Realization of the characteristics method with reference to system of the movement's equations of anisotropic piezoactive elastic medium

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Abstract. *The present work presents the obtained equation of characteristics (equation of removable discontinuities) for the system of equations of the anisotropic elastic medium motion with consideration of piezoelectric effect. The system of bicharacteristics is built on the basis of the characteristic equation's solution, and this system determines the coordinates of the elastic medium's points, which were reached by the energy of wave perturbation, which is spread from a point nonstationary source, up to an undefined moment of time. There are examples of the construction of the three-dimensional fronts of quasilongitudinal and quasitransversal waves for the piezoelectric crystals, which belong to the cubic and trigonal systems of symmetry.*

1 Introduction.

The objectives of the deformed solid body mechanics, which are connected with the study of wave fields in anisotropic media, are among the most complicated and important ones both in the theoretical sense and from a practical point of view. Their topicality is conditioned by the constantly growing need of practical application of the results of the new scientific studies in the contemporary branches of science and engineering, and also by the necessity of formation of demonstrable physical ideas about the behavior of waves of different types in anisotropic media [1]. At the same time, up to now not so many cases of applying the analytical approaches to the solution of the dynamic problems of continuum mechanics were known, which is connected with the desire of researchers to be freed from the superfluous unwieldiness of calculations and results. This led to the significant delay in the development of the traditional methods, such as the method of characteristics (method of removable discontinuities) and the method of strong discontinuities [2, 3]. At the same time, facilities and means of the contemporary systems of computer mathematics make the solution of the highly complex dynamic problems accessible and allow to mathematically simulate wave processes in continuous media on the basis of the method of characteristics. The present paper shows the representation of the results of the method's application for the simulation of the wave motions, which are spread from the nonstationary point source in anisotropic elastic media, considering piezoelectric effect.

2 Characteristics equation.

When analyzing waves' propagation in piezoelectric media in general, the equation of motion and Maxwell equation are to be solved simultaneously. The solutions are the mixed elastic-electromagnetic waves with the velocity of propagation V, accompanied by electric field, and electromagnetic waves with the velocity of propagation $v \approx V \cdot 10^5$, accompanied by mechanical deformation [1]. For the first wave mode it is possible to disregard the magnetic field, which is created by electrical one, moving at the velocity, which is rather low in comparison with the speed of electromagnetic waves. Thus, even in the strong piezoelectric media the interaction between the elastic and electromagnetic waves proves to be weak because of a large difference in the corresponding speeds. Therefore, the propagation of waves can be examined independently, in the quasistatic approximation. In this case, the equation of motion can be represented in the following way [1]:

$$\begin{gathered}\sum_{j,k,l=1}^{3} A_{ijkl}\frac{\partial^2 u_l}{\partial x_j \partial x_k} + \sum_{i,j,k=1}^{3} e_{kij}\frac{\partial^2 \Phi}{\partial x_j \partial x_k} - \rho\frac{\partial^2 u_i}{\partial t^2} = 0, \\ \sum_{j,k,l=1}^{3} \frac{\partial^2 u_l}{\partial x_j \partial x_k} - \sum_{j,k=1}^{3} \varepsilon_{jk}\frac{\partial^2 \Phi}{\partial x_j \partial x_k} = 0,\end{gathered} \tag{1}$$

Here A_{iksq} is the elastic constants; e_{jkl} is piezoelectric modules, ε_{jk} is dielectric constants; u_l is the displacement vector components; ρ is medium density; Φ is electric potential. For the constants of elasticity the following equalities are carried out: $A_{ijkl} = A_{klij}$, $A_{ijkl} = A_{jikl}$ and $A_{ijkl} = A_{ijlk}$. Thus, the number of independent constants composes 21. Piezoelectric modules are symmetrical on two sequential indices j and k ($e_{ijk} = e_{ikj}$), therefore the number of independent piezoelectric modules is equaled to 18. The pairs of indices can take six different values, designated by the numbers α and β, according to the following rule:

$$\begin{gathered}A_{ijkl} = A_{\alpha\beta}, e_{ijk} = e_{i\alpha}, i, j, k, l = \overline{1,3}, \alpha, \beta = \overline{1,6}, \\ (1,1) \to 1, (2,2) \to 2, (3,3) \to 3, (2,3) \to 4, (1,3) \to 5, (1,2) \to 6.\end{gathered}$$

