A Model of Solving of Pattern Recognition Problems Based on Resolution Method

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Abstract: *Deductive and inductive approaches to solving of pattern recognition problems are considered. Logical and precedent-related encoding models of prior information are used in these approaches correspondingly. Algebra of objects is built and its isomorphism to Boolean algebra is shown. Algorithms for mutual conversion of encoding models are developed. A modification of the resolution method for solution of pattern recognition problems is suggested. An algorithm combining the resolution method and a parametric family of recognition algorithms is developed.*

***Keywords*:** pattern recognition, resolution method, Boolean algebra.

1. INTRODUCTION

The theory of pattern recognition has passed in its evolution through two main stages. At the first stage, applied problems were considered mainly, and the studies on them consisted of solving the corresponding pattern recognition problems. The majority of pattern recognition applications is connected to fields of science which are hard to formalize, such as medicine, chemistry, sociology, etc. For this reason, at this stage of development of pattern recognition theory heuristic algorithms were used. These algorithms were grounded on convergence principle on the set of objects in a general sense.

The second stage of development of pattern recognition theory is characterized by changing from separate algorithms to models of solving of applied problems. At this stage, the heuristics was not the selection of an algorithm, but the principle, according to which algorithms could be built in a standard way. Then schemas, which were based on these heuristic algorithms, were built in order to reduce the disadvantages of these algorithms. These schemas included the logical correction schemas, algebraic models, etc. and were grounded on accuracy requirements such as, e.g. correct algorithms.

At present, the development of pattern recognition theory has reached the third stage. This stage can be described as follows. There exist classic algorithms, which grounds are thoroughly studied, e.g. the resolution method, which has the deductive nature. It would be useful to build a recognition algorithm, which is able to use the advantages of the resolution method.

It is this view of the problem of grounds os recognition algorithms that is considered in this paper. The general pattern recognition problem is formulated in two ways, which correspond to deductive (the resolution method) and inductive (recognition algorithms) approaches to its solution. The objectives of this paper are to compare these two approaches and to build an algorithm, which is able to use the advantages of both of them. These objectives are reached as follows. Traditionally, each way of formulating of pattern recognition problem has its own encoding model of prior information: logical model for the problems, which are solved by deductive methods, and precedent-related model for the problems which are solved by inductive methods. It is shown in this paper that these two models are equivalent under some constraints. Due to this fact, the resolution method is used to solve pattern recognition problems. Then an algorithm, which is based on these results and combines the resolution method with a recognition algorithm, is developed. It is shown that in the worst case this algorithm works at least as good as two algorithms from which it is combined.

2. PROBLEM DEFINITION

Let us consider the following problem in the general definition formulated in [1]:

The finite set X of objects and the finite number of subsets (classes) $X_1,\dots,X_l$ are given. Prior information I_0, which describes membership of objects from the subset $X^0 \subset X$ to these classes, is also given. One need to find an algorithm, which is defined on X and for an arbitrary object $x \in X$ calculates its membership to $X_1,\dots,X_l$, based on I_0.

The specific variants of the formulation of this problem are defined by several factors: the encoding models of X and I_0, the quantity of classes l, etc. In this paper the following two variants are considered:

1) Problem Z_1:

Prior information I_0 is represented in a logical way: by means of predicates (rules, logical formulas), which are used simultaneously for the description of objects and the function of membership to classes. Information about membership of objects, which satisfy the rules, is given. For the given object x one need to determine whether x is deducible form the rules which describe the class X_i, where $i = \overline{1,l}$.

The problem of classification of formulas in the sentential calculus is an example of problem Z_1. These problems are solved by deductive methods. The standard method of solving problems, which are formulated in the similar way to formulation of Z_1, is the resolution method [2].

2) Problem Z_2:

Prior information I_0 is represented in a precedent-related way: for each class $X_1,\dots,X_l$ objects, which belong to this class, are explicitly specified (i.e. for each object its information vector [3] is given). One need to find an algorithm which calculates the membership of an arbitrary object $x \in X$ to $X_1,\dots,X_l$.

