REAL BUSINESS CYCLE MODEL FOR LITHUANIAN ECONOMY

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Abstract

This paper develops and calibrates a small open economy dynamic stochastic general equilibrium model for Lithuania. Approximate solutions together with policy functions are calculated using local and global numerical methods. The impact of different methods to approximate solutions are assessed according second moments and Euler equation residuals.

1 Introduction

During the last decade, dynamic stochastic general equilibrium (DSGE) models became very popular among international institutions worldwide. The latter models are used to generate forecasts and policy scenarios that provide the basis for monetary policy decisions. The main advantage of DSGE models against the alternative models (for example [6]) is their microeconomic foundations.

As a result of the development of faster computer technology, complex stochastic general equilibrium models have been recently estimated and tested against the data. Most common way for calibration or estimation of DSGE are maximum likelihood or method of moments when nonlinear specifications are transformed into a linear ones [1]. In this paper we develop and calibrate a small open economy DSGE model for Lithuania. Approximate solutions together with policy functions are calculated using local and global numerical methods [2]. The impact of different methods to approximate solutions are assessed according second moments and Euler equation residuals. Obtained results we compare with those of other authors ([5], [4], [3]).

2 DSGE model for Lithuania

For the assessment the impact of numerical methods to approximate solutions and policy functions, we consider the following stochastic Ramsey model in which the representative agent solves

$$\max_{C_0, L_0} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta}(1-L_t)^{\theta(1-\eta)}}{1-\eta} \right] \tag{1}$$

$$\beta \in (0,1), \theta \geq 0, \eta > \theta/(1+\theta),$$

subject to

$$
\begin{aligned}
K_{t+1} + C_t &\leq Z_t(A_t L_t)^{1-\alpha} K_t^{\alpha} + (1-\delta)K_t, \alpha \in (0,1), && (2)\\
A_{t+1} &= aA_t, a \geq 1, && (3)\\
lnZ_{t+1} &= \rho lnZ_t + \epsilon_{t+1}, \rho \in (0,1), \epsilon_t \sim N(0,\sigma^2), && (4)\\
0 &\leq C_t && (5)\\
0 &\leq K_{t+1}, && (6)
\end{aligned}
$$

K_0, Z_0 are given and the variables are

A_t	-	efficiency factor in the period t,
C_t	-	consumption in the period t,
K_{t+1}	-	capital at the beginning of the period $t+1$,
Z_t	-	total factor productivity in the period t,
L_t	-	labour in the period t,
β	-	time discount factor,
δ	-	depreciation rate of capital,
θ	-	preference parameter of the final choice,
η	-	parameter of the elasticity of the marginal utility of consumption,
α	-	parameter of the production technology,
a	-	parameter for labour augmenting technical progress,
ρ	-	autocorrelation parameter of the shock.

From the Lagrangean we derive the following first-order conditions

$$
\begin{aligned}
0 &= C_t^{-\eta}(1-L_t)^{\theta(1-\eta)} - \Lambda_t && (7)\\
0 &= \theta C_t^{1-\eta}(1-L_t)^{\theta(1-\eta)-1} - \Lambda_t(1-\alpha)Z_t A_t (A_t L_t)^{-\alpha} K_t^{\alpha} && (8)\\
0 &= K_{t+1} - (1+\delta)K_t + C_t - Z_t(A_t L_t)^{1-\alpha}K_t^{\alpha} && (9)\\
0 &= \Lambda_t - \beta \mathbb{E}_t \Lambda_{t+1}(1-\delta+\alpha Z_{t+1})(A_{t+1}L_{t+1})^{1-\alpha}K_{t+1}^{\alpha-1}. && (10)\\
& && (11)
\end{aligned}
$$