Let us preset initial conditions to system (1) on the surface $z(x_1, x_2, x_3, t) = 0$ and switch over to new variables according to the following circuit:

$$g_i = z_i(x_1, x_2, x_3, t), g = z(x_1, x_2, x_3, t), i = \overline{1,3}. \tag{2}$$

Expressing derivatives regarding the old variables through the derivatives regarding the new variables, we will obtain:

$$\begin{gathered}\frac{\partial f}{\partial x_k} = \sum_{m=0}^{3} \frac{\partial f}{\partial g_m}\frac{\partial z_m}{\partial x_k}, \frac{\partial^2 f}{\partial x_k \partial x_l} = \sum_{m,n=0}^{3} \frac{\partial^2 f}{\partial g_m \partial g_n}\frac{\partial z_m}{\partial x_k}\frac{\partial z_n}{\partial x_l} + \sum_{m=0}^{3} \frac{\partial f}{\partial g_m}\frac{\partial^2 z_m}{\partial x_k \partial x_l}, \\ g \equiv g_0, z \equiv z_0, t \equiv x_0, i = \overline{1,3}, k, l = \overline{0,3}.\end{gathered} \tag{3}$$

Let us substitute expressions (3) by equations (1) and extract those terms, which contain the derivatives of the second order, $\frac{\partial^2 u_i}{\partial g^2}$ and $\frac{\partial^2 \Phi}{\partial g^2}$, as only they will be important for the following steps. As a result, we will have:

$$\sum_{j,k,l=1}^{3} A_{ijkl}\frac{\partial^2 u_l}{\partial g^2}p_jp_k + \sum_{k,i,j=1}^{3} e_{kij}\frac{\partial^2 \Phi}{\partial g^2}p_jp_k - \rho\frac{\partial^2 u_i}{\partial g^2}p_0^2 + \ldots = 0,$$
$$\sum_{j,k,l=1}^{3} e_{jkl}\frac{\partial^2 u_l}{\partial g^2}p_jp_k - \sum_{j,k=1}^{3} \varepsilon_{jk}\frac{\partial^2 \Phi}{\partial g^2}p_jp_k + \ldots = 0, i = \overline{1,3}. \quad (4)$$

Here $p_i = \frac{\partial z}{\partial x_i}$, $p_0 = \frac{\partial z}{\partial t}$. From the initial conditions of system (1) it is possible to find all partial derivatives of the second order, except the derivative of the second order on g. The missing derivatives can be defined from the four equations (4), which can be considered as a system of algebraic equations, relative to the derivatives $\frac{\partial^2 u_i}{\partial g^2}$ and $\frac{\partial^2 \Phi}{\partial g^2}$, $i = \overline{1,3}$. In order to simplify system (4) let us express from it the fourth equation and the partial derivative of the second order of the electric potential Φ and let us substitute it by the first three equations. After simple conversions we will have the following:

$$\frac{\sum_{j,k,l=1}^{3} e_{jkl}\frac{\partial^2 u_l}{\partial g^2}p_jp_k}{\sum_{j,k=1}^{3} \varepsilon_{jk}p_jp_k} \sum_{k,i,j=1}^{3} e_{kij}p_jp_k +$$
$$+ \sum_{j,k,l=1}^{3} A_{ijkl}\frac{\partial^2 u_l}{\partial g^2}p_jp_k - \rho\frac{\partial^2 u_i}{\partial g^2}p_0^2 + \ldots = 0, i = \overline{1,3}. \quad (5)$$

