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# WAVE FRONT TRANSFORMATION OF OPTICAL VORTICES WITH MULTIWAVE INTERACTIONS IN RESONANT MEDIA

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*The transformation of singular light beams is analyzed theoretically for multiwave interactions on amplitude and phase dynamic holograms in media with a resonant nonlinearity. The possibility of multiplying topological charge using different diffraction orders is demonstrated and optimal conditions for parametric energy exchange are determined which ensure retention of the structure of the wave front of optical vortices.*

Keywords: *singular optics, optical vortex, resonant medium, multiwave interaction.*

**Introduction.** The major topics of current research on singular optics involve studies of the nonlinear interactions of optical vortices [1], their use as "optical tweezers" for manipulation of microscopic particles [2], and the creation of optical waveguides for other light beams [3]. The methods that have been developed for transformation of the topological structure of optical vortices, including second harmonic generation [4] and three- and four-wave interactions [5, 6], offer promise for use in optical computations [7, 8].

Several features of the propagation of beams of light in media with a resonant nonlinear mechanism are related to the modulation of their wave front as the refractive index varies nonlinearly. Thus, for example, self focussing (defocussing) effects, which show up when the intensity of a light beam is comparable to the saturation intensity of a resonant transition, lead to the formation of localized spatial structures in the form of dark solitons [9]. The processing of light fields by nonlinear interference and holographic techniques based on multiwave interactions in resonant media [10] also offers some promise for solving problems involving transformation of the wave fronts of vortical optical beams. Thus, a theoretical method has been developed [11] for multiplying the topological charge of singular light beams during nonlinear multiwave mixing. We note that, in the case of a traditional four-wave interaction, optical vortices can be used for direct detection of wave-front inversion based on the inversion of topological charge [12].

In this paper we analyze the possibility of inverting the sign of the topological charge and multiplying it in multiwave interaction schemes on amplitude and phase dynamic gratings. Given that efficient parametric energy exchange between light beams occurs with substantial modulation of the refractive index, under these conditions it is appropriate to examine the energy efficiency and the quality of the transformation of the structure of the wave front of optical vortices, as well as to optimize the interaction geometry (thickness of the medium, convergence angle of the optical beams, etc.). These questions are studied by numerical simulation of a system of truncated wave equations taking into account the self- and cross-modulation of light beams during nonlinear recording of amplitude and phase dynamic gratings in a geometry of four-, six-, and eight-wave mixing.

**Theoretical Model.** A nonlinear dependence of the variation in the absorption coefficient and (or) the refractive index of resonant media on the intensity of the interacting radiation is known [10] to lead to a distortion in the groove profile of light-induced dynamic diffraction gratings. For analyzing the diffraction characteristics of these dynamic structures it is appropriate to use a Fourier series expansion of the spatial distribution of the nonlinear phase and amplitude responses of the medium in terms of the spatial harmonics of the grating. When the volumicity conditions for dynamic holograms are satisfied, the angular selectivity of the grating makes it possible to recover, independently, the waves that have been diffracted in different orders by changing the direction of propagation of the readout wave. In the Bragg regime, a readout beam directed at an angle corresponding to the *M*-th diffraction order is

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scattered in the corresponding spatial harmonic of the grating. Then the diffracted wave $E_D$ is determined by the nonlinear polarization $P = \chi^{(N-1)}(E_1 E_S^*)^M E_2$, and $N$-wave mixing ($N = 2(M + 1)$) takes place in the ($N - 1$)-st order of the nonlinearity.

Our theoretical analysis of the transformation of the wave front of singular light beams during multiwave interactions will be for the example of the readout of transmission dynamic gratings in resonant media modelled by a two-level scheme. The system of equations for the formation of the wave $E_D$ during diffraction of an $E_2$ wave at the grating formed by $E_1$ and $E_S$ waves in the standard $N$-wave interaction regime ($N$ = 4, 6, 8, ...) can be written in the form [11]

$$\left(\frac{\partial}{\partial z} \mp \gamma_1 \frac{\partial}{\partial x} + \frac{\Delta_\perp}{2ik}\right) E_{1,S} = \frac{i2\pi\omega}{cn_0}\left[\chi_0 E_{1,S} + \chi_{\pm 1} E_{S,1}\right], \tag{1}$$

