LIMIT THEOREMS FOR THE CRITICAL GALTON-WATSON BRANCHING PROCESS WITH STATE-DEPENDENT IMMIGRATION

J.B.Azimov
Institute of Mathematics and Information Technologies, Uzbek Academy of Sciences
Tashkent, Uzbekistan
e-mail: jakhongir20@rambler.ru

Abstract

Asymptotic behaviors of critical Galton-Watson branching process with state-dependent immigration are studied.

Let μ_n be a number of particles of the Galton-Watson (G-W) branching process at the moment n ($n = 0, 1, \ldots, \mu_0 = 1$) with the generating function (g.f.)

$$F(x) = \sum_{j=0}^{\infty} p_j x^j, \quad p_j \geq 0, \ j = 0, 1, \ldots, \quad \sum_{j=0}^{\infty} p_j = 1, \ |x| \leq 1.$$

The zero state is absorbing for the process μ_n , that is, if $\mu_N = 0$ for some $N > 0$, then $\mu_n = 0$ for all $n > N$. In [1], J.H.Foster considered G-W process modified to allow immigration of particles whenever the number of particles is zero. If $\mu_n = 0$, then, at the moment n,ξ_n particles immigrate to the population, where the number of particles evolves by the law of the G-W process with g.f. $F(x)$. The asymptotic behavior of branching processes with state-dependent immigration were studied by many authors (see, for instance, [1] [3]). Assume that the intensity of the immigration decreases tending to 0, when the number of descendents increases. Limit theorems for such processes have been studied in [4]. We consider the case when immigration takes place as $\mu_n = k$, $0 \leq k \leq m$ where m is some nonnegative integer. Thus, the immigration is given with g.f.

$$g_{k,n}(x) = \sum_{j=0}^{\infty} q_{kj}(n) x^j, \ |x| \leq 1, \ k = 0, 1, \ldots, m, \ q_{kj} \geq 0,$$

$$\sum_{j=0}^{\infty} q_{kj}(n) = 1, \ n = 0, 1, 2, \ldots$$

Let Z_n be a number of particles of this process at the moment n.
Denote

$$\alpha_n = \max_{0 \leq k \leq m} g'_{kn}(1), \quad \beta_n = \max_{0 \leq k \leq m} g''_{kn}(1).$$

We suppose that

$\sup_{0 \leq n < \infty} \alpha_n < \infty, \qquad \sup_{0 \leq n < \infty} \beta_n < \infty,$
$0 < \alpha_n \to 0, \quad \beta_n \to 0 \quad \text{as } n \to \infty.$
$F'(1) = 1, \quad 0 < 2b = F''(1).$

Theorem 1. Assume that $\alpha_n \sim \frac{l(n)}{n^r}$, $\beta_n = o(\alpha_n \log n)$, $n \to \infty$ where $0 \leq r < 1$ and $l(n)$ is a s.v.f. as $n \to \infty$.

Then for all $0 < x < 1$

$$\lim_{n\to\infty} P\left\{\frac{\log Z_n}{\log n} < x/Z_n > 0\right\} = x;$$

Theorem 2. Let $\alpha_n \sim \frac{l(n)}{n}$, $\quad \beta_n = o(\alpha_n \log n)$, $\quad n \to \infty$ and

$$\lim_{n\to\infty} \frac{l(n)\log n}{L(n)} = a, \quad a \geq 0.$$

where

$$L(n) = \sum_{k=1}^{n} \alpha_k \sim \sum_{k=1}^{n} \frac{l(k)}{k}$$

is s.v.f. as $n \to \infty$. Then

a) for all $0 < x < 1$

$$\lim_{n\to\infty} P\left\{\frac{\log Z_n}{\log n} < x/Z_n > 0\right\} = \frac{ax}{1+a};$$

b) for $x \geq 0$

$$\lim_{n\to\infty} P\left\{\frac{Z_n}{bn} < x/Z_n > 0\right\} = \frac{a}{1+a} + \frac{1-e^{-x}}{1+a}.$$

References

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