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Chapter 1. Mathematical Modeling in Physics and Engineering

Crown Forest Fire Mathematical Model Realization in Wolfram Mathematica

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Abstract. *Adapted mathematical model of running crown forest fire propagation is considered. Simplifying assumptions, equations of the model, initial and boundary conditions, finite difference approximations are introduced. Aspects of software realization of mathematical model, numerical results graphical visualization and storage in a database are discussed.*

1 Introduction

Interest in mathematical models of forest fires is caused by a large number of scientific difficulties. It is noted in [1, 4, 6, 11] that forest fire models have been developed since 1940 to the present, but a lot of chemical and thermodynamic questions related to fire behaviour are still to be resolved. Forest fires are divided into underground (peatbog) fires, surface fires, active crown fires, running crown fires (also called independent crown fires), and mass fires [1, 6]. A number of researchers notice, that running crown fires possess the greatest speed of propagation. They are extremely dangerous and very difficult to fight, thus mathematical modelling is represented as an important problem [1, 6, 11]. The state-of-the-art of forest fire modelling is reviewed in detail in [11]. It's indicated that there are only a few ([11], table 7) successful computer software for crown forest fire calculation, based on the crown fire spread empirical model of Rothermel and the Forestry Canada Fire Danger Group's semiempirical model.

It is possible to note several weaknesses of existing forest fire models: a lack of theoretical justification of accepted mathematical descriptions of physicochemical reactions and transformations; unavailability of analytical reference solutions of boundary value problem; the need of efficient numerical approximations and high speed algorithms construction.

The current state of IT opens the opportunity to develop and formulate forest fire mathematical models, which take into account processes of heat and mass transfer, radiation, chemical reactions and transformations. Such models require significant computing power; their use involves quite lengthy calculation time even on high speed computers. I.e. their direct application in real-time systems is impossible. The practical use of forest fire mathematical models in decision support systems can be realized in two directions: filling of a knowledge base [3] of computed forest fire propagation forecasts or creation of simplified (empirical) models for real-time calculations. Realization of similar problems using high-level programming languages (C++, Delphi, etc.) requires the involvement of experts from various fields of IT, namely: programming of analytic transformations and numerical calculation algorithms, graphical visualization of results, database formation and maintenance. On the other hand all these problems can be solved using modern computer algebra systems, which include graphical representation and database management tasks as standard services and user defined algorithms can be programmed either.

The most important theoretical model of the crown fire spread is developed by Grishin ([11]). It is based on the fundamental laws of physics, conservation; theoretical justifications are provided. In the present article we are going to show the implementation of this model in the software complex. Theoretical and methodological foundations of running crown forest fire model realization and of according knowledge base formation are described. We suggest Grishin's mathematical model extension and adaptation, refined descriptions of some model's components. New method of finite difference approximation for the boundary value problem is used.

The strongest change in environmental condition parameters occurs in the zone called fire front, which propagates at some speed across the territory, covered with forest, and is visually observed as a zone captured by a flame. As a rule the effects of smoke generation over a big territory and clouds formation over a fire zone, because of condensation of the water vapor released during combustible forest materials (CFM) burning, take place.

Describing the geometry of forest fire propagation there are usually mentioned in a horizontal direction: a region of the burnt area, a burning zone, not burnt area; in a vertical cut: a bottom forest layer, a flame, a forest canopy (tree crowns), boundary layer of the atmosphere ([1, 7, 9, 10]). Among physical and chemical processes of CFM burning, the following are defined: CFM heating, drying, pyrolysis and burning, oxidation of the gaseous, condensed and disperse products of pyrolysis, aeration of soot particles from coke, aeration of ashes particles and a smoke formation. The general scheme of change of an aggregate state and a chemical composition in a forest fire zone is described, for example in [7]: the heat supply due to convection, heat conductivity and radiation causes change in condition and composition of CFM. During combustion are formed and present in gaseous and dispersed conditions: carbon monoxide (CO), hydrogen (H_2), methane (CH_4), carbon dioxide (CO_2), water vapor (H_2O), soot particles (C). In a solid phase during generation of coke, which includes carbon (C) and a mineral part of CFM, are taking place: formation of ashes because of oxidation of coke and smoke particles in gaseous and dispersed phases, generation of soot particles due to pyrolysis and aeration.