The partial derivatives of the second order can have gaps on the surface $z(x_1, x_2, x_3, t) = 0$ only when equality to zero definitions, comprised of the coefficients with these derivatives in system (5), is fulfilled:

$$det \parallel w_{il} \parallel_{3\times3} = 0, \quad (6)$$

were $w_{il} = \sum_{j,k=1}^{3}(A_{ijkl}p_jp_k + e_{jkl}e_{kij}Sp_j^2p_k^2) - \rho p_0\delta_{il}$, $\frac{1}{S} = \sum_{j,k=1}^{3}\varepsilon_{jk}p_jp_k$, $i, l = \overline{1,3}$, δ_{il} - Kroneker's symbol. Revealing the determinant (6), after simple conversions we will obtain a nonlinear differential first order equation:

$$\frac{q_0p_0^6}{c_2^6} + \frac{q_1p_0^4}{c_2^4} + \frac{q_2p_0^2}{c_2^2} + q_3 = 0. \quad (7)$$

The coefficients of equation (7) take the following form:

$$q_0 = -1, q_1 = \sum_{i,j,k=1}^{3}(a_{ijki}p_jp_k + sK_{jki}K_{kij}p_j^2p_k^2),$$

$$q_2 = \sum_{j,k=1}^{3}\Bigg(\Bigg(\sum_{i,l=1}^{3}(a_{ijkl}a_{ljki} - a_{ijki}a_{ljkl}) - s\Bigg(\sum_{i,l=1}^{3} a_{ijki}K_{klj}K_{jkl} -$$

$$-\sum_{i=1}^{3} a_{ijki}K_{kij}K_{jki} - \left(\sum_{i,l=1}^{3} a_{ijkl}K_{kij}K_{jkl} - \sum_{i=1}^{3} a_{ijki}K_{jki}K_{kij}\right)\Bigg)p_j p_k\Bigg)p_j^2 p_k^2\Bigg),$$

$$q_3 = \sum_{j,k=1}^{3}\left(\frac{1}{3}\sum_{i,l,m=1}^{3} a_{ijkl}a_{ljkm}a_{ijkm} - \frac{1}{2}\sum_{i,l,m=1}^{3} a_{ijkl}^2 a_{mjkm} + \frac{1}{6}\sum_{i,l,m=1}^{3} a_{ijki}a_{ljkl}a_{mjkm}\right)\times$$

$$\times p_j^3 p_k^3 - \frac{s}{2}\sum_{j,k=1}^{3}\left(\sum_{i,l,m=1}^{3} a_{ijkl}^2 K_{jkm}K_{kmj} + 2\sum_{i=1}^{3} a_{ijkl}^2 K_{jki}K_{kij} - \right.$$

$$\left. - \sum_{i,l=1}^{3}(a_{ijkl}^2 K_{jkl}K_{klj} + a_{ijki}^2 K_{jkl}K_{klj} + a_{ijkl}^2 K_{jki}K_{kij})\right)p_j^4 p_k^4 +$$

$$+\frac{s}{2}\sum_{k,j=1}^{3}\left(\sum_{i,m,l=1}^{3} a_{ijki}a_{ljkl}K_{jkm}K_{kjm} + \sum_{i,m=1}^{3}(2a_{ijki}a_{mjkm}K_{jki}K_{kji} - \right.$$

$$\left. - a_{ijki}^2 K_{jkm}K_{kjm}) + 2\sum_{i=1}^{3} a_{ijki}^2 K_{jki}K_{kji}\right)p_j^4 p_k^4 + \sum_{k,j=1}^{3}\left(\sum_{i,l,m=1}^{3} a_{ijkl}a_{mjkl}K_{jkm}K_{kij} - \right.$$

$$- \sum_{i,m=1}^{3}(a_{ijkm}^2 K_{jki}K_{kij} + a_{ijki}a_{ijkm}K_{jkm}K_{kij} + a_{ijki}a_{ijkm}K_{jki}K_{kmj}) +$$

