In conclusion we note, that the concrete algorithm for realization of two-level hierarchical minimax program terminal control process in discrete-time dynamical system (1) – (5) can be described on the base of the algorithms for solving program terminal control problem which are proposed in work [4].

The results obtained in this report are based on [1] – [4] and can be used for computer simulation and for optimal digital controlling systems designing for actual technical, economic, and other multi-level control processes. Mathematical models of such systems were considered, for example, in works [1] – [3].

This work was supported by the Russian Basic Research Foundation, Project no. 07-01-00008.

References

1. *Krasovskii N.N. and Subbotin A.I.* Game-Theoretical Control Problems. Springer, Berlin, 1988.
2. *Kurzhanskii A.B.* Control and Observation under Uncertainty, Nauka, Moscow, 1977.
3. *Shorikov A.F.* Minimax Estimation and Control in Discrete-Time Dynamical Systems. Urals State University Publisher, Ekaterinburg, 1997.
4. *Shorikov A.F.* Algorithm for Solving a Problem of ϵ-Optimal Program Control for Descrete-Time Dynamical System // Control Theory and Theory of Generalized Solutions of Hamilton–Jacobi Equations. Abstracts of International Seminar. Urals State University Publisher, Ekaterinburg, 2005. P. 176–178.

STUDY OF REVERSE MAGNUS EFFECT IN THE CASE OF SPINNING ROUGH DISK MOVING IN A RARIFIED MEDIUM

T. Tchemisova[1], **A. Plakhov**[2]

[1] University of Aveiro, Aveiro 3810, Portugal
tatiana@ua.pt

[2] University of Wales - Aberystwyth, Aberystwyth SY23 3BZ, Ceredigion, UK;
on leave from University of Aveiro, Portugal
axp@aber.ac.uk

We are concerned with a spinning solid body moving in a homogeneous medium. The medium is extremely rarefied, so the free path length of the medium particles is much larger than the body's size. In such a case, the interaction of the body with the medium can be described in terms of *free molecular flow*, where a flow of point particles falls on the body's surface; each particle interacts with the body and does not with other particles. We suppose that the medium particles stay at rest, that is, the absolute temperature of the medium equals zero. In a frame of reference moving forward together with the body, we have a parallel flow of particles falling on the body.

We neglect the angular momentum of particles; each particle is identified with a mass point that approaches the body, makes several (maybe none) reflections from its surface, and then goes away. All reflections are supposed to be *absolutely elastic*.

In this paper, we restrict ourselves to the two-dimensional case. Consider a body contained in a circle of radius r and containing the concentric circle of radius $r - \varepsilon$ with $\varepsilon \ll r$. One can imagine a circle of radius r slightly damaged near the boundary, resulting in a *rough circle*. The shape of the body cannot change in time: it can only be translated or rotated. Denote by $\varphi(t)$ the rotation angle at the moment t, by $\omega(t)$ the angular velocity of the body, $\omega(t) = d\varphi/dt$, and let $\vec{v}(t)$ be the velocity of the body's center of mass. Let us agree to count off the rotation angle and the angular velocity clockwise.

Here we consider two problems as follows: (i) determine the force of the medium resistance acting on the body, find the moment of this force with respect to the body's center of mass, and investigate their dependence on the "shape of the roughness", and (ii) analyze the motion of the body in the medium, that is, study the functions $\omega(t)$ and $\vec{v}(t)$. Problem (i) is primary with respect to problem (ii). In the paper, we will devote the main attention to problem (i), having just touched upon problem (ii); we will restrict ourselves with deducing equations of motion and solving these equations for some simple particular cases.

Passing to more exact formulations, denote by B the position of the body at the initial moment of time. B is supposed to be a connected set with piecewise smooth boundary such that $B_{r-\varepsilon}(a) \subset B \subset B_r(a)$, $a \in \mathbb{R}^2$, $\varepsilon \ll r$, where $B_r(a)$ stays for the circle of radius r with the center at a. The mass of the body is distributed inside B with some density; we suppose that the center of mass coincides with the center of both circles. Denote by M the mass of the body, and by I its moment of inertia. One has $I \leq Mr^2$. We pay special attention to the two particular cases: (a) the body's mass is uniformly distributed in B, so that $I = Mr^2/2$; and (b) the mass is concentrated on the boundary ∂B, so that $I = Mr^2$.

The resistance force affecting the body B, $\vec{R}(B, \omega, \vec{v}, \varphi)$, and the moment of this force, $R_I(B, \omega, \vec{v}, \varphi)$, depend on the shape of the body, the angle of rotation φ, the angular velocity ω, and the velocity of translation $\vec{v}$.

In terms of the values of the averaged resistance force and the averaged moment of the force, $\vec{R}(B, \omega, \vec{v}) = \frac{1}{2\pi} \int_0^{2\pi} \vec{R}(B, \omega, \vec{v}, \varphi)\, d\varphi$, and $R_I(B, \omega, \vec{v}) = \frac{1}{2\pi} \int_0^{2\pi} R_I(B, \omega, \vec{v}, \varphi)\, d\varphi$, the equations of dynamics have the form

$$M \frac{d\vec{v}}{dt} = \vec{R}(B, \omega, \vec{v}), \quad I \frac{d\omega}{dt} = R_I(B, \omega, \vec{v}).$$

We shall see that the study of the values $\vec{R}$ and R_I is naturally reduced to the study of the billiard in the exterior of the rotating body B.

