On decomposing one relator products of cyclic groups into free products with amalgamation

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This paper deals with the problem of decomposing some finitely generated groups into non-trivial free products with amalgamation. We consider this problem for so called one-relator products of cyclic groups. Recall that a one-relator product of a family of groups $\{G_i\}$, $i \in I$ is the group $G = (*G_i)/N(S)$ where S is a cyclically reduced word in the free product $*G_i$ and $N(S)$ is its normal closure. S is called the relator. In general one-relator products share many properties with one-relator groups [6]. We consider the case where G_i's are cyclic and the relator is a proper power, i.e. $S = R^m$ where R is cyclically reduced in $(*G_i)$ and $m \geq 2$.

Definition 1 *A group G of the form*

$$G =< a_1, ..., a_n \mid a_1^{l_1} = ... = a_n^{l_n} = R^m(a_1, ..., a_n) = 1 >$$

where $n \geq 2$, $m \geq 2$, $l_i = 0$ or $l_i \geq 2$ for all i and $R(a_1, ..., a_n)$ is a cyclically reduced word in the free group on $a_1, ..., a_n$ is called one relator product of cyclics.

The first general result about decompositions of such groups into non-trivial free products with amalgamation was obtained by B. Fine, F. Levin and G. Rosenberger [3].

Theorem 1 **(**B. Fine, F. Levin, G. Rosenberger [3]) *If G is an one-relator product of n cyclics, $n \geq 3$, then G is a non-trivial free product with amalgamation.*

The proof of this theorem uses the techniques of representation and character varieties. We recall that if Γ is a finitely generated group then for any algebraic group H the set $R(\Gamma, H)$ of all representations $\rho : \Gamma \to H$ is known to have the natural structure of an algebraic variety, and endowed with this structure is called the variety of representations of Γ in H (cf. [7]). The group H acts on $R(\Gamma, H)$ by conjugation and if H is reductive then one can consider the categorical quotient $R(\Gamma, H)/H = X(\Gamma, H)$ which is usually called the character variety of Γ in H. M. Culler and P. Shalen [2] proved that if $\dim X(\Gamma, \mathrm{SL}_2(\mathbb{C})) > 0$ then Γ is either a non-trivial free product with amalgamation or a n0n-trivial HNN extension.