$$\left. +2\sum_{i=1}^{3} a_{ijki}^2 K_{jki}K_{kij}\right)p_j^4 p_k^4 -$$

$$-s\sum_{k,j=1}^{3}\left(\sum_{i,l,m=1}^{3} a_{ijkl}a_{mjkm}K_{jkl}K_{kij} - \sum_{i,m=1}^{3}(a_{ijki}a_{mjkm}K_{jki}K_{kij} + \right.$$

$$\left. + a_{ijki}a_{ijkm}K_{jki}K_{kmj} + a_{ijki}a_{ijkm}K_{jkm}K_{kij}) + 2\sum_{i=1}^{3} a_{ijki}^2 K_{jki}K_{kij}\right)p_j^4 p_k^4.$$

Here $s = \sum_{i,j=1}^{3} k_{ij}p_j p_k$, $a_{ijkl} = \frac{A_{ijkl}}{A_{2323}}$, $K_{ijk} = \frac{e_{ijk}}{\sqrt{A_{2323}\varepsilon_{11}}}$ are constants of electromechanical coupling, $k_{ij} = \frac{\varepsilon_{ij}}{\varepsilon_{11}}$ are nondimentional dielectric, $c_2 = \sqrt{\frac{A_{2323}}{\rho}}$. Let us examine the equation of characteristics (7) as the algebraic equation of the third order relative to p_0^2. After simple conversions we will obtain:

$$p_0^{(n)} = c_2\sqrt{2\sqrt{-\frac{p}{3}}\cos\left(\frac{\Lambda + 2\pi(4-n)}{3}\right) - \frac{q_1}{3q_0}},$$
$$\Lambda = \arccos\left(-\frac{q}{2}\sqrt{-\left(\frac{3}{p}\right)^3}\right), \tag{8}$$

where $p=-\frac{q_1^2}{3q_0^2}+\frac{q_2}{q_0}$; $q=\frac{2q_1^3}{27q_0^3}-\frac{q_1q_2}{2q_0^2}+\frac{q_3}{q_0}$, $n=\overline{1,3}$, superscript indicates the type of the elastic wave. For constructing the discontinuity surface (front) of the wave on the basis of differential equations (8) in the partial derivatives of the first order, one should build the system of bicharacteristics (rays), which corresponds to the desired wave front [3]. In this case the bicharacteristics are defined as the solutions of the following systems of ordinary differential equations [2, 3]:

$$\frac{dx_s^{(n)}}{dt}=\frac{\partial p_0^{(n)}}{\partial p_s}, n,s=\overline{1,3}. \tag{9}$$

Substituting expressions (8) into equations (9) and taking into account the fact that their right side does not depend on time, we will obtain formulas for the dimensionless coordinates $\frac{x_s^{(n)}}{c_2t}$, $s,n=\overline{1,3}$ medium points, which were reached by the wave perturbation:

$$\begin{gathered}\frac{x_s^{(n)}}{c_2t}=\frac{1}{v_n}\left(\frac{1}{2\sqrt{-3\hat p}}\left(\frac{2\hat q_1q_{1s}}{3q_0^2}-\frac{q_{2s}}{q_0}\right)\cos\left(\frac{\hat\Lambda+2\pi(4-n)}{3}\right)-\right.\\ -\frac{1}{3}\sqrt{-\frac{\hat p}{3}}\sin\left(\frac{\hat\Lambda+2\pi(4-n)}{3}\right)\sqrt{\frac{\hat p^3}{4\hat p^3+27\hat q^2}}\times\\ \times\left(\left(\frac{2\hat q_1^2q_{1s}}{9q_0^3}-\frac{\hat q_2q_{1s}+\hat q_1q_{2s}}{3q_0^2}+\frac{q_{3s}}{q_0}\right)\sqrt{-\left(\frac{3}{\hat p}\right)^3}-\right.\\ \left.\left.-\frac{9\hat q\sqrt{3}}{2}\sqrt{-\left(\frac{1}{\hat p}\right)^5}\left(\frac{2\hat q_1q_{1s}}{3q_0^2}-\frac{q_{2s}}{q_0}\right)\right)\right).\end{gathered} \tag{10}$$

