FREE BOUNDARY PROBLEM FOR QUASILINEAR HYPERBOLIC SYSTEM IN THE SCHAUDER CANONIC FORM

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In the domain $G_T^a = \{(x,t) \in \mathbf{R}^2 : 0 \leqslant t \leqslant T,\ a_1(t) \leqslant x \leqslant a_2(t)\}$ with unknown boundary $a(t) := (a_1(t), a_2(t))$ we shall consider quasilinear hyperbolic system of equations

$$\sum_{j=1}^{n} l_{ij}(x,t,u)\Big\{\frac{\partial u_j}{\partial t} + \lambda_i(x,t,u)\frac{\partial u_j}{\partial x}\Big\} = f_i(x,t,u), \quad u := (u_1,\ldots,u_n), \quad i = \overline{1,n}, \tag{1}$$

and the behaviour of the domain G_T^a boundary shall be limited by the system of differential equations

$$\frac{da_k(t)}{dt} = h_k(t, a(t), u(a(t), t)), \quad k = 1, 2, \quad u(a(t), t) := u(a_1(t), t), u(a_2(t), t). \tag{2}$$

The initial values of unknown functions are the set with conditions

$$a_k(0) = a_k^0, \quad k = 1, 2, \tag{3}$$

$$u_i(x, 0) = g_i(x), \quad a_1^0 \leqslant x \leqslant a_2^0. \tag{4}$$

Let us determine the sets

$$I_k = \{i : sgn[\lambda_i(a_k^0, 0, g(a_k^0)) - h_k(0, a^0, g(a^0))] = (-1)^{k+1}\}, \quad k = 1, 2,$$

and write the conditions at the limits of the domain

$$u_i(a_k(t), t) = H_k^i(t, a(t), \{u_s(a_k(t), t)\}_{s \in I_k}), \quad i \in I_{3-k}, \quad k = 1, 2. \tag{5}$$

In the vector-function space $(u, a) : G_T^a \times [0, T] \to \mathbf{R}^{n+2}$, $u \in (C(G_T^a))^n \cap (Lip_x(G_T^a))^n$, $a \in$ $\in (C^1[0, T])^2$ a theorem is proved on the existence and uniqueness of local and global solution to the problem (1)–(5).

The classical solvability of this problem is also considered.

The proof is based on the characteristics method and Banach theorem on contractions,applying the research in the works [1, 2].

References

1. *Andrusjak R.V., Kırılıch V.M., Myshkis A.D.* Local and global solvability of the quasilinear hyperbolic Stefan problem on the line(in the Russian) // Differentsialnye uravnenija. 2006. V. 42, No. 4. P. 489–503.

2. *Bassanini P., Turo I.* Generalized solutions to free boundary problems for hyperbolic systems of functional partial differential equations // Annali li matematika pura ed applicata. (IV), V.CLVI (1990). P. 211–230.