COEFFICIENT INVERSE PROBLEM FOR A PARABOLIC EQUATION IN A FREE BOUNDARY DOMAIN

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In the domain $\Omega_T = \{(x,t): \ h_1(t) < x < h_2(t), \ 0 < t < T\}$, where $h_1 = h_1(t)$, $h_2 = h_2(t)$ are unknown functions, we consider the inverse problem of finding unknown coefficients $b = b(t)$, $c = c(t)$ in the parabolic equation

$$u_t = a(x,t)u_{xx} + b(t)u_x + c(t)u + f(x,t), \quad (x,t) \in \Omega_T, \tag{1}$$

with the initial condition

$$u(x,0) = \varphi(x), \quad x \in [h_1(0), h_2(0)], \tag{2}$$

boundary conditions

$$u(h_1(t),t) = \mu_1(t), \quad u(h_2(t),t) = \mu_2(t), \quad t \in [0,T], \tag{3}$$

and overdetermination conditions

$$h_1'(t) = u_x(h_1(t),t) + \mu_3(t), \quad h_2'(t) = -u_x(h_2(t),t) + \mu_4(t),$$

$$\int\limits_{h_1(t)}^{h_2(t)} u(x,t)dx = \mu_5(t), \quad \int\limits_{h_1(t)}^{h_2(t)} x\,u(x,t)dx = \mu_6(t), \quad t \in [0,T], \tag{4}$$

where $h_1(0) = h_{01}$ is given.

Change the variables $y = \dfrac{x - h_1(t)}{h_2(t) - h_1(t)}$, $t = t$ we reduce the problem (1)–(4) to the problem with unknown functions $(b(t),\ c(t),\ h_1(t),\ h_3(t) = h_2(t) - h_1(t),\ v(y,t) = u(yh_3(t) + h_1(t), t))$ in the domain $Q_T = \{(y,t): \ 0 < y < 1, \ 0 < t < T\}$.

Theorem 1. *Suppose that the following conditions hold:*

1) $a \in C^{2,0}([h_{01},\infty) \times [0,T])$, $f \in C^{1,0}([h_{01},\infty) \times [0,T])$, $a(x,t) > 0$, $f(x,t) \geqslant 0$, $x \in [h_{01},\infty)$, $t \in [0,T]$, $\varphi \in C^1[h_{01},\infty)$, $\varphi(x) \geqslant \varphi_0 > 0$, $x \in [h_{01},\infty)$, $\mu_i \in C^1[0,T]$, $\mu_i(t) > 0$, $i = 1,2,5,6$, $\mu_2(t) - \mu_1(t) \geqslant 0$, $\mu_5(t) - H_1\mu_2(t) > 0$, $t \in [0,T]$, $\mu_j \in C[0,T]$, $j = 3,4$, *where* $H_1 = 2\max\limits_{[0,T]}\mu_5(t)(\min\{\min\limits_{[h_{01},h_{02}]}\varphi(x),\ \min\limits_{[0,T]}\mu_1(t),\ \min\limits_{[0,T]}\mu_2(t)\})^{-1}$, $h_{02} = h_2(0)$ *is a solution of the equation* $\int_{h_{01}}^{h_2(0)}\varphi(x)dx = \mu_5(0)$;

2) $\varphi(h_{01}) = \mu_1(0)$, $\varphi(h_{02}) = \mu_2(0)$.

Then we can indicate a number T_0, $0 < T_0 \leqslant T$, *depending on the data that there exists a unique solution* $(b,c,h_1,h_3,v) \in (C[0,T_0])^2 \times (C^1[0,T_0])^2 \times C^{2,1}(Q_{T_0}) \cap C^{1,0}(\overline{Q}_{T_0})$, $h_3(t) > 0$, $t \in [0,T_0]$ *of the problem (1)–(4).*

The proof of the theorem 1 are based on Shauder fixed-point theorem and the properties of Volterra integral equations of the second kind.