Here v_1 is the speed of quasilongitudinal piezoactive elastic wave propagation, v_2 and v_3 are the speeds of quasitransversal piezoactive elastic waves propagation. The speeds v_n are determined by formulas, analogous to relationships (8). Expressions for the coefficients $\hat q_s$, $\hat p$, $\hat q$ and $\hat\Lambda$ we obtain from the coefficients q_s, p, q and Λ accordingly by the replacement of the parameters p_k to the direction cosines of normal to the characteristic surface $n_k=\cos(\alpha_k)$ (α_k the angle between the normal to the wave surface and the coordinate axis x_k). The expressions for q_{ij}, $i,j=\overline{1,3}$ have the form of:

$$q_{1j}=\sum_{i,k=1}^{3}(a_{ijkl}n_k+sK_{jki}K_{kij}n_jn_k^2(2-sk_{jk}n_jn_k)),$$

$$q_{2j}=2\sum_{j,k=1}^{3}\left(\left(\sum_{i,l=1}^{3}(a_{ijkl}a_{ljki}-a_{ijki}a_{ljkl})n_jn_k^2-\right.\right.$$

$$-\left(\sum_{i,l=1}^{3}(a_{ijki}K_{klj}K_{jkl}+a_{ijkl}K_{kij}K_{jkl})-\sum_{i=1}^{3}a_{ijki}K_{kij}K_{jki}\right)\times$$

$$\times\left(1-\frac{1}{2}n_jn_ksk_{jk}\right)\Bigg)\Bigg)sn_j^2n_k^3\Bigg),$$

$$q_{3j}=3\sum_{j,k=1}^{3}\left(\sum_{i,l,m=1}^{3}(a_{ijkl}(\frac{1}{3}a_{ljkm}a_{ijkm}-\frac{1}{2}a_{ijkl}a_{mjkm})+\frac{1}{6}a_{ijki}a_{ljkl}a_{mjkm})\right)n_j^3n_k^3-$$

$$-\frac{1}{2}\sum_{j,k=1}^{3}\Bigg(\sum_{i,l,m=1}^{3}a_{ijkl}K_{jkm}K_{kmj}+2\sum_{i=1}^{3}a_{ijki}^2K_{jki}K_{kij}-$$

$$-\sum_{i,l=1}^{3}(a_{ijkl}^2(K_{jkl}K_{klj}+K_{jki}K_{kij})+a_{ijki}^2K_{jkl}K_{klj})\Bigg)(4-k_{jk}n_jn_ks)n_j^3n_k^4s+$$

$$+4\sum_{j,k=1}^{3}\Bigg(\sum_{i,l,m=1}^{3}a_{ijkl}a_{mjkl}K_{jkm}K_{kij}-$$

$$-\sum_{i,m=1}^{3}(a_{ijkm}^2K_{jki}K_{kij}+a_{ijki}a_{ijkm}K_{jkm}K_{kij}+a_{ijki}a_{ijkm}K_{jki}K_{kmj})+$$

$$+2\sum_{i=1}^{3}a_{ijki}^2K_{jki}K_{kij}\Bigg)n_j^3n_k^3-\sum_{j,k=1}^{3}\Bigg(\sum_{i,l,m=1}^{3}a_{ijkl}a_{mjkm}K_{jkl}K_{kij}-$$

$$-\sum_{i,m=1}^{3}(a_{ijki}a_{mjkm}K_{jki}K_{kij}+a_{ijki}a_{ijkm}K_{jki}K_{kmj}+a_{ijki}a_{ijkm}K_{jkm}K_{kij})+$$

$$2+\sum_{i=1}^{3}a_{ijki}K_{jki}K_{kij}\Bigg)(4-sk_{jk}n_jn_k)sn_j^3n_k^4,\ s=\sum_{j,k=1}^{3}k_{jk}n_jn_k.$$

In formulas (8) and (10) the superscript $n=1$ it corresponds to quasilongitudinal wave, $n=2,3$ - to quasitransversal waves.

