We believe that the methods developed in the paper can be extended to the three-dimensional case and to the case of media with nonzero absolute temperature.

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NECESSARY CONDITIONS OF OPTIMALITY FOR QUASILINEAR SYSTEMS WITH INCOMMENSURABLE DELAYS AND MIXED RESTRICTIONS

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Introduction. In the presented paper the necessary conditions of optimality for quasilinear systems with mixed restrictions and incommensurable delays in phase and control variables are given. In different of general problems, witch are considered in [1] , the investigated problem includes mixed restrictions (3). Unlike [2], the approach used by us, allows problem research were the mapping describing restrictions of a task has infinite codimension, that is typical for systems with the mixed restrictions. As against [3], necessary conditions of optimality in case of continuous initial function are proved.

1. Problem formulation. We consider the following problem

$$I=\int_{t_0}^{t_1} f^0\left(x\left(t\right),x\left(t-\Theta\right),u\left(t\right),u\left(t-\tau\right)\right)dt\rightarrow inf \tag{1}$$

under the restrictions

$$\dot{x}(t)=f\left(x\left(t\right),x\left(t-\Theta\right),u\left(t\right),u\left(t-\tau\right)\right), \tag{2}$$

$$g\left(x\left(t\right),u\left(t\right)\right)\leq 0, t\in[t_0,t_1], \tag{3}$$

$$\chi\left(u\left(t\right)\right)\leq 0, t\in[t_0-\tau,t_0], \tag{4}$$

$$x\left(t_1\right)=x_1, \tag{5}$$

$$x\left(t\right)=\varphi\left(t\right), t\in[t_0-\Theta,t_0], \tag{6}$$

where vector function φ is continuous on $[t_0-\Theta,t_0]$, $x_1\in R^n$ is fixed, $t_0,t_1,\Theta>0,\tau>0$ are fixed numbers, the scalar function f^0 and the vector functions $f\in R^n, g\in R^m$ are continuously differentiated with respect to all their arguments, $\chi\in R^s$ is linear. The conditions (2) - (3) and (4) are fulfilled accordingly for almost all $t\in[t_0,t_1]$ and almost all $t\in[t_0-\tau,t_0]$ and the restrictions (3) fulfilled the conditions of generality: for any (x,u), satisfying (3), the system of vectors $grad_u g^j(x,u), j\in J(x,u)$ is linearly independent. Here by $J(x,u)$ we denote the set of indices $j\in\{1,2,\cdots,m\}$, for which $g^j(x,u)=0$.

2. Necessary conditions of optimality. Let vector function $\varphi(t)$ is fixed, the vector function $x(t)$ is absolutely continuous, the vector function $u(t)$ is integrable on $[t_0-\tau,t_1]$, i.e. $x(t)\in W^n_{1,1}[t_0,t_1]$, $u(t)\in L^r_1[t_0-\tau,t_1]$, the functions $f^0,f\in R^n, g\in R^m$ are linear with respect to control parameters and satisfying condition of "convexity" (see [2]), the restrictions (4) fulfil the conditions of generality, then using the Lagrange Principle of taking restrictions off from [4], we obtain the following theorem:

Theorem 1. *Let $(x(t),u(t))$ is a solution of the problem (1) - (6). Then there exist multipliers $\psi_0\geq 0$, $\psi(t)\in W^n_{1,1}[t_0,t_1]$,$\nu(t)\in L^s_\infty[t_0-\tau,t_0]$ and $\mu(t)\in L^m_\infty[t_0,t_1]$, such that, the following conditions are fulfilled*

$$\mu_j(t)\geq 0, \mu_j(t)\, g^j(x(t),u(t))=0, j=\overline{1,m}, t\in[t_0,t_1] \tag{7}$$

$$\nu_k(t)\geq 0, \nu_k(t)\,\chi^k(u(t))=0, k=\overline{1,s}, t\in[t_0-\tau,t_0], \tag{8}$$

$$H[t+\tau,u(t+\tau)]=\min_{u\in\{u|\chi(u)\leq 0\}} H[t+\tau,u], t\in[t_0-\tau,t_0] \tag{9}$$

$$H[t,u(t)]+H[t+\tau,u(t+\tau)]=\min_{u\in\{u|g(x(t),u)\leq 0\}} H[t,u]+H[t+\tau,u], t\in[t_0,t_1-\tau], \tag{10}$$

$$H[t,u(t)]=\min_{u\in\{u|g(x(t),u)\leq 0\}} H[t,u], t\in[t_1-\tau,t_1], \tag{11}$$

$$\frac{d\psi}{dt}=\frac{\partial\Re(x(t),x(t-\Theta),u(t),\psi_0,\psi(t),\mu(t))}{\partial x(t)}+$$

