Theorem 1 is obtained in collaboration with A.S. Mamontov.

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THE BLOCK STRUCTURE OF UNIPOTENT ELEMENTS FROM SUBSYSTEM SUBGROUPS OF TYPE A_3 IN SPECIAL MODULAR REPRESENTATIONS FOR GROUPS OF TYPE A_n

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For $p \geqslant 11$ the Jordan block structure of regular unipotent elements from a subsystem subgroup of type A_3 in p-restricted irreducible representations of the group of type A_n over fields of characteristic p whose highest weights have three consequent zero coefficients is described.

Let $\mathbb{C}$ be a field of complex numbers, $\mathbb{N}$ be a set of positive integers, $\mathbb{N}_a^b = \{i \in \mathbb{N} \mid a \leqslant i \leqslant b\}$, let K be an algebraically closed field of characteristic $p > 0$, $G = A_n(K)$, $n > 3$, and let ω_i ($1 \leqslant i \leqslant n0$ be the fundamental weights of G. A subsystem subgroup of G is generated by root subgroups associated with all roots from a certain subsystem of a root system of G. Further $z \in G$ is a regular unipotent element from a subsystem subgroup of type A_3. For a representation ϕ of an algebraic group S (for a S-module $M0$ and a unipotent element $u \in S$ denote by $J_\phi(u)$ the set of Jordan block sizes of a representation ϕ without their multiplicities. A dominant weight $\omega = a_1\omega_1 + \ldots + a_n\omega_n$ and an irreducible representation ϕ of G with such highest weight are called p-restricted if all $a_i < p$. Put $s(\phi) = 1 + 3a_1 + 4a_2 + \ldots + 4a_{n-1} + 3a_n$, $m(\phi) = \min(p, s(\phi))$ and $\omega^* = a_n\omega_1 + \ldots + a_1\omega_n$. It is well known that ω^* is a highest weight of a representation dual to ϕ.

Theorem 1. *Let $p \geqslant 11$, ϕ be a p-restricted irreducible representation of G with the highest weight $\omega = a_1\omega_1 + \ldots + a_n\omega_n$. Suppose that $a_k = a_{k+1} = a_{k+2} = 0$ for some $i < n - 1$ and $m(\phi) = s(\phi)$. Then $J_\phi(z)$ equals to the same set for an irreducible representation of $A_n(\mathbb{C})$ with the highest weight ω and either $J_\phi(z) = \mathbb{N}_1^{m(\phi)}$, or one of the following conditions holds:*

1) $\omega = a_1\omega_1 + a_n\omega_n$, $a_1 a_n \neq 0$, $a_1 + a_n > 2$, $J_\phi(z) = \mathbb{N}_1^{m(\phi)} \setminus \{3a_1 + 3a_n\}$;

2) ω *or* $\omega^* = a_1\omega_1$, $a_1 > 2$, $J_\phi(z) = \mathbb{N}_1^{m(\phi)} \setminus \{3a_1, 3a_1 - 1, 3a_1 - 4, 2\}$;

3) $\omega = \omega_1 + \omega_n$, $J_\phi(z) = \{7, 5, 4, 3, 1\}$;

4) ω *or* $\omega^* = 2\omega_1$, $J_\phi(z) = \{7, 4, 3, 1\}$;

5) $\omega = \omega_j$, $1 < j < n$, $J_\phi(z) = \{5, 4, 1\}$;

6) ω *or* $\omega^* = \omega_1$, $J_\phi(z) = \{4, 1\}$.

Theorem 2. *Let p, ϕ, ω are the same as above, but $m(\phi) < s(\phi)$. Then $|J_\phi(z)| \geqslant p - 3$ and one of the following conditions holds:*

1) $J_\phi(z) = \mathbb{N}_1^p$;
2) $p \equiv 2(\mod 3)$, ω *or* $\omega^* = \frac{p+1}{3}\omega_1$, $J_\phi(z) = \mathbb{N}_3^{p-4} \cup \{1, p-1, p\}$;
3) ω *or* $\omega^* = a_k\omega_k + \ldots + a_l\omega_l$, $k \leqslant l < n-2$, $a_k a_l \neq 0$. a_l *or* $a_{l-1} + a_l = p-1$, $a_j + a_{j+1} + + a_{j+2} \neq 0$ *for* $k < j < l$, a_k *or* $a_{k+1} + a_k = p-1$ *for* $k > 3$, $\mathbb{N}_1^p \setminus \{2, p-2\} \subset J_\phi(z)$;
4) ω *or* $\omega^* = a_1\omega_1$, $a_1 > \frac{p+1}{3}$, $\mathbb{N}_1^p \setminus \{2, p-2\} \subset J_\phi(z)$;
5) $p \equiv 1(\mod 3)$, ω *or* $\omega^* = \frac{p-4}{3}\omega_1 + \omega_j$, $1 < j < n$, $\mathbb{N}_1^p \setminus \{p-1\} \subset J_\phi(z)$.

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ON $\mathbb{Q}$-CONIC BUNDLES

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The talk is based on joint works with Shigefumi Mori [1–3].

A $\mathbb{Q}$-conic bundle is a proper morphism from a threefold with only terminal singularities to a normal surface such that fibers are connected and the anti-canonical divisor is relatively ample. We study the structure of $\mathbb{Q}$-conic bundles near their singular fibers. The complete classification of $\mathbb{Q}$-conic bundles is obtained under the additional assumption that the base surface is singular. In particular, we show that the base surface of every Q-conic bundle has only Du Val singularities of type A (a positive solution of a conjecture by Iskovskikh). Under бertain additional assumptions we prove M. Reid's general elephant conjecture.

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EXISTENCE AND CONJUGACY OF HALL SUBGROUPS IN FINITE GROUPS

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The term "group" always means a finite group. In what follows π is a set of primes, π' is its complement in the set of all primes, $\pi(n)$ is the set of all prime divisors of a rational integer n. A positive integer n is called a π-number if all its prime divisors are in π. For a group G we set $\pi(G)$ to be equal to $\pi(|G|)$. A subgroup H of G is called a π-Hall subgroup if $\pi(H) \subseteq \pi$ and $\pi(|G : H|) \subseteq \pi'$.

According to P. Hall, we say that G *satisfies* E_π (or briefly $G \in E_\pi 0$, if G contains a π-Hall subgroup. If $G \in E_\pi$ and every two π-Hall subgroups are conjugate, we say that G *satisfies* C_π ($G \in C_\pi 0$. If $G \in C_\pi$ and each π-subgroup of G is included in a π-Hall subgroup of G, we say that G *satisfies* D_π ($G \in D_\pi 0$. Let A, B, H be subgroups of G such that $B \trianglelefteq A$ and