STABILITY OF CANONICAL PERIODIC MATRIX IMPULSIVE DIFFERENTIAL EQUATIONS

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In our report we consider canonical periodic matrix impulsive differential equation

$$dZ/dt - i\mathcal{J}\mathcal{A}(t)Z = 0, \quad t \neq t_j; \quad \triangle(Z) = i\mathcal{J}\mathcal{D}_j X, \quad t = t_j, \tag{1}$$

where i is complex identity, $Z \in C_2^{n\times m}$, $C^{n\times m}$ is the space of complex $n \times m$ matrices, $Z = (X,Y)^T$, $X, Y \in C^{n\times m}$. $\mathcal{J} = \mathcal{J}^*$. $\mathcal{J}^{-1} = \mathcal{J}$, $\mathcal{J} = \mathcal{P}_1 - \mathcal{P}_2$, $\mathcal{P}_i$ are projection operator in $C_2^{n\times m}$, $\mathcal{P}_1 Z = X$, $\mathcal{P}_2 Z = Y$, $\mathcal{A} = \begin{pmatrix} [A_{11}] & [A_{12}] \\ [A_{21}] & [A_{22}] \end{pmatrix}$, $\mathcal{D}_j = \begin{pmatrix} 0 & 0 \\ 0 & -[D_j] \end{pmatrix}$, $\mathcal{A}^*(t) = \mathcal{A}(t)$, $[D_j]Y = D_j Y \widetilde{D}_j$, $[A_{i1}]X = A_{i1} X \widetilde{A}_{i1}$, $[A_{i2}]Y = A_{i2} Y \widetilde{A}_{i2}$, $A_{i1}, D_j \in C^{n\times n}$, $\widetilde{A}_{i2}, \widetilde{D}_j \in C^{m\times m}$, $i = 1,2$, $\|Z\| = \sqrt{\mathrm{Tr}\,(X^*X) + Tr(Y^*Y)}$. The equations (1) may be rewritten as one impulsive equation [1] in double phase space $C_2^{n\times m} = C^{n\times m} \oplus C^{n\times m}$

$$dZ/dt = i\mathcal{J}\big(\mathcal{A}(t) + \sum_j \mathcal{D}_j \delta(t - t_j)\big)Z. \tag{2}$$

In more general case $\mathcal{J} = \mathrm{sign}\,\mathcal{W} = \mathcal{W}|\mathcal{W}|^{-1}$, $|\mathcal{W}| = (\mathcal{W}^*\mathcal{W})^{(1/2)}$. In real double Hilbert space $\mathcal{H}^{(2)} = \mathcal{H} \oplus \mathcal{H}$ the role of operator $(i\mathcal{J})$ play operator $\mathcal{J}_\Gamma = \begin{pmatrix} 0 & [I] \\ -[I] & 0 \end{pmatrix}$, so-called symplectic identity in real double Hilbert space $\mathcal{H}^{(2)}$. The equation

$$dZ/dt = \mathcal{J}_\Gamma\big(\mathcal{A}(t) + \sum_j \mathcal{D}_j \delta(t - t_j)\big)Z, \tag{3}$$

than is named Hamiltonian equation.

The monodromy operator $\mathcal{U}(T)$ of equation (1) is $\mathcal{J}$-unitary, i.e.

$$\mathcal{U}^*(T)\mathcal{J}\mathcal{U}(T) = \mathcal{J}. \tag{4}$$

The stability of equation (1) means that the monodromy operator $\mathcal{U}(T)$ is stable [2].

Theorem 1. *For the equation (1) to be stable necessary and sufficient that the double Hilbert space* $\mathcal{H}^{(2)}$ *be decomposed to* $\mathcal{J}$*-orthogonal subspaces* $\mathcal{H}_1$ *and* $\mathcal{H}_2$; $\mathcal{H}^{(2)} = \mathcal{H}_1 \oplus \mathcal{H}_2$, *which are invariant for the monodromy operator* $\mathcal{U}(T)$ *and subspace* $\mathcal{H}_1$ *be* $\mathcal{J}$*-positive, subspace* $\mathcal{H}_2$ *be* $\mathcal{J}$*-negative.*

Corollary 1. *If the canonical periodic matrix impulsive equation is stable than it is reducible.*

References

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