$$\left(\frac{\partial}{\partial z} \mp \gamma_2 \frac{\partial}{\partial x} + \frac{\Delta_\perp}{2ik}\right) E_{2,D} = \frac{i2\pi\omega}{cn_0}\left[\chi_0 E_{2,D} + \chi_{\mp M} E_{D,2}\right], \tag{2}$$

where $\gamma_1$ and $\gamma_2$ are the angles between the wave vectors $\mathbf{k}_1$, $\mathbf{k}_S$ and $\mathbf{k}_2$, $\mathbf{k}_D$ and the $z$ axis; $M = (N - 2)/2$ is the diffraction order; $k = \omega n_0/c$ is the wave vector; $n_0$ is the nonresonant component of the refractive index of the medium (determined by the refractive index of a solvent or buffer gas); $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse laplacian, which describes the diffraction of spatially bounded light beams. The components of the Fourier expansion $\chi_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \chi_{nl}$ $\times \exp\,[-in((\mathbf{k}_1 - \mathbf{k}_S)\mathbf{r}))]d((\mathbf{k}_1 - \mathbf{k}_S)\mathbf{r})$, $n = 0, \pm 1, \pm 2, \ldots$, of the nonlinear susceptibility of the medium are determined by the spectroscopic characteristics of the resonance transition and the conditions under which the waves interact. For a faint readout wave $E_2$ (relative to the $E_1$ and $E_S$ waves used to record the dynamic grating), in the approximation of a two-level model for the resonant medium, the spatial components for the dynamic grating are given by [10]

$$\chi_0(\omega) = \frac{n_0\kappa_0}{2\pi}\left[\frac{\hat{\Theta}_{12}}{B_{12}} + \frac{\hat{\alpha}}{\alpha}\frac{1 - A_0}{A_0}\right], \tag{3}$$

$$\chi_{\pm 1}(\omega) = -\frac{n_0\kappa_0}{2\pi}\frac{2\hat{\alpha}\sqrt{I_1 I_S}}{A_0(1 + \alpha(I_1 + I_S) + A_0)}\exp\left[\pm i(\varphi_1 - \varphi_S)\right], \tag{4}$$

$$\chi_{\pm M}(\omega) = -\frac{n_0\kappa_0}{2\pi}\frac{\hat{\alpha}(-2\alpha\sqrt{I_1 I_S})^M}{\alpha A_0(1 + \alpha(I_1 + I_S) + A_0)}\exp\left[\pm iM(\varphi_1 - \varphi_S)\right], \tag{5}$$

where $A_0 = [1 + 2\alpha(I_1 + I_S) + \alpha^2(I_1 - I_S)^2]^{1/2}$ and $\kappa_0$ is the linear extinction coefficient. The complex nonlinearity parameter for a two-level system is given by $\hat{\alpha} = a + i\alpha = (\hat{\Theta}_{12} + \hat{\Theta}_{21})/vp_{21}$, where $\hat{\Theta}_{ij}(\omega) = \Theta_{ij}(\omega) + iB_{ij}(\omega)$, and the coefficients $\Theta_{ij}(\omega)$ are coupled to the Einstein coefficients $B_{ij}(\omega)$ for induced transitions between the levels $i$–$j$ by dispersion relations; $p_{21}$ is the overall probability of spontaneous and nonradiative transitions, and $v = c/n_0$ is the speed of light in the medium. The intensity of the light waves is normalized to the saturation intensity of the resonance transition ($I_{sat} = \alpha^{-1}$, with $\alpha = (B_{12} + B_{21})/vp_{21}$). In the following we examine the case of congruent absorption and luminescence profiles ($\Theta_{12} = \Theta_{21}$).