This variant of problem definition is typical for pattern recognition and machine learning problems. A lot of algorithms are developed for solving of problem Z_2, but it can hardly be spoken about their grounds.

The next question arises: is it possible to use advantages of the resolution method while solving problem Z_2? First it is necessary to convert encoding models, which are used in problems Z_1 and Z_2, to a unified model. For this purpose, an algebra of objects, which is described later, is considered.

3. ALGEBRA OF OBJECTS

Let us show that logical and precedent-related encoding models are equivalent in the following way: assuming that the number of objects and the number of values of their signs are finite, any prior information can be encoded using any of these two models in order to describe the initial data both for problem Z_1 and problem Z_2. For this purpose, let us consider algebra of objects for the case of precedent-related description of information. Further, it is shown that this algebra and Boolean algebra, which is used is logical case of precedent-related description of information, are isomorphic.

The way of describing of objects proposed in [4] is used in this paper. Let an object have a finite number of signs. Let S_j^p be the set of signs from which the j th sign of the object p is chosen. It is supposed that all signs are binary, i.e. the set of their values is $D=\{0,1\}$. The set D is chosen for simplifying the description of objects; all the results of this chapter can be easily generalized for the case when the signs possess an arbitrary finite number of values.

It is also possible that the value of some sign is unknown. In that case, let us introduce a special symbol for the unknown value, i.e. "?", and denote this extended set of values as $\overline{D}=\{0,1,?\}$.

Definition 1. An object is the following mapping:

$p: S_1^p \times S_2^p \times \ldots \times S_n^p \to \overline{D}^n$,

where n is the number of signs of the object p, $\overline{D}^n = \underbrace{\overline{D}\times\ldots\times\overline{D}}_{n}$.

An object p, which has signs $s_j^p \in S_j^p$, $j=\overline{1,n}$, with correspondent values $d_j^p \in \overline{D}$, is written as follows:

$p(s_1^p, s_2^p, \ldots, s_n^p) = (d_1^p, d_2^p, \ldots, d_n^p)$.

Definition 2. Objects p and q are considered as equal, if

1) they have equal number of signs;

2) there exist such a permutation σ of indexes of signs of the object q that

2.1) $\forall j\ S_j^p = S_{\sigma(j)}^q$;

2.2) $p(s_1^p, s_2^p, \ldots, s_n^p) = (d_1^p, d_2^p, \ldots, d_n^p) =$
$= (d_{\sigma(1)}^q, d_{\sigma(2)}^q, \ldots, d_{\sigma(n)}^q) = q(s_{\sigma(1)}^q, s_{\sigma(2)}^q, \ldots, s_{\sigma(n)}^q)$

where n is the number of signs of the objects p and q.

In other words, two objects are equal, if sets of their signs are equal, and so do corresponding values of their signs.

Definition 3. A collection of objects P is a set of objects, which has the following properties:

1) all objects have equal number of signs;

2) for each pair of objects $p, q \in P$ there exists such a permutation σ of indexes of signs of q that $\forall j\ S_j^p = S_{\sigma(j)}^q$.

I.e. a collection of objects is a set of objects where all objects have the same signs.

The collection P consisting of $p_1, p_2, \ldots, p_r$ is written as follows: $P=\{p_1, p_2, \ldots, p_r\}$.

Definition 4. Collections P and Q are equal, if

1) $|P|=|Q|$;

2) $\forall p \in P\ \exists q \in Q\ p=q$;

3) $\forall q \in Q\ \exists p \in P\ p=q$.

Definition 5. An object-sign is an object, for which a value of only one of its n signs is known.

Let us consider an arbitrary object p, where the value of the j th sign is unknown. If there are no additional value constraints, then this sign can potentially take any value from D. Therefore the following interpretation is used in this paper: the object p, where the value of the j th sign is unknown, is considered as a collection of 2 different objects, where the value of j th sign of one object is 0 and the value of j th sign of the other object is 1; all the values of all other signs are equal to values of correspondent signs of p:

$$p=(\ldots,\overset{j}{?},\ldots)=\{(\ldots,\overset{j}{0},\ldots),(\ldots,\overset{j}{1},\ldots)\}.$$

Therefore, when necessary, it is assumed, that all signs take values from D.

