ON SOME PROBLEMS OF STATISTICAL ACCEPTANCE CONTROL

Sh.K. Formanov[1], T.A. Formanova[2], L.D. Sharipova[3]
[1]*Institute of Mathematics and Information Technologies*
Tashkent, UZBEKISTAN
e-mail: shakirformanov@yandex.ru
[2]*Tashkent Automobile and Roads Institute*
Tashkent, UZBEKISTAN
e-mail: fortamara@mail.ru
[3]*Tashkent Institute of Railway Engineering*
Tashkent, UZBEKISTAN
e-mail: slola@mail.ru

Abstract

Problems of optimal plans for statistical acceptance control were considered by R.A. Van der Warden and A. Hald in the case when the indication of a checking product is a random variable ξ having the normal distribution. In the present work, more general conditions are imposed upon the density of ξ, moreover the Sheppard corrections are taken into account.

Mathematical theory of statistical acceptance control (SAC) was created by A.N. Kolmogorov [1] and S.Kh. Sirajdinov [2].

Optimal plans of SAC in minimax statement were investigated by R.A. Van der Warden [3], and bayesian plans of SAC were introduced by A. Hald [4].

The most general problem of SAC in a widely known form is contained in the following. Let there be a totality containing N products. Quality of each product from this population is characterized by an indication ξ (for example, a product is considered as a suitable one with respect to a given limit T if $\xi < T$). The question is on quality of a set (party) of N products on the whole basing on a random sample consisting of n independent observations over ξ.

SAC gives possibility to realize current control of manufacturing process, preventing in time output of poor-quality products, on the one hand, and allows to carry out acceptance of output ready products with least outlay, on the other hand. The last is reached by means of choosing plans of SAC. On the base of plans of SAC, it will be known how much pieces of products will be necessary to choose randomly for control and under what conditions solution on acceptance or rejection of products will be taken.

According to what has been said above, results of control over ξ represent a sample $(\xi_1, \xi_2, \ldots, \xi_n)$ consisting of n independent random variables (r.v.'s) with the common distribution function $F(x)$. Suppose that $F(x)$ is a known function up to values of parameters $\mu = \mathbf{E}\xi$, $\sigma^2 = D\xi$. Such assumption is typical for a lot of problems of mathematical statistics.

Let $T > 0$ be a given number (admissibility level). A product is considered as a suitable one if its indication $\xi < T$ (otherwise, it is considered as a defect product). A part of defect products in the totality is the probability

$$p = P(\xi > T) = P\left(\frac{\xi-\mu}{\sigma} > \frac{T-\mu}{\sigma}\right) = 1 - F\left(\frac{T-\mu}{\sigma}\right).$$

The following economical parameters are used to count total outlay connected with any kind of plans of SAC. Let a be a damage of a defect product in accepted party, b be a damage of a defect product in rejected party, c be a cost of checking of a product in the sample. Parameters a and b don't depend on a kind of control and realization of SAC makes sense if $a > b + c$.

Let ξ as a random variable have a continuous distribution with the density function $f(x)$. Further, we suppose that the density $f(x) = F'(x)$ satisfies the following smoothness conditions and boundedness:
A) $f(x)$ and its first derivatives of the order $2s$ ($s \geq 1$) are continuous for any x,
B) for some $A > 0$

$$\sup_x |x|^5 f^{(k)}(x) \leq A < \infty, \quad k = 0, 1, 2, \ldots, 2s.$$

Let $\delta > 0$ and ξ_0 be a r.v. independent on any $\xi_1, \xi_2, \ldots, \xi_n$ and having the uniform distribution on the interval $(-\delta/2, \delta/2)$.

