THE INVESTIGATION OF STATIONARY THREE-DIMENSIONAL FLOWS OF INCOMPRESSIBLE RIVLIN-ERICKSEN VISCOUS FLUID BY THE METHODS OF COMPLEX ANALYSIS

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The system of four partial differential equations with respect to the three components of velocity vector V_1, V_2, V_3 and the hydrostatic pressure P, which includes the equations of motion and incompressibility condition is considered. Because of the dependence of the strain tensor on the Rivlin – Ericksen [1] tensors, the equations of motion take form of third-order partial differential equations. The flow is considered in a simply connected domain. The boundary conditions are as follows. On one part of the domain boundary the velocity vector is specified, the other part of the boundary is free.

In order to solve the boundary-value problem the equations are complexified. Two complex-valued functions $W_1 = V_1 + iV_2, W_2 = V_3 + iV_4$ as well as the complex-valued hydrostatic pressure P are introduced. Complex variables are determined through real variables as follows: $z_1 = x_1 + + ix_2, z_2 = x_3 + ix_4$. Because the sought complex-valued functions are assumed independent of the x_4 coordinate, the complexified system is supplemented by the conditions of independence of the functions W_1, W_2 and P from x_4 :

$$\frac{\partial P}{\partial z_2} + \frac{\partial P}{\partial \bar{z}_2} = 0; \tag{1}$$

$$\frac{\partial W_1}{\partial z_2} + \frac{\partial W_1}{\partial \bar{z}_2} = 0; \quad \frac{\partial \bar{W}_1}{\partial z_2} + \frac{\partial \bar{W}_1}{\partial \bar{z}_2} = 0; \quad \frac{\partial W_2}{\partial z_2} + \frac{\partial W_2}{\partial \bar{z}_2} = 0; \quad \frac{\partial \bar{W}_2}{\partial z_2} + \frac{\partial \bar{W}_2}{\partial \bar{z}_2} = 0; \tag{2}$$

For the sake of simplicity, the domain of variation of the two complex variables $(z_1, z_2) \in D \subset \subset C^2$, in which the obtained complexified system is considered is assumed to be a domain having a bicircular capsule of holomorphy. It is also assumed that the cross-section of this domain by the $x_4 = 0$ hyperplane is the considered three-dimensional domain of fluid flow. The solution is constructed in the following form:

$$P = \sum \psi_{k1,k2}(z_1, z_2)\bar{z_1}^{k1}\bar{z_2}^{k2}; \tag{3}$$

$$W_1 = \sum \varphi^1_{k1,k2}(z_1, z_2)\bar{z_1}^{k1}\bar{z_2}^{k2}; W_2 = \sum \varphi^2_{k1,k2}(z_1, z_2)\bar{z_1}^{k1}\bar{z_2}^{k2}; \tag{4}$$

$$\bar{W}^s_1 = \sum \varphi^{s1}_{k1,k2}(z_1, z_2)\bar{z_1}^{k1}\bar{z_2}^{k2}; \bar{W}^s_2 = \sum \varphi^{s2}_{k1,k2}(z_1, z_2)\bar{z_1}^{k1}\bar{z_2}^{k2}; \tag{5}$$

here $\psi_{k1,k2}; \varphi^1_{k1,k2}; \varphi^{s1}_{k1,k2}; \varphi^2_{k1,k2}; \varphi^{s2}_{k1,k2}$ are the holomorphic functions of two complex variables in the D domain.

The relationship between these holomorphic functions which is obtained from the complexified equations is supplemented by the conditions

$$\|ImP\|_D = 0; \quad \left\|W_1 - \bar{W}_1^s\right\|_D = 0; \quad \left\|W_2 - \bar{W}_2^s\right\|_D = 0; \qquad (6)$$

and by the demand of the minimum of the residual of the boundary conditions.

References

1. *Truesdell C.* A first course in rational continuum mechanics. The Johns Hopkins university, Baltimore, Maryland, 1972.