To each set B of a certain shape we assign a measure ν_B characterizing the law of billiard scattering on B. The values $\vec{R}$ and R_I are defined to be functions of the values ν_B, ω, and $\vec{v}$, where ν_B is a measure characterizing the law of billiard scattering on B. We calculate the exact values $\vec{R}$ and R_I for some special values of ν_B (and thus for some special kinds of rough bodies). In general, we define the set of all possible values of $\vec{R}$, R_I, when ω and $\vec{v}$ are fixed and ν_B takes all possible values. In other words, we answer the following question: what values can take the resistance force and its moment acting on a rough unit circle?

While we look through all possible "shapes of roughness the values of vector ($\vec{R}$, R_I) cover a fixed three-dimensional set. The problem of determination of this set is formulated in terms of a vector-valued Monge-Kantorovich problem and is solved numerically for some fixed values of the parameter $\lambda = \omega/v$. Here we use numerical simulation of the problem using a succession of transportation problems that are solved using Xpress solver.

If on a spinning body moving in a medium acts a transversal force resulting in deflection of the body's trajectory, then we deal with the so called *Magnus effect*. Usually there are considered a sphere or a cylinder moving in the direction orthogonal to its symmetry axis. If the direction of the transversal force coincides with the instantaneous velocity of the front point of the body, then the proper Magnus effect takes place. If these directions are opposite, then *reverse* Magnus effect occurs.

We show that although both reverse and direct Magnus effects are possible in our model, the reverse effect is more common and is more strongly expressed that the direct one.

There exist a number of papers, see [1, 2, 4, 5], that state that in the cases of movement in extremely rarefied media the reverse Magnus effect takes place, and study this phenomenon. In these studies the medium is supposed so rarefied that the description in terms of free molecular flow is possible.

In our opinion, the reverse Magnus effect in such kind of media is caused by two factors:

(i) Non-elastic collisions of particles with the body. A part of the tangential component of the momentum of particles is transmitted to the body, resulting in creation of a transversal force.

(ii) Multiple collisions of particles with the body originating from the fact that that the body's surface is not convex but contains microscopic cavities.

We believe that the methods developed in the paper can be extended to the three-dimensional case and to the case of media with nonzero absolute temperature.

References

1. *Borg K. I., Söderholm L. H., Essén H.* Force on a spinning sphere moving in a rarefied gas. Physics of Fluids. 2003. 15. P.736-741.
2. *Ivanov S. G., Yanshin A. M.* Forces and moments acting on bodies rotating around a symmetry axis in a free molecular flow. Fluid Dyn. 1980. 15. P.449.
3. *Plakhov A.* Billiards and two-dimensional problems of optimal resistance. Accepted in Arch. Ration. Mech. Anal.
4. *Wang C.-T.* Free molecular flow over a rotating sphere. AIAA J. 1972. 10. P.713.
5. *Weidman P. D., Herczynski A.* On the inverse Magnus effect in free molecular flow. Physics of Fluids. 2004. 16. L9-L12.

NECESSARY CONDITIONS OF OPTIMALITY FOR QUASILINEAR SYSTEMS WITH INCOMMENSURABLE DELAYS AND MIXED RESTRICTIONS

Z.A. Tsintsadze

Muskhelishvili Institute of Computational Mathematics, National Academy of Sciences of Georgia
8 Akuri str., 380093 Tbilisi, Georgia,
zutsints@rambler.ru

Introduction. In the presented paper the necessary conditions of optimality for quasilinear systems with mixed restrictions and incommensurable delays in phase and control variables are given. In different of general problems, witch are considered in [1] , the investigated problem includes mixed restrictions (3). Unlike [2], the approach used by us, allows problem research were the mapping describing restrictions of a task has infinite codimension, that is typical for systems with the mixed restrictions. As against [3], necessary conditions of optimality in case of continuous initial function are proved.

1. Problem formulation. We consider the following problem

$$I = \int_{t_0}^{t_1} f^0\left(x\left(t\right), x\left(t-\Theta\right), u\left(t\right), u\left(t-\tau\right)\right) dt \to inf \tag{1}$$

under the restrictions

$$\dot{x}(t) = f\left(x\left(t\right), x\left(t-\Theta\right), u\left(t\right), u\left(t-\tau\right)\right), \tag{2}$$

$$g\left(x\left(t\right), u\left(t\right)\right) \leq 0, t \in [t_0, t_1], \tag{3}$$

$$\chi\left(u\left(t\right)\right) \leq 0, t \in [t_0-\tau, t_0], \tag{4}$$

$$x\left(t_1\right) = x_1, \tag{5}$$

$$x\left(t\right) = \varphi\left(t\right), t \in [t_0-\Theta, t_0], \tag{6}$$

where vector function φ is continuous on $[t_0-\Theta, t_0]$, $x_1 \in R^n$ is fixed, $t_0, t_1, \Theta > 0, \tau > 0$ are fixed numbers, the scalar function f^0 and the vector functions $f \in R^n, g \in R^m$ are continuously differentiated with respect to all their arguments, $\chi \in R^s$ is linear. The conditions (2) - (3) and (4) are fulfilled accordingly for almost all $t \in [t_0, t_1]$ and almost all $t \in [t_0-\tau, t_0]$ and the restrictions (3) fulfilled the conditions of generality: for any (x, u), satisfying (3), the system of vectors $grad_u g^j(x, u), j \in J(x, u)$ is linearly independent. Here by $J(x, u)$ we denote the set of indices $j \in \{1, 2, \cdots, m\}$, for which $g^j(x, u) = 0$.