GENERALIZED GEVREY ULTRADISTRIBUTIONS

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We first introduce a new differential algebras of generalized Gevrey ultradistributions denoted $\mathcal{G}^{\sigma}(\Omega)$. We show that $\mathcal{G}^{\sigma}(\Omega)$ contains the space of Gevrey ultradistributions of order $(3\sigma - 1)$. We then develop a Gevrey microlocal analysis adapted to these algebras. The starting point of the Gevrey microlocal analysis in the framework of the algebra $\mathcal{G}^{\sigma}(\Omega)$ consists first in introducing the algebra of regular generalized Gevrey ultradistributions $\mathcal{G}^{\sigma,\infty}(\Omega)$ then to define, with the help of the Fourier transform, the generalized Gevrey wave front of $f \in \mathcal{G}^{\sigma}(\Omega)$, denoted $WF_g^{\sigma}(f)$, and further to give its main properties.

Finally, we give an application of the introduced generalized Gevrey microlocal analysis. The product of two generalized Gevrey ultradistributions always exists, but there is no final description of the generalized wave front of this product. Such problem is also still posed in the Colombeau algebra. The well-known Hörmander's result on the wave front of the product of two distributions, has been extended to the case of two Colombeau generalized functions. We show this result in the case of two generalized Gevrey ultradistributions, namely we obtain the following result : let $f, g \in \mathcal{G}^{\sigma}(\Omega)$, satisfying $\forall x \in \Omega,\ (x,0) \notin WF_g^{\sigma}(f) + WF_g^{\sigma}(g)$, then

$$WF_g^{\sigma}(fg) \subseteq \left(WF_g^{\sigma}(f) + WF_g^{\sigma}(g)\right) \cup WF_g^{\sigma}(f) \cup WF_g^{\sigma}(g).$$

References

1. *Antonevich A.B., Radyno Ya. V.* On a general method of constructing algebras of new generalized functions. Soviet. Math. Dokl. 1991. Vol. 43:3. P. 680- 684.

2. *Benmeriem K., Bouzar C.* Generalized Gevrey ultradistributions. Preprint.

3. *Colombeau J.F.* Elementary introduction to new generalized functions. North Holland, 1984.

4. *Grosser M., Kunzinger M., Oberguggenberger M., Steinbauer R.* Geometric theory of generalized functions with applications to general relativity. Kluwer Publishing, 2001.