2 Equations of mathematical model

Let us examine a problem of running crown forest fire propagation. We will assume that wind velocity, ambient temperature, geometrical, structural and reactionary properties of the forest canopy, temperature and the sizes of the ignition center are known.

The following assumptions are usually made: environment is considered to be a five-phase porous medium, consisting of dry organic substance, dispersed water, solid pyrolysis product (coke), ash and a gas phase; the gas phase consists of oxygen, combustible pyrolysis product components, inert air components and also water vapor and inert products of burning; the gradient of temperature across the forest canopy is small in a comparison with a temperature gradient in a longitudinal direction; influence of Coriolis force and centrifugal force is small in comparison with gravity ([7]). The mathematical model taking into account the listed assumptions is described by the system of equations, concerning which the author notices complexity and that its practical application is not always possible.

In the work presented here, the model of running crown forest fires propagation is studied with following additional assumptions: pressure is considered constant; wind velocity in the forest canopy basically depends on forest phytocenosis structure parameters and poorly depends on coordinates and characteristics of the forest fire front itself, it is accepted to be equal to the equilibrium velocity, calculated by the formula (4.2.7) [7]; differences in thermal (and diffusive) streams between the top and bottom borders of a forest canopy are approximated by the Newton's formulas [7, 12]; heat inflow into a forest canopy due to radiation from a flame torch is negligible.

Then the one-temperature mathematical model of a running crown fire can be written down in the form of the following system of the partial differential equations:

$$\frac{\partial \varphi_1}{\partial t} = \Phi_{\varphi 1}(\varphi_1, T), \tag{1}$$

$$\frac{\partial \varphi_2}{\partial t} = \Phi_{\varphi 2}(\varphi_2, T), \tag{2}$$

$$\frac{\partial \varphi_3}{\partial t} = \Phi_{\varphi 3}(\varphi_1, \varphi_3, c_1, c_2, T), \tag{3}$$

$$\frac{\partial \varphi_4}{\partial t} = 0, \tag{4}$$

$$\frac{\partial c_1}{\partial t} + (V, grad c_1) - \frac{1}{\rho_5} div(\rho_5 D_T grad c_1) = \Phi_{c1}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{5}$$

$$\frac{\partial c_2}{\partial t} + (V, grad c_2) - \frac{1}{\rho_5} div(\rho_5 D_T grad c_2) = \Phi_{c2}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{6}$$

$$\frac{\partial T}{\partial t} + \frac{\rho_5 c_{p5}(V, grad T) - div(\lambda_T grad T)}{\rho_5 c_{p5} + \sum_{j=1}^{4} \rho_j \varphi_j c_{pj}} = \Phi_T(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T), \tag{7}$$

$$\Phi_{\varphi 1}(\varphi_1, T) = -\frac{R_1}{\rho_1}, \qquad \Phi_{\varphi 2}(\varphi_2, T) = -\frac{R_2}{\rho_2}, \tag{8}$$

$$\Phi_{\varphi 3}(\varphi_1, \varphi_3, c_1, c_2, T) = \frac{\alpha_c R_1}{\rho_3} - \frac{M_c}{M_1}\frac{R_3}{\rho_3}, \tag{9}$$

$$\Phi_{c1}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{1}{\rho_5}\left(R_{51} - c_1 Q - \frac{\alpha}{c_{p5}\Delta h}(c_1 - c_{1\infty})\right), \tag{10}$$

$$\Phi_{c2}(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{1}{\rho_5}\left(R_{52} - c_2 Q - \frac{\alpha}{c_{p5}\Delta h}(c_2 - c_{2\infty})\right), \tag{11}$$

