuning of interferometer for the signal wave. An analytical solution of the equation (3) and coupled differential equations (5) with the boundary conditions (7) makes it possible to describe the bistable mode of multiwave interaction. Fig.4 illustrates the calculated dependences of the signal $I_S(z=l)$, $I_D(z=0)$ and phase-conjugated $I_C(z=0)$, $I_F(Z=l)$ waves on the intensity of the pump wave. The range of optical bistability for all the interacting waves is determined by the bistability range for pumping waves and is found from the equation (3). The control of a bistable response for signal and conjugate waves may be realized by changing the initial phase detuning of the resonator for a signal wave through a change in the angle between the light beams at the interferometer input. Such a possibility is demonstrated in Fig.4 (a-c) for low values of the optical density in a nonlinear medium. Fig.4a represents a case when the characteristics of all signal and conjugate waves follow the S-shaped bistable function for pumping waves. Fig.4b demonstrates the N-type bistability corresponding to down-switching of signal and conjugate waves upon an increase in the pump wave intensity. Realization of the butterfly-type bistability is demonstrated in Fig.4c; in this case both increased and decreased intensity of the pump wave is associated with switching from the state of large transmission of all signal and conjugate waves into the state with lower transmission. An increase in the optical density of a nonlinear layer results in a significant change of the character of a bistable response due to the increased nonlinear phase shifts on scattering from the dynamic gratings. As seen from Fig.4d, this allows for simultaneous realization of bistable responses of all types with upward and downward switching.

In summary, we derive a simple model of multiwave mixing in nonlinear Fabry-Perot interferometer and show the possibilities for realization of optical bistability of S-, N- and butterfly-type as well as asymmetric modes in a scheme for symmetric propagating of two light beams within the resonator. Investigations of multiwave mixing in the process of recording dynamic gratings in nonlinear Fabry-Perot interferometer also look promising for the development of multifunctional optical logical elements.

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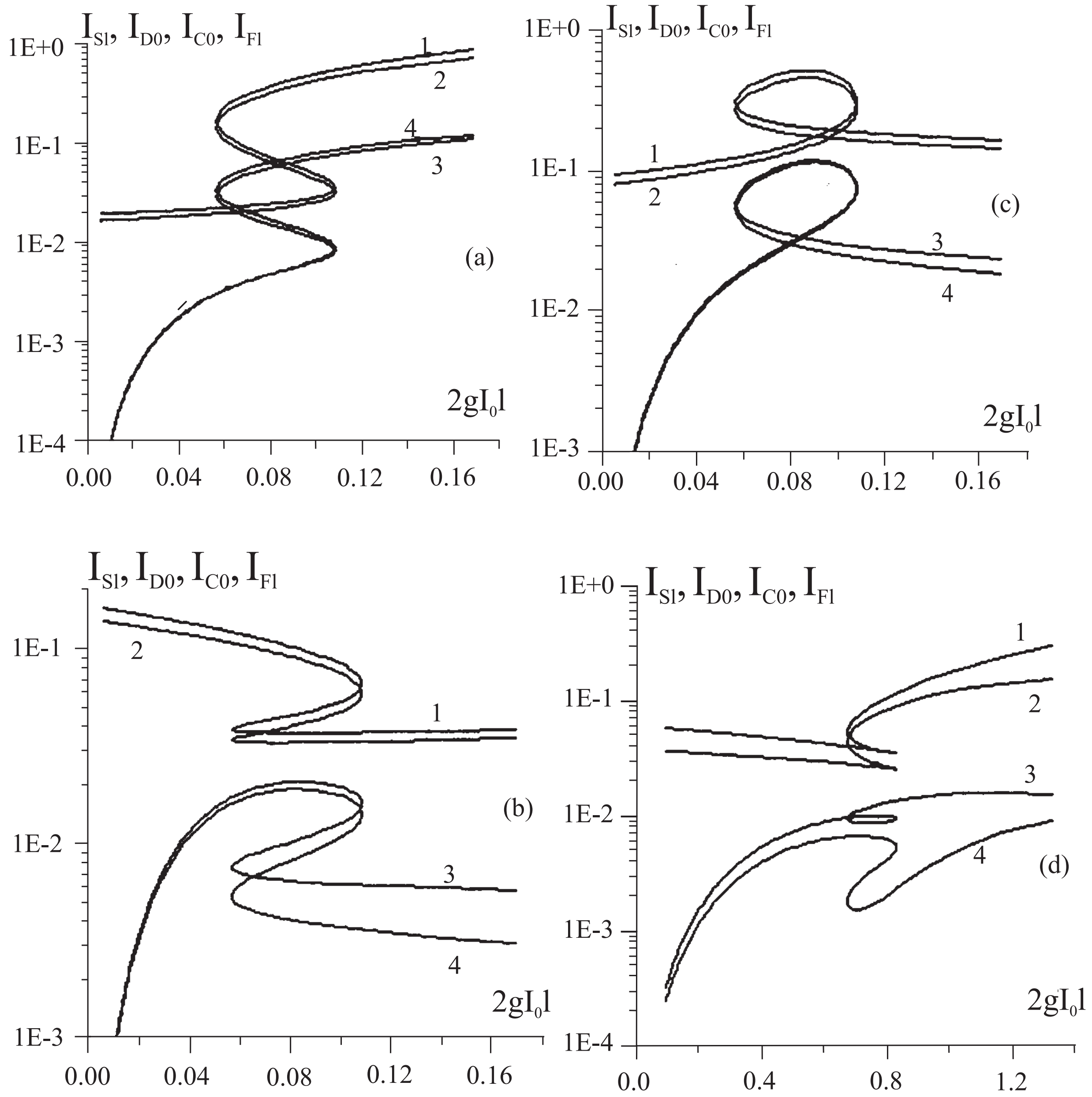

FIG. 4. Output intensities of signal (I_{Sl} - curve 1,I_{DO} - curve 2) and phase-conjugated (I_{C0} - curve 3, I_{Fl} - curve 4) waves as a function of the pump wave intensity I_0. Intensities of signal and conjugate waves are normalized to the intensity of the incident signal wave I_{SO}. $(a) - kl = 0.05,\ R = 0.9,\ F0 = 0.25,\ FSo = 0.37$; $(b) - kl = 0.05,\ R = 0.9,\ F0 = 0.25,\ F_{SO} = -0 - 1$; $(c) - kl = 0.05,\ R = 0.9,\ F_0 = 0.25,\ F_{so} = 0 - 15$; $(d) - kl = 0.3,\ R = 0.86,\ F_0 = 0.53,\ F_{so} = -0.13$