Using unknown values of signs, it is possible to simplify description of objects in order to make all objects have the same set of signs. Note that the order of enumeration of signs is not significant for description of objects and their collections. Let $S=\{s_1, s_2, \ldots, s_n\}$ be a set of all signs which are used in field where the problem Z_1 or Z_2 is considered. Suppose that the value of the sign s_j of object p is unknown. Let us replace p by a collection of 2 objects which differ only in values of j th sign. After this procedure has been repeated for all unknown values, all values are defined. Thus without loss of generality we can further suppose that all objects have all n signs from the set $S=\{s_1, s_2, \ldots, s_n\}$.

Let us consider now main operations on objects and their collections. Let 0_n be an empty collection containing no objects, and 1_n be a collection of all possible objects which have n signs from the set S.

Given two arbitrary collections P and Q, the following operations are considered:

1) *Negation*

Negation $\overline{P}$ of the collection P is defined as follows: $\overline{P}=1_n \setminus P$

2) *Multiplication*

Objects p and q are called compatible if for any sign s_j at least one of the following conditions is satisfied:

a) $d_j^p = ?$

b) $d_j^q = ?$

c) $d_j^p = d_j^q$

Multiplication is defined only for compatible objects.

The product $p \wedge q$ (or pq) of compatible objects p and q is an object whose values of signs are defined as follows:

$$d_j^{pq} = \begin{cases} d_j^q, d_j^p = ? \\ d_j^p, d_j^p \neq ? \end{cases}$$

The product $P \wedge Q$ (or PQ) of collections P and Q is a collection consisting of all possible pairwise products of objects from P and Q:

$$P \wedge Q = \bigcup_{p\in P, q\in Q} \{pq\}$$

3) *Addition*

The sum $P \vee Q$ of collections P and Q is defined as follows:

$$P \vee Q = (PQ) \bigcup (\overline{P}Q) \bigcup (P\overline{Q})$$

It is shown in [4] that any collection can be represented as a sum of objects, which belong to that collection, and any object can be represented as a product of its objects-signs:

$$P = \{p_1, \ldots, p_r\} = \mathop{\wedge}_{i=1}^{r} p_i = \mathop{\wedge}_{i=1}^{r} \mathop{\vee}_{j=1}^{n} (?, \ldots, d_j^{p_i}, \ldots, ?).$$

For short objects-signs are denoted as follows:

$$d'^{p}_{j} = (?, \ldots, d_j^p, \ldots, ?)$$

Then $P = \{p_1, \ldots, p_r\} = \mathop{\wedge}_{i=1}^{r} p_i = \mathop{\wedge}_{i=1}^{r} \mathop{\vee}_{j=1}^{n} d'^{p_i}_{j}$.

Thus the algebra of objects $G_n =< P_n, \{\neg, \wedge, \vee\} >$ is built. The underlying set of this algebra is the set P_n of collections of objects, which have n signs, and the underlying operations are operations $\neg, \wedge, \vee$ on this set [5].

Let N be the number of all objects which have n signs (it is easy to see that $N = \prod_{j=1}^{n} |D| = 2^n$). Let $B_N =< C_N, \{\neg, \wedge, \vee\} >$ be Boolean algebra of N-dimensional binary vectors, where C_N is a set of these vectors. Our next goal is to establish the interconnection between G_n and B_N.

Let us create for every object and collection its corresponding code which is a sequence of 0 and 1. Let all objects be numbered from 1 to N. For every object p consider its following code:

$$c(p) = (\underbrace{0\ldots0 \quad 1 \quad 0\ldots0}_{N}),$$

where 1 occupies the position which number is equal to the number of p.

For every collection of objects consider its following code:

$$c(P) = \mathop{\vee}_{p\in P} c(p)$$

E.g. the collection 0_n has the code $C(0_n) = \underbrace{0\ldots0}_{N}$, and the collection 1_n has the code $c(1_n) = \underbrace{1\ldots1}_{N}$. Since all codes of collections have 1's in different positions, c establishes one-to-one correspondence between C_N and P_n.