$$\Phi_T(\varphi_1, \varphi_2, \varphi_3, c_1, c_2, T) = \frac{q_5 R_5 - q_2 R_2 + q_3 R_3 - \frac{\alpha}{\Delta h}(T - T_\infty) - 4\kappa_R \sigma T^4}{\rho_5 c_{p5} + \sum_{j=1}^{4} \rho_j \varphi_j c_{pj}}, \tag{12}$$

$$\sum_{\nu=1}^{3} c_\nu = 1, \quad \rho_5 = \frac{\rho_\infty T_\infty}{M_\infty T}\left(\sum_{\nu=1}^{3}\frac{c_\nu}{M_\nu}\right)^{-1}, \quad Q = (1-\alpha_c)R_1 + R_2 + \frac{M_C}{M_1}R_3. \tag{13}$$

Here t is the time, x—coordinate in the system of coordinates connected with the center of an initial fire, the x-axis is directed towards unperturbed wind velocity parallel to a horizontal spreading surface; the coordinate z is counted upwards from a spreading surface; $V = (u, v, w)$—equilibrium wind velocity vector; T—temperature (in Kelvins), T_∞—unperturbed ambient temperature (in Kelvins); λ_T—turbulent thermal conductivity; φ_j, $j = 1, 2, 3, 4$—volume fractions of the multiphase reactive medium, where φ_1 corresponds to dry organic substance, φ_2—water in liquid-drop state combined with CFM, φ_3—coke (condensed pyrolysis product), φ_4—mineral part of CFM (ash); ρ_j, $j = 1, 2, 3, 4$—j^{th} phase density; ρ_5—density of a gas phase (a mix of gases), ρ_∞—unperturbed density of a mix of gases (air density); c_ν, $\nu = 1, 2, 3$—mass concentration of components of a gas phase, where c_1 corresponds to oxygen (O_2), c_2—to combustible gases (combustible pyrolysis product components), c_3—mixes of other gases (inert components of air, water vapor, inert products of reactions of pyrolysis, coke burning and of combustible gases oxidation); $c_{1\infty}$ and $c_{2\infty}$—mass concentrations of oxygen and combustible gases in unperturbed atmosphere; M_ν, $\nu = 1, 2, 3$—molecular mass of gas phase components; M_C—molecular mass of carbon, M_∞—molecular mass of air; R_1, R_2, R_3—mass rates of reactions of dry CFM pyrolysis (chemical decomposition of substance by heating with allocation of combustible gases), of moisture evaporation from CFM (drying), coke burning; R_{51}, R_{52}, Q—mass rates of generation (disappearance) of oxygen, combustible gases, gas phase; R_5—mass rate of reaction of burning (oxidation) of combustible gases; c_{pj}, $j = 1, 2, 3, 4$—j^{th} phase thermal capacity; c_{p5}—thermal capacity of a gas phase; q_2, q_3 and q_5—heat effects of processes of evaporation, of burning of the condensed fuel and of gaseous combustible pyrolysis products accordingly; D_T—diffusion coefficient; Δh—crown height ($\Delta h = h_3 - h_2$, where h_3 and h_2 are heights of the top and bottom borders of forest canopy accordingly); α—coefficient of heat exchange between atmosphere and a forest canopy; α_c—the coke number of CFM; κ_R—integrated absorptance; σ—Stefan-Boltzmann constant.

Equations (1)–(13) are written above in a form which differs from the traditional one ([7]), some changes took place, some equation members are grouped together to identify the physical processes described. The form presented (derivative in time, convection, diffusion, and the right-hand side) simplifies the understanding of finite difference approximations which are used below. In such mathematical notation the algorithm is realized in the computer program made in ***Wolfram Mathematica*** system. The accepted form of the equations is also convenient because it makes more understandable where and what nonlinearities take place.