$$+\frac{\partial\Re(x(t+\Theta),x(t),u(t+\Theta),\psi_0,\psi(t+\Theta),\mu(t+\Theta))}{\partial x(t-\Theta)}, t\in[t_0,t_1-\Theta], \tag{12}$$

$$\frac{d\psi}{dt}=\frac{\partial\Re(x(t),x(t-\Theta),u(t),\psi_0,\psi(t),\mu(t))}{\partial x(t)}, t\in[t_1-\Theta,t_1], \tag{13}$$

and

$$(\psi_0,\psi(\cdot))\neq(0,0), \tag{14}$$

where

$$H[t,u(t)]\equiv H(x(t),x(t-\Theta),u(t),\psi_0,\psi(t))\equiv$$

$$\equiv\psi_0 f^0(x(t),x(t-\Theta),u(t),u(t-\tau))-\sum_{i=1}^n\psi_i(t)\,f^i(x(t),x(t-\Theta),u(t),u(t-\tau)),$$

$$\Re(x(t),x(t-\Theta),u(t),u(t-\tau),\psi_0,\psi(t),\mu(t))\equiv$$

$$H(x(t),x(t-\Theta),u(t),u(t-\tau),\psi_0,\psi(t))+\mu(t)\,g(x(t),u(t))$$

and the restrictions (7)-(13) are fulfilled for almost all t.

P r o o f. The problem (1) – (6) can be presented as a partial case of an extreme problem

$$f_0(w)\to inf \mid F(w)=0, f_i(w)\leq 0, (i=\overline{1,n}), w\in W, \tag{15}$$

considered in [4]. Using Theorem 1 from [4] we have: for any solution $\widehat{w}$ of the problem (15), there exist numbers $\lambda_i\geq 0, i=\overline{0,n}$ and an element y^* of the conjugate space Y^*, such that, the conditions

a) $(\lambda_0,\lambda_1,...,\lambda_n,y^*)\neq(0,...,0)$,

b) $\lambda_i f_i(\widehat{w})=0, i=\overline{1,n}$,

c) $L(\widehat{w},\lambda_0,\lambda_1,...,\lambda_n,y^*)=\min_{w\in W} L(w,\lambda_0,\lambda_1,...,\lambda_n,y^*)$,

d) $\sum_{i=0}^n\lambda_i\frac{\partial f_i(\widehat{w})}{\partial w}+(F'(\widehat{w}))^*y^*=0$

are fulfilled. From standard transformations (see [3]) of these conditions we receive conditions (7)-(14).

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МЕТОД РЕШЕНИЯ ИНТЕГРАЛЬНОЙ ЗАДАЧИ ЛЯПУНОВСКОГО ТИПА

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Рассмотрим интегральную задачу ляпуновского типа [1]

$$\int_T g_0(u(t),t)dt \to min, \quad u \in V, \tag{1}$$

$$\int_T g_i(u(t),t)dt = a_i, \quad i = \overline{1,m},$$

$$V = \{u \in L^r_\infty : \ u(t) \in U, \ t \in T\}.$$

Представим задачу в дифференциальной форме, используя переменные

$$y_i = \int_{t_0}^t g_i(u(\tau),\tau)d\tau, \quad i = \overline{0,m}.$$

В результате получаем динамический вариант интегральной задачи (1)

$$\dot{y}_i(t) = g_i(u,t), \quad y_i(t_0) = 0, \quad i = \overline{0,m},$$

$$y_0(t_1) \to min, \quad y_i(t_1) = a_i, \quad i = \overline{1,m}, \quad u \in V.$$

Пусть $Y \subset R^{m+1}$ - множество достижимости y - системы в момент времени t_1 (выпуклый компакт). В пространстве R^{m+1} переменных $y = (y_0, y_1, ..., y_m)$ задача представляется в конечномерной форме

$$y_0 \to min, \quad y_i = a_i, \quad i = \overline{1,m}, \quad y \in Y. \tag{2}$$

Для численного решения этой задачи возьмем за основу метод параметризации целевой функции [2]. Приведем основные соотношения метода.

Пусть $y^* = (y^*_0, y^*_1, ..., y^*_m)$ - решение задачи (2). Образуем функцию - свертку с параметром β

$$S(y,\beta) = (y_0 - \beta)^2 + \sum_{i=1}^m (y_i - a_i)^2$$

и сформулируем вспомогательную задачу проектирования на множество Y

$$S(y,\beta) \to min, \quad y \in Y. \tag{3}$$

Пусть $y(\beta) = (y_0(\beta), y_1(\beta), ..., y_m(\beta))$ - решение этой задачи,

$$\beta_* = min\{\beta : \ S(y(\beta),\beta) = 0\}$$