A numerical simulation of the system of Eqs. (1) and (2) was carried out under the assumption that the signal light beam contains a helical phase dislocation of the topological charge $m$:

$$E_S(z = 0, r, \varphi) = E_{S0}\left[(r - r_S)/r_0\right]^{|m|}\exp\left[-(r - r_S)^2/2r_0^2 + im\varphi\right]. \tag{6}$$

As reference $E_1$ and readout $E_2$ beams we used beams with a flat wave front and a gaussian intensity distribution:

$$E_1\,(z=0, r, \varphi) = E_{10}\exp\left[-(r-r_1)^2/2r_{01}^2\right], \tag{7}$$

$$E_2\,(z=L, r, \varphi) = E_{20}\exp\left[-(r-r_2)^2/2r_{02}^2\right], \tag{8}$$

with a half width of three times that of the signal wave ($r_{01} = r_{02} = 3r_0$) to ensure efficient overlap of the beams within the volume of the medium. The half width of the signal light beam was $r_0 = 1$ mm, the initial absorption coefficient of the nonlinear medium was $k_0 = 1$ cm$^{-1}$, the length $L$ of the nonlinear medium was varied over 0.25–1.00 cm, the peak intensity of the reference beam was $\alpha I_0 = 0.1$–$5.0$, $\lambda_{rad} = 0.5$ μm, and the detuning of the radiation frequency from the center of the absorption band was $\delta = (\omega - \omega_{12})/\Delta = 0$; $\pm 1.5$, where $\Delta$ is the half width of the gaussian absorption profile.

**Numerical Simulation and Discussion.** Before proceeding to the numerical simulation of the system of Eqs. (1) and (2), let us analyze the phase structure of the diffracted wave $E_D$, using Eqs. (4) and (5). Thus, for a four-wave interaction employing plane reference and readout waves ($\varphi_1 + \varphi_2 = \text{const}$), the phase of the diffracted wave $E_D$ is the inverse of the phase of the signal wave ($\varphi_D = -\varphi_S$, the inversion of the wave front). If a signal beam $E_S$ contains a helical phase dislocation of order $m$, then the wave front of the inverted light beam $E_D$ must contain a dislocation of the opposite sign, i.e., have a topological charge of $-m$. Multiwave (six-, eight-, etc.) interactions make it possible to carry out various operations for transformation of the wave front of a singular light beam $E_S$. When plane reference and readout waves ($M\varphi_1 + \varphi_2 = \text{const}$) are used, the phase of the diffracted wave becomes a multiple of the phase of the signal wave with the opposite phase ($\varphi_D = -M\varphi_S$); this makes it possible to invert the sign of the topological charge of the singular light beam with simultaneous multiplication by a factor of $M$, which determines the order of Bragg diffraction for the readout beam.

These features are illustrated in Fig. 1, which shows the intensity and phase distributions of a signal beam $E_S$ with topological charge $m = 1$, as well as the intensity and phase distributions of the diffracted beam $E_D$, obtained by numerical simulation of the system of coupled wave equations (1) and (2) for four-, six-, and eight-wave mixing. The calculations were done for excitation of a two-level resonance system in the center of the absorption band ($\delta = 0$), when energy exchange between the light beams is determined by diffraction on amplitude dynamic gratings. It can be seen from Fig. 1b that, owing to diffraction of a readout wave $E_2$ on an amplitude grating written by a reference $E_1$ beam and a signal $E_S$ beam containing a helical dislocation with topological charge $m = 1$, an inverted wave $E_D$ is formed whose wave front contains a dislocation of opposite sign relative to the signal wave ($m = -1$). Under the conditions of six- and eight-wave interactions, the sign of the topological charge of the optical vortex ($m = 1$) changes to the opposite with simultaneous doubling ($m = -2$, Fig. 1c) or trebling ($m = -3$, Fig. 1d) of the topological charge.

The above results were obtained for frequencies ..... of the interacting waves equal to the frequency of a resonant transition at which nonlinear modulation of the refractive index is absent. At the same time, an analysis of the system of Eqs. (1) and (2) shows that phase self- and cross-modulation in a nonlinear medium owing to light induced changes in the refractive index can induce significant changes in the transformation behavior for coherent images formed by the use of singular light beams. To study these effects, we shall examine three cases of multiwave interactions in resonant media: transformation of a converging singular beam under conditions for writing amplitude dynamic gratings which cause spatial modulation of the absorption coefficient (Fig. 2I), as well as under conditions with a nonlinear variation in the refractive index when the frequencies of the light waves in the short wavelength region ($\lambda < \lambda_{12}$, focussing nonlinearity, Fig. 2II) or long wavelength region ($\lambda > \lambda_{12}$, defocussing nonlinearity, Fig. 2III) of the spectrum are detuned relative to the center of the absorption band of the resonant medium.