Set

$$I_\alpha = \left(\xi_0 + \alpha\delta - \frac{\delta}{2},\ \xi_0 + \alpha\delta + \frac{\delta}{2}\right), \quad \alpha = 0, \pm 1, \pm 2, \ldots$$

The system of intervals $\{I_\alpha,\ \alpha = 0, \pm1, \pm2, \ldots\}$ forms a separation of the number axis, moreover, the origin of reference ξ_0 is chosen randomly. If n_α is the number of elements of the sample $(\xi_1, \xi_2, \ldots, \xi_n)$ located in the interval I_α, then the value

$$S_{2,\delta} = \frac{1}{n} \sum_\alpha n_\alpha \left(\xi_0 + \alpha\delta\right)^2$$

represents a 2-sample moment when elements of the sample $(\xi_1, \xi_2, \ldots, \xi_n)$ round off up to the nearest divisible by δ. It is clear, exact expressions of $\mathbf{E}\xi^2$ can be obtained adding some corrections to $\mathbf{E}S_{2,\delta}$ expressed in terms of powers of δ. They are called the Sheppard corrections (see [5]). For example,

$$\mathbf{E}S_{1,\delta} = \mathbf{E}\xi, \quad \mathbf{E}S_{2,\delta} = \mathbf{E}\xi^2 + \frac{\delta^2}{12}.$$

The plan of SAC, corresponding to the case of the variance σ^2 is known but its mean μ is unknown, is denoted (Z_δ, σ) where

$$Z_\delta = S_{1,\delta} + k\sigma,$$

k is a number called "the factor" of control. For the plan (Z_δ, σ), a solution of the control is the following:

if $Z_\delta < T$, the totality is accepted;

if $Z_\delta \geq T$, the totality is rejected.

A plan of the control (Z_δ, σ) is considered as an optimal one if the sample volume n minimizes the maximal by p value of the "residual" risk function $R(p,n)$ (see definition of this function in [3, 4]). Thus, for the optimal sample volume $n = n_{opt}$, the following equality holds:

$$R\,(n_{opt}) = \min_{1\le n\le N}\max_{0\le p\le 1} R(p,n). \qquad (*)$$

According to $(*)$, choice of the plan of SAC is a minimax statement of the problem of organization of SAC, and it is uncertain to expect obtaining of exact expressions for $n_{opt}(N)$. On the other hand, in most cases for not large values of the totality volume N necessity in SAC is eliminated. Therefore it is appeared necessity to study a problem on asymptotical behavior of the optimal sample volume $n_{opt}(N)$ as $N \to \infty$.

Theorem. *Let the density function $f(x)$ of a r.v. ξ satisfies to the conditions A) and B). Then for large values of N and small values of δ ($N \to \infty$, $\delta \to 0$), the following relation*

$$\frac{n_{opt}(N)}{N^{2/3}} = 0.193\left[\frac{a-b}{c}f(u_0)\right]^{2/3} + O\left(\max\left(\frac{1}{N^{2/3}}, \frac{\delta^2}{N^{2/3}}, \delta^{2s}\right)\right)$$

holds for the optimal volume of the plan (Z_δ, σ) where $u_0 = F^{-1}(1-p_0)$, $p_0 = \dfrac{c}{a-b}$ is the part of "indifference".

Proof of theorem is based on results of work [6] where estimate for the rate of convergence is obtained in central limit theorem taking account of Sheppard corrections.

References

[1] Kolmogorov A.N. (1950). Unbiased estimates, *Izvestiya AN SSSR, ser. matem.* Vol. **14**, No. **4**, pp. 303-326.

[2] Sirajdinov S.Kh. (1955). Single statistical control. *in: Trudi Inst. Matem. AN UzSSR* Vol. **15**, pp. 41-56.

[3] Van der Warden R.A. (1960). Sampling inspection as a minimum loss problem. *Annals of Math. Stat.* Vol. **31** (2), pp. 269-384.

[4] Hald A. (1968). Bayesian single sampling attribute plans for continuous prior distributions. *Tehnimetrica.* Vol. **10**, pp. 667-683.

[5] Willks S.S. (1963). *Mathematical Statistics.* Wiley, New York.

[6] Formanov Sh.K., Formanova T.A. (2005). Optimal plans of statistical acceptance control with Sheppard corrections. *Communications in dependability and quality management*, Vol. **8**, No. **3**, pp. 44-48.