3 Initial and boundary conditions

Boundary conditions are set as follows ([7]):

$$T|_{t=0} = T_0(P), \qquad T(\pm\infty) = T_\infty; \tag{14}$$

$$c_1|_{t=0} = c_{10}(P), \qquad c_1(\pm\infty) = c_{1\infty}; \tag{15}$$

$$c_2|_{t=0} = c_{20}(P), \qquad c_2(\pm\infty) = c_{2\infty}; \tag{16}$$

$$c_3|_{t=0} = 1 - c_{10}(P) - c_{20}(P), \qquad c_3(\pm\infty) = 1 - c_{1\infty} - c_{2\infty}; \tag{17}$$

$$\varphi_1|_{t=0} = \varphi_{10}(P), \qquad \varphi_1(-\infty) = \varphi_{1K}, \qquad \varphi_1(+\infty) = \varphi_{1H}; \tag{18}$$

$$\varphi_2|_{t=0} = \varphi_{20}(P), \qquad \varphi_2(-\infty) = \varphi_{2K}, \qquad \varphi_2(+\infty) = \varphi_{2H}; \tag{19}$$

$$\varphi_3|_{t=0} = \varphi_{30}(P), \qquad \varphi_3(-\infty) = \varphi_{3K}, \qquad \varphi_3(+\infty) = \varphi_{3H}; \tag{20}$$

$$\varphi_4|_{t=0} = 0. \tag{21}$$

Here P—forest fire zone, $-\infty$ and $+\infty$ indicate burnt and unburned areas accordingly removed from fire zone to adequate distance (unperturbed values $c_{1\infty}$, $c_{2\infty}$ and T_∞).

Reasons about the nature of reactions, phase transitions, physical and chemical transformations in a part of forest untouched by fire processes require addition of the following conditions ([7]):

$$R_1 = 0, \qquad R_2 = 0, \qquad R_3 = 0, \qquad R_5 = 0 \qquad \text{at } T_\infty \le T \le T*, \tag{22}$$

where $T*$—given value of temperature. In other words, processes described in model do not occur, until the temperature reaches the certain level.

For a given modeled forest region it is possible to define the density of dry organic substance ρ_1, the bulk density ρ_0 of typical layer of CFM and it's moisture content W, calculated under the formula:

$$W = \frac{m_0 - m_1}{m_1}, \tag{23}$$

where m_0 and m_1—mass of CFM in natural and absolutely dry conditions accordingly. Then the initial values of volume fractions are calculated by the formulas:

$$\varphi_{1H} = \frac{\rho_0}{\rho_1}, \qquad \varphi_{2H} = \frac{\rho_0}{\rho_2}(1-\zeta)W, \qquad \varphi_{3H} = 0. \tag{24}$$

ζ - ash content of combustible forest materials. According to help tables [7] values of ash content range within $0.001 < \zeta < 0.01$. It is noticed, that ash content account on modeled processes is of a little influence, therefore further we will accept $\varphi_4 = 0$. $\varphi_{3H} = 0$ since in an unburnt zone coke is not formed yet. Using the assumption that CFM are completely burnt down, relations for final volume fractions can be described by the formulas:

$$\varphi_{1K} = 0, \qquad \varphi_{2K} = 0, \qquad \varphi_{3K} = \frac{\rho_1}{\rho_3}\alpha_c\varphi_{1H}. \tag{25}$$

For a case of one-dimensional process when x-axis focuses the wind direction, the types of initial distributions T_0, c_{10}, c_{20}, φ_{10}, φ_{20}, φ_{30} assumed in the performed calculations are presented in Fig. 1. Such initial distributions of temperature, volume fractions and component concentrations are confirmed by the results of physical modelling of crown forest fires propagation, and also by the results of calculations on the "self-coordinated mathematical model" given in [7]. The formulated initial conditions correspond to characteristics of forest fires in case when ignition has occurred in the zone of the finite size, and further burning extends by different possible scenarios: a switch to the established regime or to an extinction of a fire.