In order to form a curved wave front, the signal light beam is initially directed onto a phase transparency of the form $n = n_0 - \Delta n(r/r_0)^2$ which introduces a nonuniform phase delay along the transverse profile. The solution of the corresponding truncated wave equation can be used to calculate the spatial profile of the converging singular beam on entry into the nonlinear medium (Fig. 2I,b, curve 1). The curvature of the wave front of this beam along the radial coordinate $r$ is characterized by a linear variation in the derivative $\partial\varphi/\partial r \equiv \varphi'$ (Fig. 2I,c, curve 1). With multiwave interactions in a resonant medium with a nonlinear variation in the absorption coefficient, a diverging diffracted light beam $E_D$ is formed (Fig. 2I,b, curves 2–4) whose wave front has a curvature that depends on the diffraction order $M$

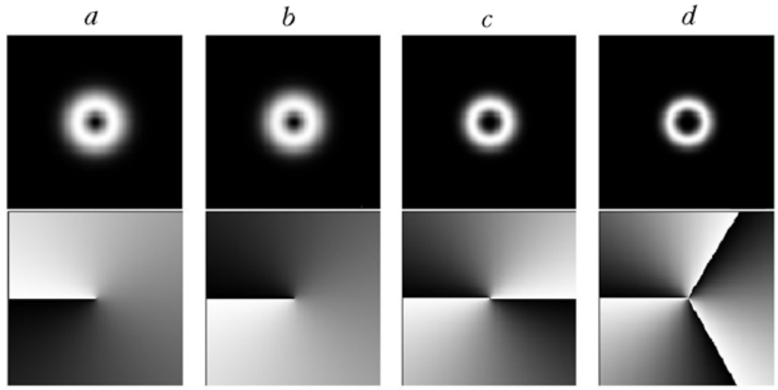


Fig. 1. The intensity (upper row) and phase (lower row) distributions in the transverse cross section of the signal (a) and diffracted light beams for four- (b), six- (c), and eight-wave (d) interactions; $\delta = 0$, $k_0L = 0.25$, $\alpha I_1 = 1$, $\alpha I_S = 0.1$.

and coincides with the curvature of the signal singular beam in the case of a four-wave interaction (Fig. 2I, c, curve 2). In the cases of six- (Fig. 2I,b, c, curves 3) and eight-wave (curves 4) mixing, the curvature of the wave front of the diffracted beam increases compared to the signal beam by factors of two and three, respectively.

Multiwave interactions in a resonant medium with a focussing-type nonlinear variation in the refractive index (e.g., $\delta = 1.5$) lead to significant distortions in the wave front of the diffracted light beam. Thus, for the parameters of the nonlinear medium and light considered here, in the case of a four-wave interaction the wave front of the diffracted beam in the axial region is essentially flat (Fig. 2II,b, c, curves 2); this is explained by compensation of the phase distortions of the wave front of the diffracted wave (a diverging light beam) owing to nonlinear phase cross-modulation which arises during propagation of the light beam in a medium with an induced focussing lens. In addition, there is a phase shift whose magnitude is determined by the ratio of the contributions to the nonlinear susceptibility from the modulation in the absorption coefficient and the refractive index. In the case of a four-wave interaction, it can be estimated beginning with $\Delta\varphi = \arg\tan\,[a/\alpha] = \arg\tan\,[\theta_{ij}/B_{ij}]$. In the case of six- (Fig. 2II,b, c, curves 3) and eight-wave (curves 4) interactions, the curvature of the wave front of the beam $E_D$ is also reduced, which can cause a significant change in the transverse dimensions of coherent images formed in various diffraction orders on a dynamic hologram.

