TOPOLOGY OF THE FREE PRODUCTS OF PARATOPOLOGICAL GROUPS

N.M. Pyrch

Ukrainian Academy of Printing, Pidgoloslo str. 19, 79020, Lviv, Ukraine
pnazar@ukr.net

By a paratopological group we understand a pair (G, τ) consisting of a group G and topology τ on G making the group opertation $\cdot : G \times G \to G$ of G continuous.

Definition 1. Let $\{G_i : i \in I\}$ be a set of paratopological groups. Then the paratopological group G is said to be a free topological product of $\{G_i : i \in I\}$, denoted by $\prod\limits_{i \in I}{}^{*} G_i$, if it has properties:

1) for each $i \in I$, G_i is a subgroup of G;

2) G is generated algebraically by $\bigcup\limits_{i \in I} G_i$;

3) if for each $i \in I$, f_i is a continuous homomorphism of G_i into paratopological group H, then there exists a continuous homomorphism f of G into H such that $f = f_i$ on G_i for each $i \in I$.

Theorem 1. *Let $\{G_i : i \in I\}$ be any set of paratopological groups. Then free topological product $\prod\limits_{i \in I}{}^{*} G_i$ exists.*

A metric d on group G is called left-invariant if $d(ax, ay) = d(x, y)$ for all $x, y, a \in G$.

Theorem 2. *Let G and H be a nontrivial paratopological groups. Then free topological product $G * H$ is metrizable by left-invariant metric if and only if G and H are discrete.*

Theorem 3. *Let G and H be a nontrivial paratopological T_0-groups. Then free topological product $G * H$ is locally compact if and only if G and H are discrete.*