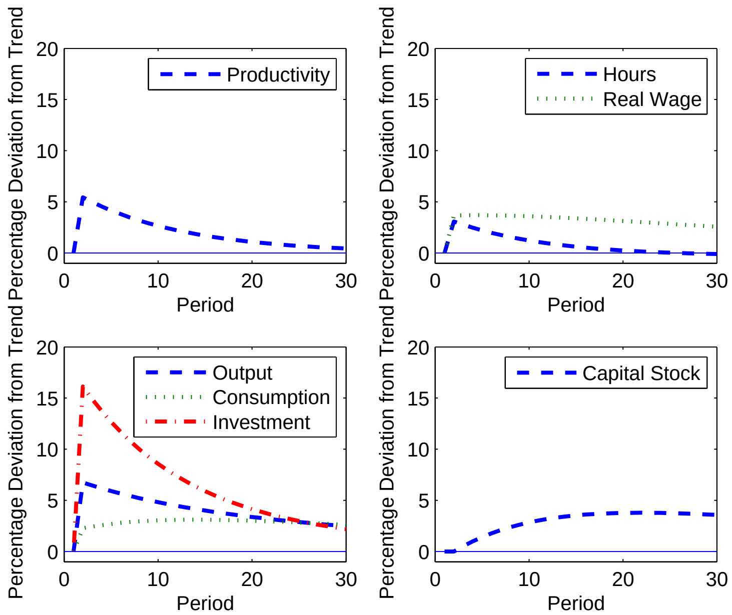
3 Empirical results

For the estimation of the parameters of above model we use calibration. Empirical analysis is conducted using seasonally adjusted quarterly Lithuanian data covering period from year 1995 to 2009. To account for the representative agent nature of the model we scale the data by the size of the population where it is appropriate. We start a calibration process with estimation of production parameters. In the stationary equilibrium, output per households grows at the rate of labour augmenting technical progress $a-1$. Therefore if we infer a from fitting a linear time trend to gross domestic product at factor prices per capital, we obtain $a = 1.0171$ that would imply a quarterly growth rate of 1.7 percent. The parameter of the production technology, α, we set equal

Table 1: Estimated or calibrated model parameters.

Preferences	Production	
$\beta = 0.9829$	a=1.0171	α=0.58
$\eta = 1.5$	δ=0.016	ρ=0.9142
θ =1.379	σ=0.0545	

Figure 1: Impulse responses



to the average wage plus 50 percent of mixed income (under this item the wage income of self-employed persons are accounted) in gross domestic product at current prices. We get a wage share of $1 - \alpha = 0.58$ that is somewhat lower to the commonly used number of 0.64, but higher than 0.5 used in [5]. The rate of depreciation δ we compute as the average ratio of quarterly real depreciation to the quarterly capital stock. As compared to the number of 0.025 typically used for the USA economy and in [5], we obtain a smaller value that is $\delta = 0.016$. Having estimated the parameters a, α and δ we use the production function to calculate the productivity shock Z_t

$$Z_t = \frac{Y_t}{((1.0171)^t H_t)^{0.58} K_t^{0.52}}$$

where H_t stands for working hours. Given that Z_t follows AR(1) process we estimate autoregression parameter $\rho = 0.9142$ and $\sigma = 0.0545$. According the methodology in [2] and results of [4] and [3], we estimate the value of the discount factor $\beta = 0.9829$ and set the elasticity of the merginal utility consumption equal to 1.5, corresponding to $\theta = 1.379$. The calibrated values are summarised in Table 1.

Only very special DSGE models admit an exact solution therefore we apply numerical methods that provide approximate solutions. The latter procedure consists of several steps and leads to the calculation of impulse responses, simulations and second moments. Impulse responses are the deviations of the model's variables from their stationary solution that occur after a one-time shock. In this abstract we show the response of several variables to one standard deviation productivity shock in Figure 1 measured in percentage deviations from their stationary values.

We see that increased productivity raises the real wage and therefore the representative household substitutes leisure for consumption that leads to increase in working hours. The increased productivity and the additional supply of labour boost output where the investment expenditures show the strongest reaction. The above average supply of the capital explains why real wages remain high even after the productivity shock has almost faded. The impact of different methods to approximate solutions assessed according second moments and Euler equation residuals will be presented at the conference and in the extended version of the abstract.

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