Let us show that coding function c retains the operations on objects and collections, i.e. the code of result of any operation on a collection (a pair of collections) is equal to the result of the correspondent Boolean operation on the code of the given collection (the given pair of collections) executed componentwise.

Theorem 1. For any collections P and Q the following equations are fulfilled:

1) $\overline{c(P)} = c(\overline{P})$

2) $c(P) \wedge c(Q) = c(P \wedge Q)$

3) $c(P) \vee c(Q) = c(P \vee Q)$

Theorem 2. $G_n \cong B_N$.

The proof of the theorem 2 is based on the following. To show the isomorphism of G_n and B_N one need to build a mapping h from P_n to C_N [5], where h must have the next properties:

1) h is a homomorphism, i.e.

$\overline{h(P)} = h(\overline{P})$

$h(P) \wedge h(Q) = h(P \wedge Q)$

$h(P) \vee h(Q) = h(P \vee Q)$

2) h is bijective.

It is easy to see that the mapping c satisfies both properties because c is a homomorphism by theorem 1 and, as it was shown earlier, c is bijective. Hence the theorem 2 is proved.

The equivalence of precedent-related and logical encoding models of prior information follows from the theorem 2.

4. ALGORITHMS FOR CONVERSION OF ENCODING MODELS OF INFORMATION

Let us consider algorithmic implementations of conversion logical encoding model to precedent-related and vice versa. Let In_{12} be an algorithm for the conversion of logical model to precedent-related one (i.e. from encoding model used in problem Z_1 to the one used in problem Z_2), and In_{21} be an algorithm for the inverse conversion.

Suppose that all objects are preliminarily represented in such a way that all their signs take values from the set $D = \{0,1\}$.

Consider logical formula φ, which describes membership of an object to a class. φ takes n sign values from D as input parameters. Let $p(s_1, s_2, \ldots, s_n) = (d_1^p, d_2^p, \ldots, d_n^p)$ be the object, which is being considered. Suppose one needs to determine whether p belongs to Y. Let 0 as a result of φ mean that $p \notin Y$, and 1 mean that $p \in Y$:

$$\varphi(d_1^p,\dots,d_n^p)=\begin{cases}1, & p\in Y\\ 0, & p\notin Y\end{cases}$$

Thus $\varphi: D^n \to D$, i.e. φ is a Boolean function, hence φ can be represented as full DNF [6]:

$$\varphi(d_1^p,\dots,d_n^p)=\bigvee_{(s_1,\dots,s_n)} s_1^{d_1^p}\dots s_n^{d_n^p}\,\varphi(s_1,\dots,s_n) \qquad (1)$$

Here $s_i^{d_i^p}=\begin{cases}s_i, & d_i^p=1\\ \overline{s_i}, & d_i^p=0\end{cases}$.

Let r denote the number of elementary conjunctions (EC) in φ. Let us denote the j th EC as

$$\varphi_j(d_{j1}^p,\dots,d_{jn}^p)=s_1^{d_{j1}^p}\dots s_n^{d_{jn}^p}.$$

Then $\varphi(d_1^p,\dots,d_n^p)=\bigvee_{j=1}^{r}\varphi_j(d_{j1}^p,\dots,d_{jn}^p)$.

From the results of the previous chapter one can state that every logical formula φ has its correspondent collection of objects P which represents the set of objects described by φ. Therefore algorithm In_{12} must build such a collection P based on the formula φ, and algorithm In_{21} must build such a formula φ based on the collection P that the following equation is satisfied:

$$p\in P \Leftrightarrow \varphi(p)=1 \qquad (2)$$

Let us describe algorithm In_{12}. Consider formula φ in the form of (1). One need to obtain collection P, for which (2) is satisfied.

Algorithm In_{12}:

Step 1. Build for each EC $\varphi_j(d_{j1}^p,\dots,d_{jn}^p)=s_1^{d_{j1}^p}\dots s_n^{d_{jn}^p}$ in (1) an object p_j which has signs $s_1,\dots,s_n$ with correspondent values $d_{j1}^p,\dots,d_{jn}^p$:

$$p_j(s_1,s_2,\dots,s_n)=(d_{j1}^p,\dots,d_{jn}^p)$$

Step 2. Build collection $P=p_1\vee p_2\vee\dots\vee p_r$.