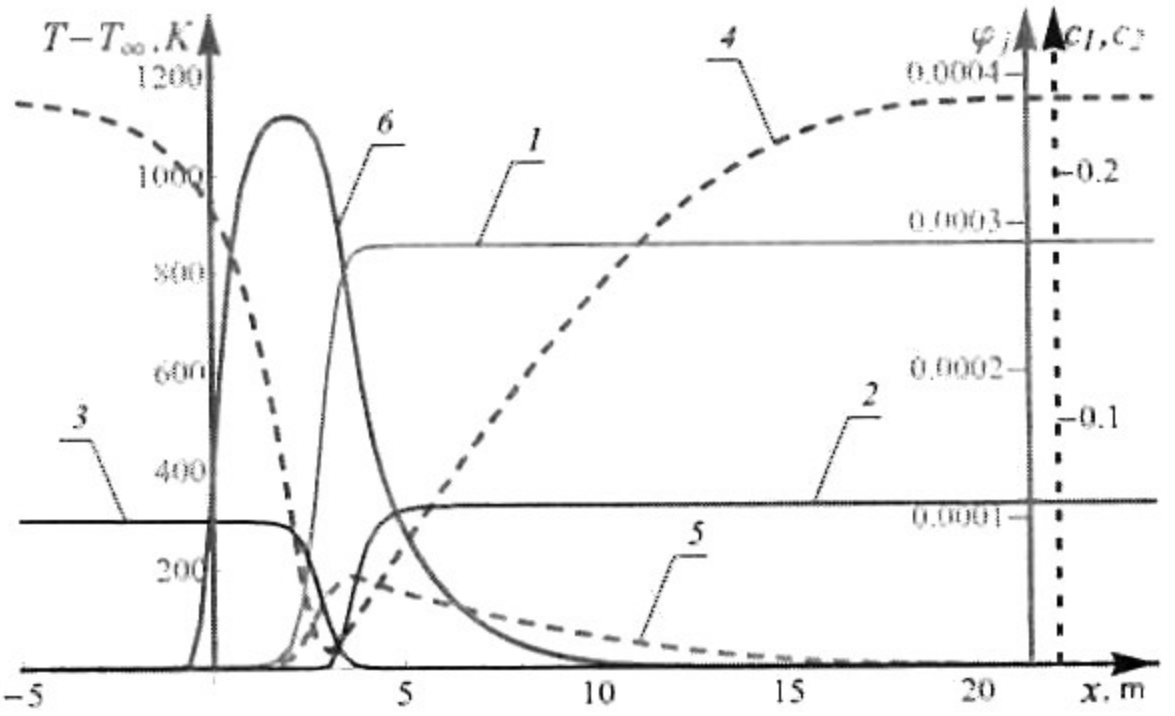

Figure 1: Initial distributions of volume fractions of the multiphase environment and concentration of gas phase components. Numbers *1*, *2*, *3* note the curves corresponding to dry organic substance of combustible forest materials, water in an liquid-drop condition, coke; *4*, *5* illustrate concentration of oxygen, carbon monoxide; *6* temperature

4 Corrected mathematical model

To complete the system of equations it is necessary to write down the dependences describing the speeds of CFM pyrolysis, drying, coke burning and chemical reactions

in a gas phase. For example, according to [7, 9]:

$$R_1 = k_{01}\rho_1\varphi_1 \exp(-\frac{E_1}{RT}), R_2 = k_{02}T^{-1/2}\rho_2\varphi_2 \exp(-\frac{E_2}{RT}), \tag{26}$$

$$R_3 = k_{03}s_\sigma\varphi_3\rho_5 c_1 \exp(-\frac{E_3}{RT}), \tag{27}$$

$$R_{51} = -R_3 - \frac{R_5 M_1}{2M_2}, R_{52} = (1-\alpha_c)\nu_1 R_1 - R_5, \tag{28}$$

where k_{01}, k_{02}, k_{03}—pre-exponential factor of chemical reactions ([6, 16]), E_1, E_2, E_3—activation energy of chemical reactions, R—universal gas constant, s_σ—specific surface of the condensed product of pyrolysis (of coke), ν_1—the proportion of gaseous combustible pyrolysis products.

