AN INVERSE PROBLEM FOR THE WEAKLY DEGENERATE PARABOLIC EQUATION IN A FREE BOUNDARY DOMAIN

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In the free boundary domain $\Omega_T = \{(x,t) : 0 < x < h(t),\ \ 0 < t < T,\}$ where $h = h(t) > 0, t \in [0,T]$ is an unknown function we consider an inverse problem of identification the time dependent coefficient $a = a(t) > 0,\ \ t \in [0,T]$ in the parabolic equation

$$u_t = a(t)\psi(t)u_{xx} + b(x,t)u_x + c(x,t)u + f(x,t), \quad (x,t) \in \Omega_T \tag{1}$$

with initial condition

$$(x,0) = \varphi(x), \quad 0 \leqslant x \leqslant h(0), \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t),\ u(h(t),t) = \mu_2(t), \quad 0 \leqslant t \leqslant T \tag{3}$$

and overdetermination conditions

$$a(t)\psi(t)u_x(0,t) = \mu_3(t), \quad 0 \leqslant t \leqslant T, \tag{4}$$

$$\int\limits_0^{h(t)} u(x,t)dx = \mu_4(t), \quad 0 \leqslant t \leqslant T. \tag{5}$$

We assume, that $\psi = \psi(t) > 0$ is an increasing function and $\psi(0) = 0$. There was investigated the case of the weak degeneration when

$$\int\limits_0^t \left(\int\limits_\tau^t \psi(\sigma)d\sigma \right)^{-1/2} d\tau \to 0 \ \text{ as } \ t \to 0.$$

A triplet of functions $(a,h,v) \in C[0,T] \times C^1[0,T] \times C^{2,1}(Q_T) \cap C^{1,0}(\overline{Q}_T)$, $a(t) > 0, h(t) > 0, t \in [0,T]$ is called a solution to the problem (1)–(5) when it satisfies the conditions (1)–(5).

We established the conditions of the existence of the solution to the problem (1)–(5) on the base of Schauder fixed point theorem and the conditions of uniqueness using the properties of the solutions to the homogeneous Volterra integral equations of the second kind.