An analysis of the case of a defocussing nonlinearity (e.g., $\delta = -1.5$) shows that for both four- (Fig. 2,III,b,c, curves 2) and six- and eight-wave (curves 3 and 4) interactions, there is an additional increase in the curvature of the wave front of a diverging diffracted singular beam $E_D$ owing to additional operation of the defocussing effect. Here an increase in the transverse dimensions of images recovered in different diffraction orders is to be expected.

When high order ($m \geq 2$) optical vortices are obtained in six- and eight-wave interaction configurations, it is also necessary to examine the stability of their spatial and topological structure as they propagate in media with a nonlinear refractive index modulation [13]. The numerical simulations show that the main factor determining the quality of the wave front of singular beams with a high topological charge $m = 2, 3, ...$ formed in a multiwave interaction and the uniformity of the intensity distribution in the transverse cross section is effective overlap of all the light beams within the volume of the resonant medium. Since the readout and diffracted beams do not generally propagate collinearly with the reference and signal beams, this requirement is met for relatively small convergence angles $\gamma = 2\gamma_1$ of the beams used to record a dynamic hologram and lengths $L$ of the nonlinear layer. Figure 3a shows the stability boundaries for the topological structure of a diffracted beam with topological charge $m = -2$ and $-3$ for the cases of six- and eight-wave interactions. The regions of stability for a light beam with topological charge $m = -2$ and $-3$ lie below curves 1 and 2. Figure 3b and c shows that spatial distributions of the intensity and phase of the optical vortices during propagation in the nonlinear medium; here the point $z = 0$ corresponds to the diffracted beam at the outlet from the medium.