An important component of the formulated mathematical model is the expression for gaseous pyrolysis products burning speed R_5. There is no uniform approach on this question in the scientific literature. Often it is specified ([7, 9, 16]) that it is possible to consider the only reaction

$$\mathrm{CO} + \frac{1}{2}\mathrm{O_2} = \mathrm{CO_2} + 68\ \mathrm{kcal/mol} \tag{29}$$

as defining, since it brings the greatest energy release during the forest fire.

In [12] the features of the mentioned reaction are marked: "Dry (without water vapor) $CO + O_2$ react with a low speed. However, speed of reaction becomes fast at the addition of a small amount of water vapor or hydrogen to a mix. Radical OH and atomic hydrogen H (and oxygen O) serve as the primary active centers. Reaction of continuation of a chain with simultaneous CO_2 generation is the reaction $CO+OH \rightarrow CO_2+H$. ... Practically during combustion of natural fuels water vapor and hydrogen are always present at mixes and provide high speed of CO burning reaction". In other words, the formulas describing speed of reaction (29) require specification.

To describe the speed of reaction (29) in [7] two different expressions taken from various sources are offered. They essentially differ. Possibility of practical use of each of them raises the doubts, moreover the analysis of the formula (5.4.19) [7] shows the discrepancy of measuring system units in the parts describing the same reaction at various concentration of oxygen.

In [9] the kind of the formula and value pre-exhibitors of reaction (29) are taken from [12], but the value of activation energy is taken from [7, 8]. The formula is:

$$R_5 = \rho_5 c_2 k_{CO} \exp(-\frac{E_{CO}}{RT}),\ k_{CO} = 7.05 \cdot 10^6 s^{-1},\ E_{CO} = 96800 \cdot R\ \mathrm{K}. \tag{30}$$

The mathematical model adapted and applied further consists of the equations (1)–(12), (26)–(28), boundary conditions (14)–(21), and also the formula for R_5. If there is not enough oxygen for total burning out of all carbon monoxide, using formula (30) for reaction R_5 description gives physically incorrect result—concentration of oxygen can reduce below zero level. At any (including very small) values c_1, speed

of burning of combustible gases and destruction of the oxygen, described by expression (30) can be very high.

The computational experiments made for model adaptation give the basis to introduce the new formula for reaction (29).

Using a method of stationary concentrations [16] for chemical reaction of carbon monoxide burning (29), concentration c_2 can be expressed through stationary concentration c_1 as follows:

$$c_2 = \frac{M_2}{2M_1} c_1. \tag{31}$$

The given formula means that for total combustion of carbon monoxide (CO) which mass concentration equals c_2 in some volume, the required mass concentration in the same volume of oxygen (O_2) is defined by the right-hand part of the expression (31).

Using (31) the adaptation of the formula (30) is offered in which process of carbon monoxide burning goes by an "excess/deficiency" principle:

$$R_5 = \rho_5 \min(c_2, \frac{M_2}{2M_1} c_1) k_{CO} \exp(-\frac{E_{CO}}{RT}). \tag{32}$$

The process description by expression (32) offered instead of expression (30) works fine also for an important case of very low oxygen concentration. Note expression (30) does not include the oxygen concentration c_1. Therefore in the mathematical model a situation of combustion without oxygen can happen which is physically impossible. It should be noted that function of minimum is not used in any known published work on the theory of combustion; the delimitation of its applicability requires further study. Performed computational experiments indicate that the use of formula (32) allows to avoid reduction of oxygen concentration below zero, and at the same time it adequately models the running crown fire propagation limiting conditions by the wind velocity, CFM density and moisture content.

