A NOTE ON THE SPIN ALGEBRA IN A FRACTIONAL SPACE

P. Miškinis

Vilnius Gediminas Technical University,
Saulėtekio Ave. 11, LT-10223, Vilnius, Lithuania
paulius.miskinis@fm.vgtu.lt

In 1927, W. Pauli proposed to consider the electron in the magnetic field as a particle that has a spin, *i.e.* a moment of its own. To the quantum value, spin s, corresponds the operator of spin $\hat{s}$ whose components are expressed by Pauli matrices [1]:

$$\hat{s}_i = \frac{1}{2}\sigma_i, \quad \sigma_i\sigma_k = -\sigma_k\sigma_i, \quad \sigma_i^2 = I. \tag{1}$$

Independently of a concrete representation, the spin algebra meets the following commutation relations [2]:

$$[\hat{s}_i, \hat{s}_j] = i\varepsilon_{ijk}\ \hat{s}_k, \quad [\hat{s}_i, \hat{s}^2] = 0. \tag{2}$$

What will be the changes the commutation relations (2) undergo in a space of non-whole dimension?

The author shows that in the case when the effective dimension of space is not integer and may acquire fractional values [3], the electron becomes a particle with the spin projection $s_z = 1/2^\alpha$, where $\alpha \in [0;1]$ and commutation relations for spin operators of fractional power

$$\left[\hat{s}_i^\alpha, \hat{s}_j^\alpha\right] = \frac{i}{2^{2(\alpha-1)}}\sin^4\left(\frac{\pi\alpha}{2}\right)\varepsilon_{ijk}\ \hat{s}_k, \quad [\hat{s}_i^\alpha, \hat{s}^2] = 0\,. \tag{3}$$

The above peculiarities of electrons in a space of fractional dimension may contribute to theory of solid body, theory of superconductivity, the quantum effect of Hall, theory of phase transitions.

References

1. *Pauli W.* Wave Mechanics (Vol. 5 of Pauli Lectures on Physics). Unabridged edition: Dover Publications, 2000.

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3. *Mandelbrot B.* The Fractal Geometry of Nature. New-York: W.H. Freeman & Co, 1983.