Step 3. Stop.

Let us describe algorithm In_{21}. Consider collection P. One need to obtain formula φ in the form of (1), for which (2) is satisfied.

Algorithm In_{21}*:*

Step 1. Build for each object $p_j(s_1,s_2,\dots,s_n)=(d_{j1}^p,\dots,d_{jn}^p)$, $p_j\in P$ an EC $\varphi_j(d_{j1}^p,\dots,d_{jn}^p)=s_1^{d_{j1}^p}\dots s_n^{d_{jn}^p}$.

Step 2. Build for collection P the next DNF:

$$\varphi(d_1^p,\dots,d_n^p)=\bigvee_{j=1}^{r}\varphi_j(d_{j1}^p,\dots,d_{jn}^p),$$

where r is the number of objects.

Step 3. Stop.

Let us show that (2) is satisfied for algorithms In_{12} and In_{21}. Let $P=In_{12}(\varphi)$ denote that collection P is the result of application of algorithm In_{12} to formula φ, and let $\varphi=In_{21}(P)$ denote that formula φ is the result of application of algorithm In_{21} to collection P.

Theorem 3. For arbitrary formula φ and object p statement (2) is satisfied, where $P=In_{12}(\varphi)$.

Theorem 4. For arbitrary collection P and object $p(s_1,s_2,\dots,s_n)=(d_1^p,d_2^p,\dots,d_n^p)$ statement (2) is satisfied, where $\varphi=In_{21}(P)$.

Corollary. The conversion made by algorithms In_{12} and In_{21} are mutually inverse: for arbitrary formula φ, collection P and object $p(s_1,s_2,\dots,s_n)=(d_1^p,d_2^p,\dots,d_n^p)$ the following statements are satisfied:

1) $\varphi(d_1^p,\dots,d_n^p)=1 \iff$

$\iff (In_{21}\circ In_{12}(\varphi))(d_1^p,\dots,d_n^p)=1$

2) $p\in P \iff p\in(In_{12}\circ In_{21}(P))$

5. MODIFICATION OF RESOLUTION METHOD FOR PROBLEM Z_2

The equivalence of precedent-related and logical encoding models lets to explore application of methods used for solution of problem Z_1 to problem Z_2.

Let us modify the resolution method in order to use it for the prior data given in precedent-related form. This modified resolution method is called object resolution method.

Consider objects p and q and sign h such that $d_h^p=1$, $d_h^q=0$. Assume h is the only sign with this property (i.e. values of other signs are equal or unknown at least in one object). An object resolvent is object r, values of whose signs are determined by the next rules:

1) $d_h^r=?$.

2) If an least in one object the value of sign s, where $s\neq h$, is unknown, i.e. $d_s^p=?$ or $d_s^q=?$, then $d_s^r=d_s^q$ or $d_s^r=d_s^p$ correspondently.

3) If in both objects the value of sign s, where $s\neq h$, is known, then $d_s^r=d_s^p\wedge d_s^q$.

Let us show that the operation of obtaining of an object resolvent is equivalent to the operation of obtaining a resolvent in the classical resolution method: the results of application of object resolution method to objects and application of classical resolution method to formulas, which describe these objects, are equal.

Theorem 5. Consider objects p and q. Suppose that $\exists! h$, $d_h^p=1$, $d_h^q=0$. Let r be the object resolvent for p and q, t be the object which corresponds to classical resolvent for formulas describing p and q. Then $t=r$.

Consider again problem Z_2, which was described at the beginning of this paper. Let us fix number i of class X_i and determine whether x belongs to X_i.

Object resolution algorithm A:

Stage 1. Forward chaining: obtaining object x from description of class X_i.

Step 1.1. Build DNF φ, which describes set X_i^0, e.g. using algorithm In_{12}:

$$\varphi=In_{12}(X_i^0)$$

Step 1.2. If $x\in\varphi$, then go to step 2.6, otherwise go to step 1.3.