3 Examples of the construction of wave fronts.

The presence of one or another symmetry of anisotropic medium leads to the appearance of specific ratios between the constants A_{ijkl} and e_{ijk}, in view of which the number of independent components becomes less than 21 and 18 respectively. A quantity of dielectric constants for the anisotropic media of different systems of symmetry does not exceed three. Let us further examine the piezoelectric crystals of the cubic and trigonal system of symmetry. In the case of the cubic system of symmetry they are the independent constants of elasticity A_{11}, A_{12} and A_{44}, by the independent piezoelectric module e_{14}. For the dielectric constants the following equality is carried out: $\varepsilon_{11}=\varepsilon_{22}=\varepsilon_{33}$. Fig. 1 represents the three-dimensional fronts of quasi-longitudinal and quasitransversal waves, which are extended from point nonstationary source in germanate of bismuth. During the construction we

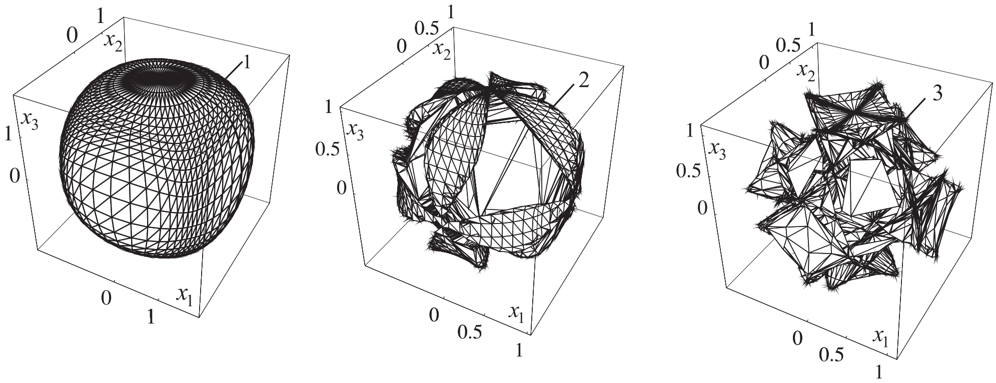

Figure 1: Three-dimensional fronts of the elastic piezoactive waves, stretching in the germinate of bismuth: 1 - a quasilongitudinal wave: 2 - a quasitransversal wave with the speed of v_2; 3 - a quasitransversal wave, having the speed v_3.

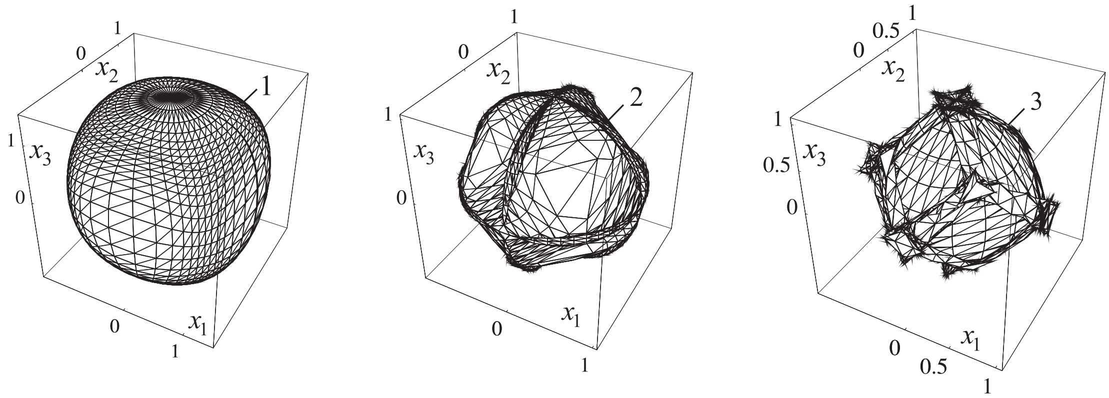

Figure 2: Three-dimensional fronts of the elastic piezoactive waves, stretching in the gallium arsenide: 1 - a quasilongitudinal wave: 2 - a quasitransversal wave with the speed of v_2; 3 - a quasitransversal wave, having the speed v_3.

assume the following material constants: $A_{11} = 128$, $A_{12} = 30.5$ and $A_{44} = 25.5$ HPa, $e_{14} = 0.99$ Cl/m^2, $\varepsilon_{11} = 3.42$ pF/m [1].

