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NUMERICAL STUDY OF EQUILIBRIUM FORMS OF MAGNETIC FLUID INCLUDING MAGNETIC FIELD DISTURBANCES

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A numerical algorithm is developed for determining equilibrium forms of a free surface of magnetic fluid taking account of magnetic field perturbation. The algorithm is approved by the problem on the shape of an isothermal magnetic fluid droplet in a uniform magnetic field. Numerical results agree with experimental data.

1. Introduction

Of greatest interest are the phenomena which take place in nonisothermal fluids in the presence of a free surface. In view of this, great attention has been rapid lately to the determination of a static shape of the free surface of magnetic fluid in a magnetic field. It is difficult to calculate the magnetic field effect on the shape of the magnetic fluid as the problem is self-consistent: the magnetic field at the surface of the fluid is determined by its shape and affects the latter itself.

It is a tradition in solving such problems to introduce a noninductive approximation by assuming that the fluid does not distort the external magnetic field. Thus, search for a free surface can be separated from finding the magnetic field structure [1,2]. In some cases, however, the noninductive approximation cannot be applied. Such a situation occurs, for example, in searching for the shape of a nonmagnetic drop in a magnetic fluid when the magnetic field at the surface is highly nonuniform [3]. Therefore, the most complete statement of the problem on equilibrium states of magnetic fluid requires that the set of essentially nonlinear ferrohydrostatic equations be solved together with Maxwell equations for the magnetic field. The difficulties of mathematical statement not only exclude the analytical solution, but also greatly hinder the application of numerical methods. In the present work, an attempt has been made to model numerically the equilibrium forms of a free magnetic fluid surface taking account of magnetic field perturbations.

2. Mathematical statement

Let us consider the problem on the shape of an imponderable magnetic fluid droplet in a uniform magnetic field of intensity H_0 surrounded by a nonmagnetic gas. The problem being considered axisymmetric, we shall formulate it in cylindrical coordinates Z and R by superposing the Z axis and the direction of the external magnetic field. Note that problem symmetry allows one to consider only one-fourth of the axial cross section of the droplet. We shall make use of the most widespread model of homogeneous nonconducting isothermal magnetic fluid [4]. The fluid magnetization M and the field strength H are related by the Langevin function. To describe the magnetic field, let us introduce the scalar potential $\Phi(R, Z)$ so that $\nabla\Phi = \boldsymbol{H}$.

The shape of the free surface is sought in the parametric form $Z(S)$, $R(S)$ with the length S of the equilibrium line are from the symmetry axis ($S = 0$) to the plane $Z = 0$ ($S = S_*$) as a parameter. Introduce dimensionless variables

$$z = Z/S_*, \quad r = R/S_*, \quad s = S/S_*, \quad \varphi = \Phi/H_0 S_*.$$

The equilibrium shape equations will be obtained

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from the equation of magnetic fluid statics and the generalized Laplace condition for the pressure jump on a free surface [4]. Thus, in case of imponderability we shall have the following dimensionless differential problem to determine $z(s)$ and $r(s)$ [1]:

$$\frac{1}{r}(rz')' = r'(Q - F_1), \tag{1}$$

$$r'' = -z'\left(Q - F_1 - \frac{z'}{r}\right), \quad 0 < s < 1, \quad ' = \mathrm{d}/\mathrm{d}s, \tag{2}$$

$$\begin{aligned} &z'(0) = 0, \quad r'(0) = 1, \quad r(0) = 0, \\ &z'(1) = -1, \quad r'(1) = 0, \quad z(1) = 0, \end{aligned} \tag{3}$$

where

$$Q = -\frac{2}{r^2(1)}(r(1) - I_2), \quad I_2 = \int_0^1 rr'F_1 \,\mathrm{d}s,$$

$$F_1 = A_1 A_3 A_2^2 \frac{(\mathrm{cth}\ \psi - 1/\psi)^2}{I_1 \psi^2}\left(r'\frac{\partial\varphi}{\partial z} - z'\frac{\partial\varphi}{\partial r}\right)^2$$

$$+ \frac{A_1}{I_1}\ln\left(\frac{1}{\psi}\mathrm{sh}\ \psi\right), \quad \psi = A_2\sqrt{\left(\frac{\partial\varphi}{\partial r}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2},$$