Step 1.3. If all pairs of objects have been considered, then go to step 2.1. Otherwise select from φ such a pair of objects p and q, that have not been considered and $\exists! h$, $d_h^p = 1$, $d_h^q = 0$.

Step 1.4. Calculate values of signs of object resolvent r from values of signs of p and q:

$$d_h^r = ?$$

$$d_s^r = \begin{cases} d_s^q, d_s^p = ? \\ d_s^p, d_s^p \neq ? \end{cases}, \ \forall s \neq h$$

Step 1.5. Replace φ by the DNF which describes set $X_i^o \cup \{r\}$:

$$In_{12}(X_i^0 \cup \{r\})$$

Go back to step 1.2.

Stage 2. Backward chaining: obtaining object o from description of set $\overline{X_i^0}$.

Step 2.1. Build DNF ψ, which describes set $\overline{X_i^0}$, e.g. using algorithm In_{12}:

$$\psi = In_{12}(\overline{X_i^0})$$

Step 2.2. If $o \in \varphi$, then go to step 2.6, otherwise go to step 2.3.

Step 2.3. If all pairs of objects have been considered, then go to step 2.6. Otherwise select from φ such a pair of objects p and q, that have not been considered and $\exists! h$, $d_h^p = 1$, $d_h^q = 0$.

Step 2.4. Calculate values of signs of object resolvent r from values of signs of p and q:

$$d_h^r = ?$$

$$d_s^r = \begin{cases} d_s^q, d_s^p = ? \\ d_s^p, d_s^p \neq ? \end{cases}, \ \forall s \neq h$$

Step 2.5. Replace ψ by the DNF which describes set $\overline{X_i^0} \cup \{r\}$:

$$In_{12}(\overline{X_i^0} \cup \{r\})$$

Go back to step 2.2.

Step 2.6. Stop.

The result of algorithm A is interpreted as follows:

1) If algorithm stopped after stage 1 due to obtaining x, it means that, by replacing symbols "?" in the description of set X_i^0 with the actual values of signs, it is possible to obtain x. Hence object x belongs to class X_i.

2) If algorithm stopped after stage 2 due to obtaining o, it means that, by replacing symbols "?" in the description of set X_i^0 with the actual values of signs, it is possible to obtain any object, i.e. $\overline{X_i^0}$ potentially contains all objects from X. Hence object x can't belong to class X_i.

3) If no one of conclusions 1) and 2) is made, then it is impossible to make any statements about belonging of object x to class X_i by using algorithm A.

6. RECOGNITION ALGORITHM

There exists a lot of approaches to solving of problem Z_2. Let us describe a parametric family of algorithms suggested in [1] and compare it with algorithm A.

Consider the next function $\mu : X^2 \to [0,1]$:

$$\mu(x, y) = \max\{0, (\sum_{s \in S} (-1)^t a_{is}) \cdot (\sum_{s \in S} a_{is})^{-1}\},$$

where $\|a_{is}\|$ is a matrix which corresponds to set $\{1,\ldots,l\} \times |S|$, $a_{is} \in R$, $\forall i, j \ (a_{ij} \geq 0)$, $\forall i \ (\sum_j a_{ij} > 0)$; S is a set of all signs of all objects; $t = \begin{cases} 1, x \neq y, \\ 2, x = y. \end{cases}$ Here it is assumed that all values of signs of all objects are known. One of possible schemes for calculation of $\|a_{is}\|$ is also suggested in [1].

Let us describe now a recognition algorithm, which is denoted as B.

Recognition algorithm B:

Step 1. For each object $x \in X$ and each $x^0 \in X^0$ calculate $\mu(x, x^0)$.

Step 2. For each $i = \overline{1,l}$ calculate $P_i^B(x) = \max_{x_j^0 \in X_i^0} \{\mu(x, x_j^0)\}$.

Step 3. Stop.

It is easy to see that $P_i^B(x) \in [0,1]$, $i = \overline{1,l}$. Here it is assumed that $P_i^B(x)$ reflects closeness of object x to class X_i; if $P_i^B(x) = 1$, then $x \in X_i$.