It is evident from figure 1 that the propagation of quasitransversal waves in lead occurs with the formation of lacunas. Thus, at the front of the quasitransversal wave, which is extended with a speed v_2, there appear twelve lacunas in the form of tapered strips. At the front of another quasitransversal wave a complex system of lacunas is formed, among which it is possible to isolate six conical lacunas with the bases, perpendicular to the coordinate axes.

Figure 2 represents wave surfaces for the elastic piezoactive waves, which are extended in the gallium arsenide. The properties of the material are described by the following constants: $A_{11} = 118.8$, $A_{12} = 53.8$ and $A_{44} = 59.4$ HPa, $e_{14} = -0.16$ Cl/m^2, $\varepsilon_{11} = 0.973$ pF/m [1].

As it follows from figure 2, the propagation of quasi-transverse waves in the

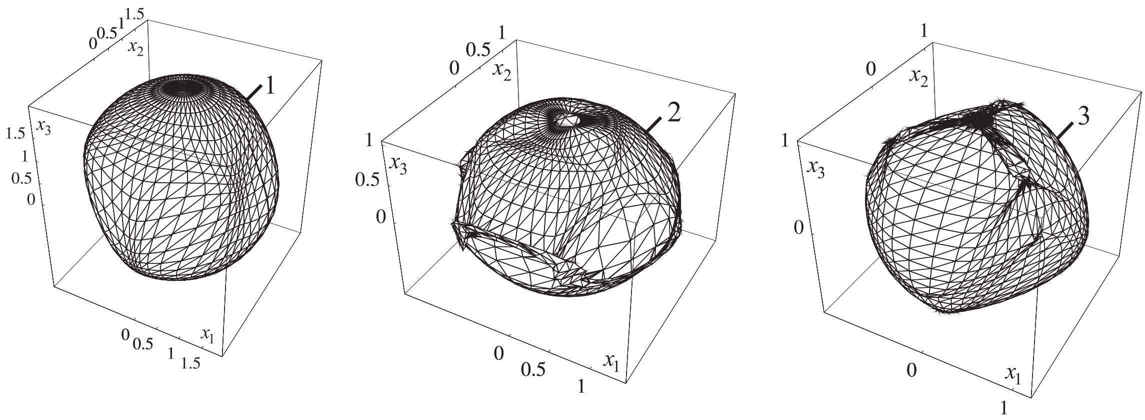

Figure 3: Three-dimensional fronts of the elastic piezoactive waves, stretching in the lithium columbate: 1 - a quasilongitudinal wave: 2 - a quasitransversal wave with the speed of v_2; 3 - a quasitransversal wave, having the speed v_3.

galena occurs with appearance of twelve (wave, which is extended with the speed v_2) and eight (wave, which is extended with the speed v_2) the lacunas.

Some of the widely used in practice piezoelectric materials are the crystals of the trigonal system of symmetry, in particular lithium columbate and α - quartz, that belong to the classes $3m$ and 32 accordingly. Figure 3 shows the wave fronts of piezoactive waves, which are extended in lithium columbate. We assume during the construction $A_{11} = 203$, $A_{12} = 53$, $A_{13} = 75$, $A_{33} = 245$, $A_{44} = 60$ and $A_{14} = 9$ HPa, $e_{15} = 3.7$, $e_{22} = 2.5$, $e_{31} = 0.2$, $e_{33} = 1.3$ Cl/m^2, $\varepsilon_{11} = 3.89$, $\varepsilon_{33} = 2.57$ pF/m [1].