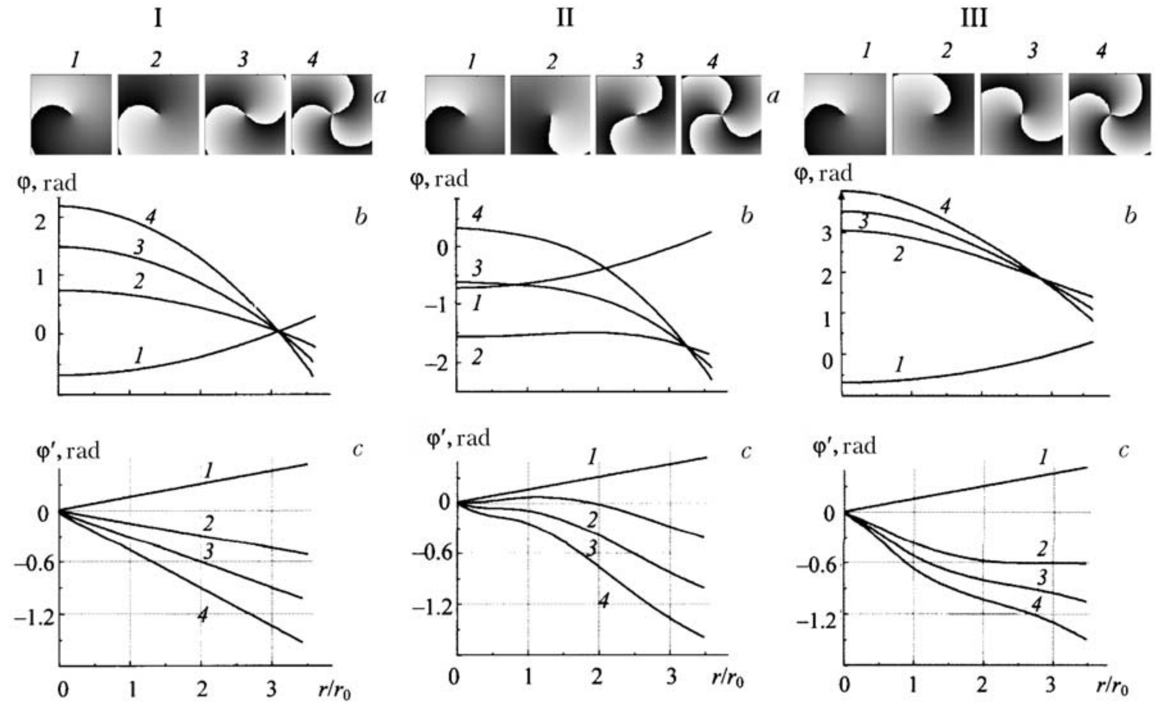


Fig. 2. Spatial distributions of the phase (a) of converging signal (1) and diffracted light beams for four- (2), six- (3), and eight-wave (4) interactions; (b) the cross section of the wave front $\varphi(r)$ in the segment [$-3.5 < x < 0$; $y = 0$], (c) the derivative $\varphi'(r)$ characterizing the curvature of the wave front of the signal (1) and diffracted (2–4) light beams; $\delta = 0$ (I), 1.5 (II) and –1.5 (III), $k_0L = 0.25$, $\alpha I_1 = 1$, $\alpha I_S = 0.1$.

Note that the simulations of the transformation of a wave front of singular light beams shown in Fig. 2 were done for $\gamma = 20$ mrad and $L = 0.25$ cm (Fig. 3a, point $A$), i.e., under conditions such that the singular beam retains its topological structure at as it leaves the nonlinear medium. At the same time, increasing the angle between the interacting beams and the length of the nonlinear layer can lead to a reduction in the quality of the transformation of the signal beam, but can also result in the destruction of its structure owing to the formation of several optical vortices with lower topological charge. Thus, for the case of a six-wave interaction (e.g., $\gamma = 120$ mrad, $L = 0.6$ cm, point $B$), an optical vortex with $m = -2$ decays into two beams with equal charges $m = -1$. When an optical vortex with $m = -3$ propagates in the nonlinear medium, as happens during an eight-wave interaction (e.g., $\gamma = 40$ mrad, $L = 1.2$ cm, point $C$), it typically breaks up into two vortices with $m = -1$ and $-2$ and the second order optical vortex then decays further.

The curves of Fig. 3 were obtained for a detuning of the interacting waves by 1.5 times the half width of the profile from the maximum of the absorption band with highly efficient energy exchange between the light beams. Reducing the nonlinear modulation of the refractive index of the resonant medium, by, for example, tuning the interaction frequency closer to the center of the absorption band or by reducing the intensities of the interacting waves, leads to a substantial extension of the stability region for the high order optical vortices.

Let us examine the effect of energy transfer between the interacting light beams on the transformation efficiency for the singular beams using a four-wave interaction as an example. The energy efficiency of the conversion of a readout light beam $E_2$ into a diffracted wave $E_D$ is given by the expression for the diffraction efficiency, $\xi_0 = |E_D(z=0)|^2/|E_2(z=L)|^2$, and, in general, depends on the intensity of the light waves, the optical depth of the nonlinear layer, and the depth of the nonlinear modulation in the absorption coefficient and the refractive index. Under optimal interaction conditions with an optical depth $k_0L \geq 1$, here the conversion coefficients can be as high as several

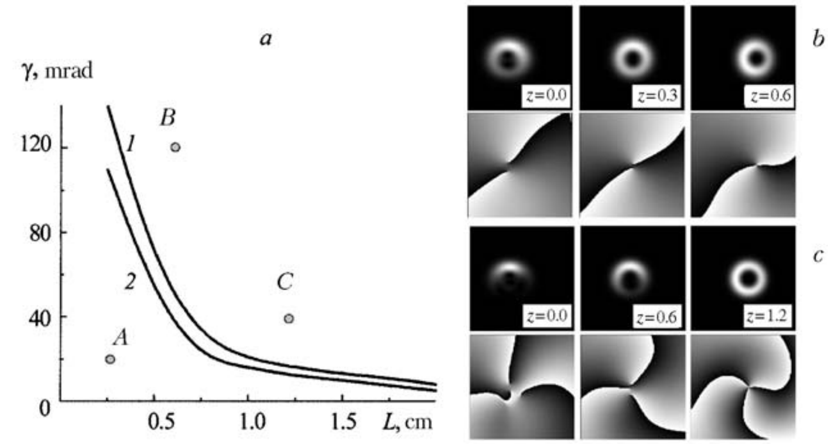


Fig. 3. Stability diagram (a) for optical vortices of order 2 (1) and 3 (2) for six- (1) and eight-wave (2) interactions: $\delta = 1.5$, $\alpha I_1 = 1$, $\alpha I_S = 0.1$; spatial distributions of the intensity and phase over the transverse cross section of the optical vortices in (b) second order, point $B$, (c) third order, point $C$, as the beam propagates in the nonlinear medium.

