Thermal Convection in a Ferromagnetic Fluid in an Inhomogeneous Magnetic Field

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Dimensionless ratios describing heat transfer in a ferromagnetic fluid are introduced. Conditions under which magnetic-field perturbations can be neglected are derived. It is shown on the basis of numerical and experimental results that ferromagnetic fluids can be used for controlled cooling of current-carrying conductors.

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We write the set of ferrohydrodynamics equations [1, 2] in dimensionless form with the standard Bussinesq assumptions:

$$\frac{\partial \vec{\vartheta}}{\partial t} + (\vec{\vartheta} \cdot \nabla)\vec{\vartheta} = -\nabla \vec{p} + \Delta \vec{\vartheta} + G_r \Theta \vec{\kappa} - G_{r_m} \nabla H , \tag{1}$$

$$\text{div}\,\vec{\vartheta} = 0, \quad \frac{\partial \Theta}{\partial t} + \vec{\vartheta} \nabla \Theta = \frac{1}{P_r} \Delta \Theta, \quad \text{rot}\,\vec{H} = 0 , \tag{2}$$

$$\text{div}\left\{\frac{\vec{H}}{\left(\frac{1}{r} + A_2\right)}[A_1 A_2 + \vec{H} - A_3 \Theta]\right\} = 0 , \tag{3}$$

where

$$\vec{H} = H_x^2 + H_y^2 + H_z^2 - A_2 , \quad \vec{p} = p + \rho^* g z - \mu_0 M^* H - \frac{\mu_0 \chi}{2}(H - H^*)^2$$

$$P_r = \frac{\nu}{\chi}, \quad G_r = \frac{\beta g \gamma \ell^4}{\nu^2}, \quad G_{r_m} = \frac{\mu_0 \chi \Gamma G \ell^4}{\rho^* \nu^2}, \quad \Theta = T - T^*$$

$$A_1 = \frac{1 + M^*/H^*}{1 + \chi}, \quad A_2 = \frac{H^*}{G \ell}, \quad A_3 = \frac{\chi \gamma}{(1 + \chi)G}, \quad \chi = \chi + \beta M^*$$

The physical quantities are the same as in [2]. The scale factors used are: for the coordinate, the characteristic dimension of the cavity ℓ; for the time, ℓ^2/ν; for velocity, ν/ℓ; for the pressure, $\frac{\rho^* \nu^2}{\ell^2}$; for the

temperature, γ^{ℓ}; and for the magnetic field $G\ell$. The equation of state for magnetization was taken in the form

$$M = M^* - \mathcal{X}(T - T^*) + \chi(H - H^*).$$

The dimensionless equations (1)-(3) describe convective heat transfer in an incompressible ferromagnetic fluid. Quantities Gr, Pr, Gr_m, A_1, A_2 and A_3 describe convective heat transfer in ferromagnetic fluids and have the following meaning. Gr and Pr are the standard Grashof and Prandtl numbers; Gr_m is the magnetic Grashof number, defining the onset of convective instability brought about by a purely magnetic mechanism. A comparison of Gr and Gr_m makes it possible to estimate the contributions of the gravitational and the magnetic mechanisms to convection. The dimensionless quantity A_3 is the ratio of the inhomogeneity of the magnetic field, produced by nonisothermicity of the ferromagnetic fluid, to the magnitude of the characteristic magnetic field gradient. Ratio A_2 expresses the degree of inhomogeneity of the magnetic field, while ratio A_1 describes the deviation of the magnetization equation of state from linearity.

It is seen by analyzing Eqs. (1)-(3) that the temperature can be neglected in the Maxwell equation if

$$A_3 \ll 1 \tag{4}$$

or

$$A_3 \ll A_1 A_2 \quad \text{and} \quad A_3 \ll A_1. \tag{5}$$

When conditions (4) and (5) are satisfied, the hydrodynamic equations can be solved independently of the Maxwell equations. This approximation is termed inductionless in magnetohydrodynamics.

Let us consider, in this approximation, convective motion in a ferromagnetic fluid in a finite vertical layer of height h and width ℓ, the sides of which are maintained at constant but different temperatures, while a linear temperature profile is specified on its horizontal faces. It is assumed that the heated vertical boundary of the layer can be treated as a thin, infinitely long conducting slab, which conducts a current I, uniformly distributed over the width h.

The above boundary layer problem was solved numerically by a monotonic second-order conservative difference scheme [3]. The components of the magnetic field of the actively conducting plate were calculated from familiar formulas [4], with ℓ as the characteristic dimension and the value of the magnetic-field gradient on the central axis of the plate as the characteristic gradient.

Figure 1 depicts the effect of the magnetic field of the actively conducting plate on convective heat transfer in a ferromagnetic fluid in a cavity with square cross section, i.e., with dimensions $h/\ell = 1$.