The comparative analysis of the corresponding three-dimensional fronts of the quasilongitudinal and quasitransversal waves, built taking into account and without taking into account piezoelectric effect, shows that the interrelation of the electrical and mechanical properties of lithium columbate substantially influences the geometry of wave front. In particular, for lithium columbate it leads piezoelectric effect to practically complete disappearance of lacunas at the fronts of quasitransversal waves.

Three-dimensional wave fronts, which are extended from a point nonstationary source in α - quartz, are represented in Fig. 4. The mechanical and electrical properties of material are described by the follow-ing constants: $A_{11} = 86.7$, $A_{12} = 7$, $A_{13} = 11.9$, $A_{33} = 107.2$, $A_{44} = 57.9$ and $A_{14} = -17.9$ HPa, $e_{11} = 0.171$, $e_{14} = -0.04$ Cl/m^2, $\varepsilon_{11} = 0.392$, $\varepsilon_{33} = 0.41$ pF/m [1].

It is evident from figure 4 that the propagation of quasitransversal waves in α - quartz occurs with the appearance of the complex system of lacunas. In particular, at the wave front of quasitransversal wave, that has phase speed equal v_2 ,there appear two conical lacunas, axis of which is the coordinate axis x_3, four lacunas of the symmetrically located relative to three axes coordinates, and also two lacunas of symmetrical relative to beginning coordinates. Lacunas, which appear at the front of the quasitransversal wave, which is extended with the phase speed v_3, take

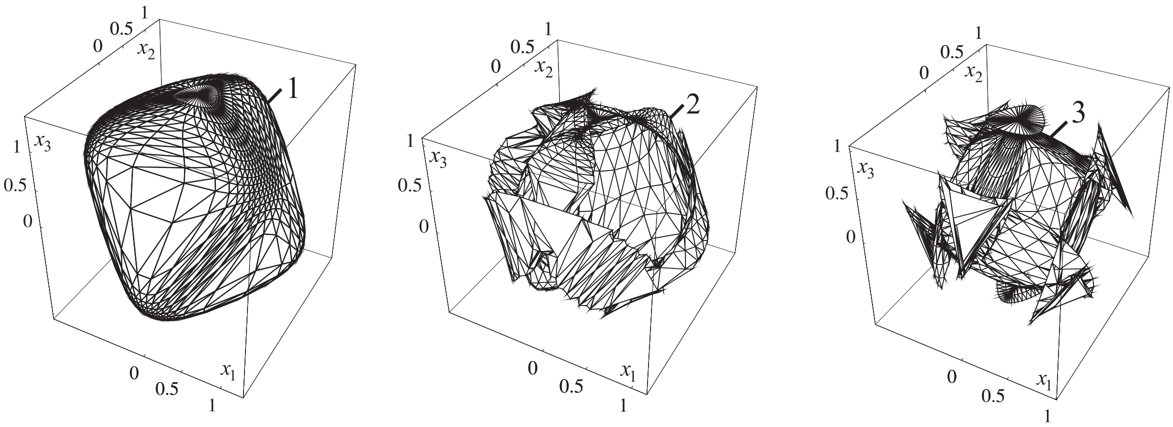

Figure 4: Three-dimensional fronts of the elastic piezoactive waves, stretching in the α - quartz: 1 - a quasilongitudinal wave: 2 - a quasitransversal wave with the speed of v_2; 3 - a quasitransversal wave, having the speed v_3.

the form of strips. Let us note that the geometric form of lacunas at the fronts of quasitransversal waves differs significantly from the conical lacunas, which have one branch point of the lines of wave front.

4 Conclusion.

The obtained results can be used for conducting of full-scale experiments regarding different physical and mechanical constants and correct interpretation of experimental data. The information about the special features of the propagation of wave fronts can be used for designing the piezoelectric structures with the assigned properties of acoustic waves, devices of working signal and different sensors. The paperwork has been performed with the financial support of the Belorussian republic fund for basic research (project F08M - 087).

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