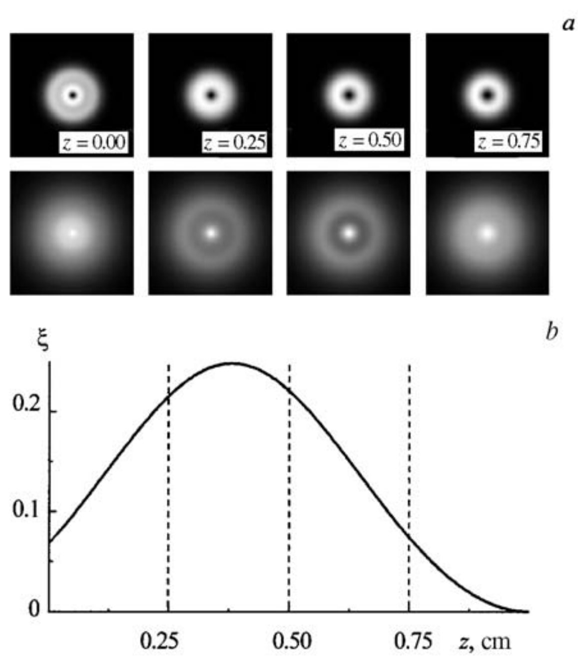


Fig. 4. The intensity distribution over the transverse cross section of the diffracted (upper row) and readout light beams (lower row) (a); the variation in the integral transformation coefficient $\xi$ for different penetration depths of the light beams into the nonlinear medium (b); $\delta = -1.5$, $k_0 L = 1$, $\alpha T_1 = 1$, $\alpha I_S = 1$.

tens of percent [14]. An examination of this problem taking into account the spatial distribution of the intensity in the transverse cross sections of the light beams would show that energy exchange between the readout and diffracted beams is substantially nonuniform over the transverse cross section, which has a significant effect on the efficiency of conversion of the readout beam $E_2$ into a diffracted singular beam $E_D$. The transverse profiles of these beams inside the volume of the nonlinear medium, calculated from the solutions of the system of Eqs. (1) and (2), are shown in

Fig. 4a. It can be seen that during formation of the diffracted beam $E_D$, there is local transfer of energy from the wide readout beam $E_2$, so that an annular dip shows up in the profile of the latter. Here the integral transformation coefficient $\xi = \int\int_{-\infty}^{\infty} |E_D(x, y)|^2\, dxdy / \int\int_{-\infty}^{\infty} |E_2(x, y)|^2 dxdy$ increases from the boundary $z = 1$, where the diffracted wave is generated, and reaches a maximum inside the nonlinear medium. Then the conversion efficiency may fall off owing to reverse transfer of energy from the diffracted beam to the readout beam (Fig. 4b); hence, it is necessary to optimize the interaction length.

**Conclusion.** Theoretical analysis and numerical simulation of the propagation and interaction of singular light beams in resonant media have shown that multiwave mixing configurations on amplitude and phase dynamic gratings can be used to perform various transformation operations on the wave front of optical vortices, including reversal of the sign and multiplication of the topological charge during image recovery in different diffraction orders. The optimal conditions for the multiwave interactions have been determined and it has been found that the energy efficiency with which a gaussian readout light beam can be converted into a diffracted singular beam is limited to about 20–30% because of spatially nonuniform energy exchange between interacting light beams with different topological structures. Here the inclusion of phase dynamic gratings in the interaction when the energy transfer is highly efficient may lead to distortion or even the destruction of singular beams with topological charges of two or more. The structural quality of the wave front of optical vortices is assured for small convergence angles of the interacting waves (several tens of milliradians) and nonlinear layer thickness of less than a centimeter (during operation with light beams with diameters of a few millimeters).

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