The effect of the magnetic field is insignificant at $Ra_m = Gr_m Pr < 0.2 Ra_0$, and convective heat transfer is governed by gravity forces. When Ra and
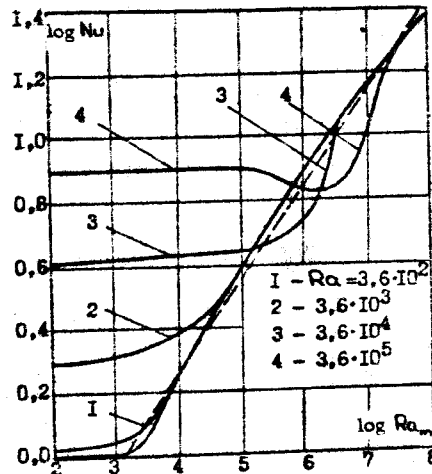
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Fig. 1.

Ra_m are in the above ratio, heat transfer through the layer is quite
satisfactorily described by the dimensionless equation

$$Nu = 0.224 (h/\ell)^{-0.25} Ra^{0.28}, \tag{6}$$

which holds in the ranges $Ra > 5 \cdot 10^3$, $Ra_m < 0.2 Ra$ $1 < h/\ell < 10$.

In the range $0.2 Ra < Ra_m < 10 Ra$ the effect of magnetic forces is
greater. But, whereas at $Ra < 10^4$ the Nusselt number increases rapidly
with Ra_m in this range, at large Ra (i.e., $Ra \simeq 10^5$) the thermal convection
is slightly less intensive, because of a specific interaction of the gravita-
tional and magnetic mechanisms of induction of convective motion in the
transitional region.

A further increase in the current in the plate at $Ra_m > 10^2 Ra$ pro-
duces a merger of all heat transfer curves, as magnetic forces become
controlling. As the field becomes more inhomogeneous, the contribution
of gravity-induced convection to convective heat transfer becomes smaller
and, tending to zero at the limit. At $Ra_m > 5 \cdot 10^3$, $1 < h/\ell < 10$, the
limiting curve may be approximated by the dimensionless equation

$$Nu = 0.126 (h/\ell)^{-0.25} Ra_m^{0.29}, \tag{7}$$

which is represented by the dashed curve in Fig. 1.

The thermal convection of ferromagnetic fluids was investigated in
an annular layer, formed by an external cylinder held at constant tem-
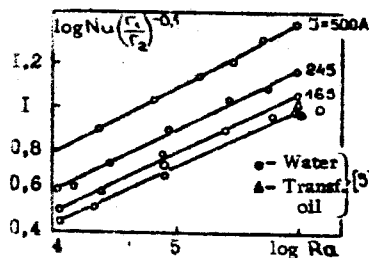perature and an inner cylinder through which direct current (0 to 500 A)
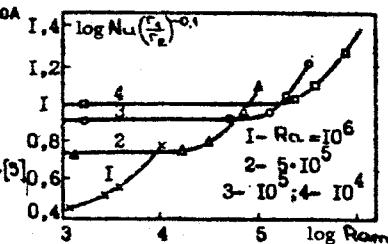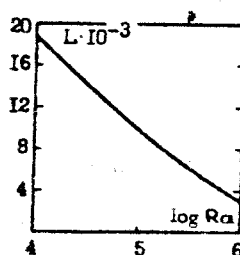
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Fig. 2.

Fig. 3.

Fig. 4.

was passed. We used a ferromagnetic fluid with the following principal properties: $\rho^* = 1234$ kg/m^3, volumetric concentration of magnetic material 8.3%, saturation magnetization $M^* = 2.1 \cdot 10^4$ A/m, $\eta = 8 \cdot 10^{-3}$ N-sec/m^2, $\lambda = 0.18$ W/m^2-$^{\circ}$C, Pr $= 97$. Under these conditions Eqs. (4) and (5) are satisfied. Hence the contribution of the magnetic convection mechanism to heat transfer was determined in terms of the magnetic Rayleigh number, in which the characteristic quantities were $G = \dfrac{\eta}{2\sigma(\Delta\tau)^2}$; $\ell = \Delta\tau$, $\gamma = \dfrac{\Delta T}{\Delta r}$.

Figure 2 presents the heat transfer data as a plot of Nu $=$ Nu(Ra) at various currents; for comparison, this graph also presents the experimental results of other workers. The data are correlated in the form of Nu = Nu(Ra$_m$) for several constant values of Ra (Fig. 3). It can be seen that when the current attains values corresponding to Ra$_m \approx$ Ra the heat transfer rate increases significantly. Starting with the inflection point the heat transfer curves can be described by the empirical expression

$$Nu = L\left(\frac{r_1}{r_2}\right)^{-0.1}(Ra_m)^{0.63},$$

where L is a function of Ra. The curve of L = L(Ra) is plotted in Fig. 4.

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It was shown as a result of numerical and experimental studies that automatic heat transfer control becomes possible when active conductors are cooled by ferromagnetic liquids, since the removal rate of the released heat increases with the current through the conductor.

NOMENCLATURE

$\Delta r = r_2 - r_1$ - difference in the radii of cylinders forming the annular layer; ΔT - temperature difference between cylinders.

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