Let $P_1,\ldots,P_l$ be predicates which characterize the true membership of objects to classes $X_1,\ldots,X_l$:

$$\forall x \in X \quad (P_i(x) \in \{0,1\} \wedge (P_i(x) = 1 \Leftrightarrow x \in X_i)).$$

Let us fix number i of class X_i and introduce the next quality functional for arbitrary algorithm A' which solves problem Z_2 on set $Y \subseteq X$

$$\Phi_{A'}(Y) = 1 - \frac{1}{|Y|} \sum_{x \in Y} \left| P_i(x) - P_i^{A'}(x) \right|$$

Given several algorithms, that one would be preferable, for which the value of this quality functional is the greatest.

Let us form two classification vectors based on results of algorithms A and B:

$$A(x) = (P_1^A(x),\ldots,P_l^A(x))$$

$$B(x) = (P_1^B(x),\ldots,P_l^B(x))$$

It is necessary to represent the results of algorithm A in numerical form. Let $P_i^A(x)$ be defined as follows:

$$P_i^A(x) = \begin{cases} 0, x \notin X_i \\ 1, x \in X_i \\ -1, \textit{otherwise} \end{cases}$$

If object x can be deduced from the description of class X_i, it is denoted as $X_i \to x$. If addition of x to class X_i leads to a logical contradiction, it is denoted as $X_i \otimes x$. It is easy to see that if $X_i \to x$, then $P_i(x) = 1$;

if $X_i \otimes x$, then $P_i(x) = 0$.

Theorem 6. Given $Y_i = \{x \mid X_i \to x\}$, $Y_i' = \{x \mid X_i \otimes x\}$, the following inequalities are satisfied:

1) $\Phi_B(Y_i \cup Y_i') \le \Phi_A(Y_i \cup Y_i')$;

2) $\Phi_A(X \setminus (Y_i \cup Y_i')) \le \Phi_B(X \setminus (Y_i \cup Y_i'))$.

From theorem 6 one can conclude that it is preferable to use algorithm A on set $Y_i \cup Y_i'$ and algorithm B on set $X \setminus (Y_i \cup Y_i')$.

7. COMBINED ALGORITHM

Let us combine algorithms A and B in order to reach the best results. Let us denote this combined algorithm as C and show that $\Phi_C(X)$ is not less than $\Phi_A(X)$ and $\Phi_B(X)$.

Object resolution and recognition algorithms have been described above in details; therefore, they are included in the description of algorithm C as its separate steps. Let us fix number i of class X_i.

Combined algorithm C:

Step 1. Choose object $x \in X$.

Step 2. Apply object resolution algorithm A to object x and class X_i.

Step 3. If $P_i^A(x) \in \{0,1\}$, then go to step 5, other wise go to step 4.

Step 4. Apply recognition algorithm B to object x.

Step 5. If all objects have been considered, then go to step 6, other wise go to step 2.

Step 6. Stop.

Form a classification vector based on results of algorithm C:

$$C(x) = (P_1^C(x), \ldots, P_l^C(x)),$$

where $P_i^C(x) \in [0,1]$.

Theorem 7. $\Phi_C(X) \ge \max\{\Phi_A(X), \Phi_B(X)\}$.

Hence algorithm C, which combines object resolution and recognition algorithms, works not worse than algorithms A and B on whole set X.

8. CONCLUSION

Two approaches to solving of pattern recognition problems are considered in this paper. The first approach has deductive nature (the resolution method modified for pattern recognition problems), while the second has inductive nature (classical pattern recognition algorithms). Logical and precedent-related encoding models of information are used in these approaches correspondently. The algebra of objects is suggested for the case of precedent-related encoding model. It is proved that this algebra of objects is isomorphic to Boolean algebra used in case of logical encoding model. Algorithms for mutual conversion of two encoding models are developed. Object resolution method is suggested as a modification of the classical resolution method for the case of precedent-related encoding model. A new algorithm combining object resolution method and a parametric family of recognition algorithms is developed.

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