5 Finite difference approximations

PDE system of mathematical model written above does not have an analytical solution. It is impossible not only for multi-dimensional case provided, but even for one-dimensional process approximation. Therefore, the problem is solved numerically using the explicit finite difference approximations:

$$\varphi_{1i}^{n+1} = \varphi_{1i}^{n} \frac{2 - z_{1i}^{n}}{2 + z_{1i}^{n}}, \qquad z_{1i}^{n} = \Delta t \cdot k_{01} \exp(-\frac{E_1}{RT_i^n}). \tag{33}$$

$$\varphi_{2i}^{n+1} = \varphi_{2i}^{n} \frac{2 - z_{2i}^{n}}{2 + z_{2i}^{n}}, \qquad z_{2i}^{n} = \Delta t \cdot k_{02} (T_i^n)^{-1/2} \exp(-\frac{E_2}{RT_i^n}). \tag{34}$$

$$\varphi_{3i}^{n+1} = \varphi_{3i}^{n} \frac{2 - z_{3i}^{n}}{2 + z_{3i}^{n}} + (\varphi_{1i}^{n+1} + \varphi_{1i}^{n}) \frac{\rho_1}{\rho_3} \frac{\alpha_c z_{1i}^{n}}{2 + z_{3i}^{n}}, \tag{35}$$

$$z_{3i}^n = \Delta t \cdot k_{03} \rho_{5i}^n s_\sigma c_{1i}^n \exp(-\frac{E_3}{RT_i^n}), \tag{36}$$

$$\rho_{5i}^n = \rho_{5i}^n(c_{1i}^n, c_{2i}^n, T_i^n) = \frac{\rho_\infty T_\infty}{M_\infty T_i^n} \left(\frac{c_{1i}^n}{M_1} + \frac{c_{2i}^n}{M_2} + \frac{1 - c_{1i}^n - c_{2i}^n}{M_3} \right)^{-1}. \tag{37}$$

Equations for concentration and temperature are approximated as follows:

$$c_{1i}^{n+1} = c_{1i}^n + \Delta t [-u \frac{c_{1i+1/2}^n - c_{1i-1/2}^n}{\Delta x} + \frac{D_T}{\rho_{5i}^n (\Delta x)^2} (\rho_{5i+1/2}^n (c_{1i+1}^n - c_{1i}^n) -$$

$$- \rho_{5i-1/2}^n (c_{1i}^n - c_{1i-1}^n)) + \Phi_{c1}(\varphi_{1i}^n, \varphi_{2i}^n, \varphi_{3i}^n, c_{1i}^n, c_{2i}^n, T_i^n)],$$

$$T_i^{n+1} = T_i^n + \Delta t (- \frac{1}{\rho_{5i}^n c_{p5} + \sum_{j=1}^{4} \rho_j \varphi_{ji}^n c_{pj}} [\rho_{5i}^n c_{p5} u \frac{T_{i+1/2}^n - T_{i-1/2}^n}{\Delta x} -$$

$$- \frac{\lambda_T}{\Delta x^2} (T_{i+1}^n - 2T_i^n + T_{i-1}^n) + \Phi_T(\varphi_{1i}^n, \varphi_{2i}^n, \varphi_{3i}^n, c_{1i}^n, c_{2i}^n, T_i^n)].$$

The approximations listed describe the one-dimensional problem; the generalization for a multi-dimensional case is obvious because the explicit difference schemes are applied. The values of half the nodes $i + 1/2$, $i - 1/2$ are calculated by the formula of the two-point upstream weighting ([14, 13]).

Methodical calculations were conducted to justify and optimize the used method of solving the boundary value problem. Coefficients and independent members quasi-linearization methods, grid fragmentation impact on the accuracy were examined. Two-point upstream weighting formula was used to improve the accuracy of computed solutions near fronts. Comparison of numerical solutions obtained with two-point, central- and backward-difference approximation formulas showed that it is preferable to use for forest fire model equations the two-point upstream weighting scheme in version [13], so as in the problems of physicochemical methods for enhancing oil recovery.