$$I_1 = \left(2\pi\int_0^1 rzr' \,\mathrm{d}s\right)^{1/3}, \quad A_1 = \frac{\mu_0 M_* H_* V^{1/3}}{\sigma},$$

$$A_2 = H_0/H_*, \quad A_3 = M_*/H_*,$$

μ_0 is the magnetic permeability of vacuum, M_*, the saturation magnetization of the magnetic fluid, σ, the surface tension coefficient; $H_* = kT/\mu_0 m$, k, the Boltzmann constant; T, the absolute temperature; m, the magnetic moment of the material of ferromagnetic particles; V, half of the droplet volume. Note that $I_1 = V^{1/3}/S_*$, whence $S_* = V^{1/3}/I_1$.

The magnetic field inside and outside the droplet is described by the Maxwell equations. The boundary conditions for the magnetic field include the continuity of normal induction components and the continuity of tangential intensity components on crossing the interface as well as symmetry conditions and conditions of field unperturbance far from the droplet. So, for the potential $\varphi(r, z)$ within $(0 \leqslant r,\ z < \infty)$ we obtain the following dimensionless equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r(1 + F_2)\frac{\partial\varphi}{\partial r}\right) + \frac{\partial}{\partial z}\left((1 + F_2)\frac{\partial\varphi}{\partial z}\right) = 0, \tag{4}$$

where $F_2 = A_3$ (cth $\psi - 1/\psi)/\psi$ inside the fluid, $F_2 \equiv 0$ outside the fluid with the boundary conditions on the symmetry lines and at infinity

$$\partial\varphi(0, z)/\partial r = 0, \quad \varphi(r, 0) = 0, \quad \varphi|_{r,z\to\infty} = z \tag{5}$$

and on the free surface

$$\varphi_\mathrm{i} = \varphi_\mathrm{e}, \quad (1 + F_2)\left(-z'\frac{\partial\varphi_\mathrm{i}}{\partial r} + r'\frac{\partial\varphi_\mathrm{i}}{\partial z}\right) = -z'\frac{\partial\varphi_\mathrm{e}}{\partial r} + r'\frac{\partial\varphi_\mathrm{e}}{\partial z}, \tag{6}$$

where the subscript i stands for the function determined inside the fluid and e, for the function outside the fluid.

Thus, the shape of the equilibrium surface of a magnetic fluid droplet is sought as solution to eq. (1), (2), (4) subject to boundary conditions (3), (5), (6) and is specified by three dimensionless parameters, A_1, A_2 and A_3.

3. Computational algorithm

Problem (1)–(6) was solved using the grid method. This method requires a confined calculation domain. Therefore, the problem stated will be considered in a square $\mathscr{D} = \{(r, z)\,|\,0 \leqslant r \leqslant r_\infty, 0 \leqslant z \leqslant z_\infty\}$. It is clear that the values of r_∞, z_∞, should be chosen large enough so as not to violate conditions (5). It should be noted that the stated problem is conjugate. So, for the region $\mathscr{D}$ there are in fact two problems to be solved to estimate the field potential inside and outside the droplet with conditions (6) on a free surface which is unknown a priori and is the solution to problem (1), (2) subject to boundary conditions (3).

The problem was approximated on a so-called consistent nonuniform net [5], i.e., $\mathscr{D}$ was split so as to have nodes on the free surface. The region beyond the droplet was split into spaces changing in geometric progression. The calculations were performed as three iteration procedures, one of them external and two internal ones. The magnetic field configuration being prescribed, each external iteration involved first the calculation of the droplet shape and then the correction of the field potential distribution inside and outside the droplet for the shape found. Relaxation parameters were introduced to improve the convergence of iterations.

4. Results

The formulated problem on the shape of a magnetic fluid droplet under imponderability is a fine model to be calculated. It was studied experimentally [6] and theoretically [1–3] using a noninductive approximation [1,2] and the assumption that the droplet shape is ellipsoidal [3].