When debugging the computer model of crown forest fire propagation the accuracy was studied by comparing with the experimental results of the Institute of Forest and Wood of the USSR Academy of Sciences and of Tomsk State University published in scientific and technical literature, and by comparing with information on Van Wagner's experiments, conducted in natural conditions at the plantations of "red pine" [7, 15]. In particular, the calculations reproduce conditions in a zone of crown fire, including the characteristics of fire front dynamics during establishing to a regime of steady propagation. Computer modelling also reproduce with high accuracy fire front rate of speed for all the cases examined, corresponding to different wind velocities.

An "internal" control was also carried out. Namely, solutions with different temporal and spatial steps were obtained and compared. The use of explicit approximations imposes severe restrictions on the time step size ([5, 13]). Control of

numerical instability and misconvergence ("strong" nonlinearity of right sides in the equations for temperature and oxygen concentration) was performed by analyzing the graphics of solutions. Application of numerical experiments conduction technology using toolkit developed for knowledge base creation and support [3] simplified this control greatly.

6 Software supporting technology

The proposed computer model involves multivariant calculations. Those are needed to identify the qualitative features of the process and in some cases specific quantitative characteristics. On the other hand even the very first calculations showed that to obtain the final results in some cases it's required to refine the grid step size at some intermediate stages. An additional aspect of the numerical scheme correction is the need to prepare self-consistent distributions of initial conditions. All these requirements can be resolved if calculation results are stored in a database. Then we can, for example, find, extract and use intermediate results from a database as initial distributions for continuing with different parameters (physical constants or grid step values). For storing the results of mathematical modelling of forest fire distribution in different moments of time the specialized organization of a relational database including three tables is offered in [2, 3] (see Fig. 2).

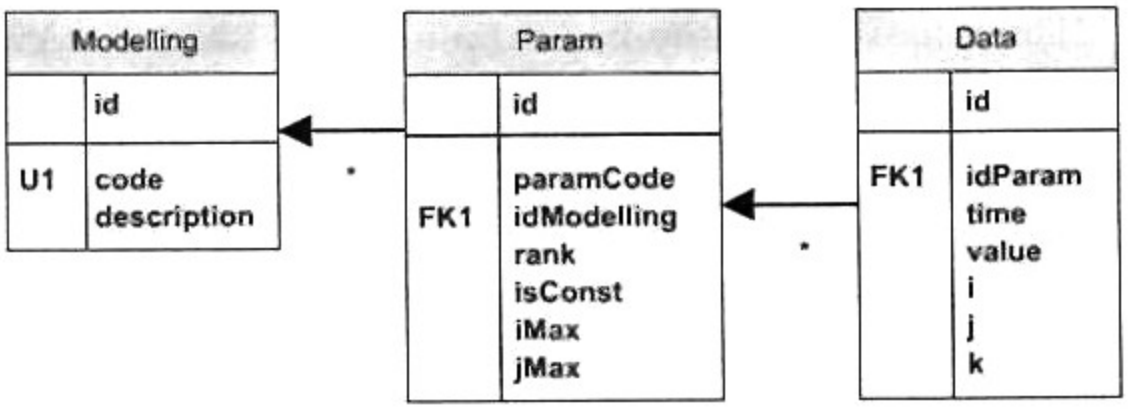

Figure 2: ER Diagram of Mathematical Modelling Results Database

For work with a database satisfying the scheme offered a special **Wolfram Mathematica** module is developed. It uses standard functions of **DatabaseLink** packages. **OpenSQLConnection/CloseSQLConnection** establishes/closes a connection to a database (ODBC source). **SQLExecute, SQLInsert, SQLSelect, SQLDelete** are used to make database requests.

It is necessary to note universality of the offered scheme of knowledge base organization, the possibility of its application for storage mathematical modelling results of various problems. Advantages of the given library are independency of a database type (Oracle, Microsoft SQL Server, Access, MySQL or any other relational database), and also simplicity in use. You do not need to learn about databases or expert systems. Methods of data storage are hidden from the user. All necessary operations of saving, searching and retrieving data from the database are carried

out by the functions of this module. To connect to a database user just indicates the name of the data source as well as a modelling result verbal code. To store and retrieve data simple functions are used.

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