The present work concerned the effect of the parameter A_2, which may be interpreted as the dimensionless intensity of the external magnetic field, on the droplet shape and potential $\varphi(r, z)$ configuration. The parameters A_1 and A_3 were prescribed to be $A_1 = 7.69$ and $A_3 = 5.7$ estimated by the experimental data [6]. Fig. 1 shows equilibrium shapes and typical isolines of the field potential for three value of A_2. It is seen that higher values of A_2 cause the droplet to elongate along the field lines. The theoretical degree of droplet elongation l, i.e., its length-to-diameter ratio, vs. external field intensity agrees with experiment [6]. We failed to obtain elongations $l > 5$ owing to computational instability at large values of A_2. Nevertheless, even for obtained elongations l, the magnetic field at the fluid surface is nonuniform, which tells on the free surface configuration. For comparison, the dashed line in fig. 1 shows the equilibrium form obtained in ref. [1] on the assumption that the magnetic field both inside and outside the fluid is uniform. The numerical analysis of the droplet structure and the magnetic field configuration has shown that, in the vicinity of the symmetry axis, the droplet top become more acute as A_2 grows. It is in this very region that the field is distorted most. Besides, the droplet shape tends to differ even more from an ellipse with growing value of A_2. Thus, at $A_2 = 1$ the maximum deviation of the free surface from an ellipse was about 10%; at $A_2 = 3$ this deviation went up to 32%.

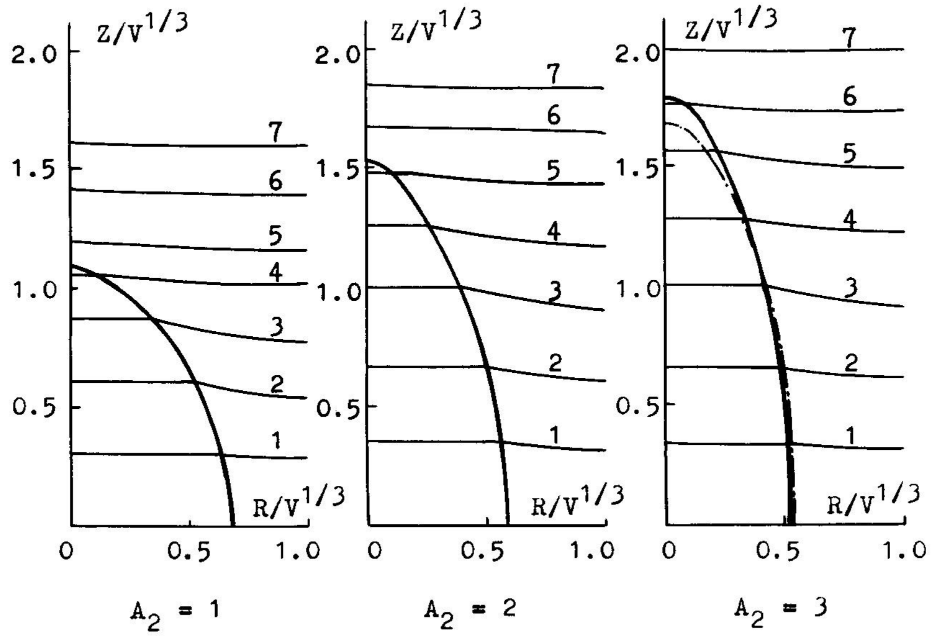

Fig. 1. Shapes of free fluid surfaces and potential isolines with changing magnetic field intensity. 1 – $\Phi/H_0V^{1/3} = 0.2$, 2 – 0.4, 3 – 0.6, 4 – 0.8, 5 – 1.0, 6 – 1.2, 7 – 1.4.

References

[1] B.M. Berkovsky and V.K. Polevikov, Magnetic Hydrodynamics, N4 (1983) 60.

[2] B.M. Berkovsky and N.N. Smirnov, Magnetic Hydrodynamics, N1 (1984) 15.

[3] E.Ya. Blum, Yu. A. Mikhailov and R.Ya. Ozols, Heat and Mass Transfer in Magnetic Field (Zinatne, Riga, 1980) p. 355.

[4] V.G. Bashtovoy, B.M. Berkovksy and A.N. Vislovich, Introduction to Magnetic Fluid Thermomechanics (IVTAN, Moscow, 1985) p. 188.

[5] A.A. Samarsky and V.B. Andreyev, Difference Methods for Elliptic Equations (Nauka, Moscow, 1976) p. 352.

[6] V.G. Bashtovoy, A.M. Budnik, V.K. Polevikov and A.G. Reks, Magnetic Hydrodynamics N2 (1984) 47.