$$t_1 = \mu(\alpha_0 + \mu\alpha_1),\, t_2 = \mu(\delta_0 + \mu\delta_1),\, \lambda_1 = 1 + \mu(\beta_0 + \mu\beta_1),$$
$$\lambda_2 = 1 + \mu(\gamma_0 + \mu\gamma_1),\, \theta = \theta_0 + \mu\theta_1,\, z = \mu(z_0 + \mu z_1). \quad (4)$$

The first approximation of the asymptotic approach gets

$$Q^s = 6g_2 + 8g_3 - \frac{8g_3{}^2}{4g_3 + 3g_1}.$$

This result is in the good agreement with the numerical one for small μ.

If $Q < Q^s$ then the asymptotic solution (4) should be constructed separately for $\varphi_s < \varphi < \pi$ and $0 < \varphi < \varphi_s$. In particular, φ_s satisfies the approximate equation

$$\sin(\varphi_s) - \varphi_s \cos(\varphi_s) = \pi Q / Q^s .$$

Numerical results show that in the post-critical equilibrium state the membrane is fully stretched and the functions t_1, t_2, λ_1 and λ_2 are sufficiently large for small values of μ. One can search the approximate solution of equations (1) and (2) for $Q < Q^s$ in the form

$$t_1 = \frac{\alpha_0}{\mu} + \alpha_1,\, \lambda_1 = \frac{\beta_0}{\mu} + \beta_1,\, \lambda_2 = \frac{\gamma_0}{\mu} + \gamma_1,\, t_2 = \frac{\delta_0}{\mu} + \delta_1,\, \theta = \theta_0 + \mu\theta_1,\, \zeta = \zeta_0 + \mu\zeta_1.$$

In the first approximation

$$\lambda_1 = \frac{\sqrt{2B_1B_2}}{\mu Q},\, \lambda_2 = \frac{B_1}{\mu Q} + \frac{\sqrt{2B_1B_2}}{Q}\cos(\varphi),\, B_1 = g_1 + g_3,\, B_1 = g_2 + g_3. \quad (5)$$

The relative error of formulas (5) in comparison with the numerical results for $g_1 = g_1 = g_1 = 0{,}5$, $Q = 1{,}5$ and $\mu < 0{,}2$ is less than 3%.

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INITIAL-VALUE PROBLEMS IN GENERAL ASYMPTOTIC THEORY FOR THIN WALLED ELASTIC STRUCTURES

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The vast majority of publications on asymptotic analysis of 3D problems for thin walled elastic structures deal only with equations of motion. Much less work is done on asymptotic justification and refinement of boundary conditions. Until recently [1] the asymptotic theory has not treated initial-value problems.

In this talk the 3D dynamic equations in linear elasticity are subject to asymptotic analysis in case of a thin plate. A typical scale of the initial data along the middle plane of the plate is assumed to be much greater than its thickness. At the same time there is no restriction on the variation of the initial data across the thickness.

A composite asymptotic procedure is developed starting from the 2D long-wave approximations of the original 3D problem. These include not only the Kirchhoff plate theory and its low-frequency refinements demonstrating polynomial displacement variation across the thickness but also high-frequency theories approximating sinusoidal variation, e.g. [2].

A hierarchy of asymptotic iterative processes is established for the initial data with arbitrary transverse variation. A variety of initial-value problems is derived including an asymptotic corrector to the classical initial conditions for a thin plate. The prospective of extending the proposed approach to a shell of general shape is addressed.

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THIN INCLUSIONS WITH DELAMINATIONS IN ELASTIC BODIES

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In the talk, we discuss new models to describe a state of equilibrium for elastic bodies with thin inclusions. It is assumed that a delamination of the inclusions takes place thus forming a crack between the elastic body and the inclusion. Presence of inclusions and cracks in elastic bodies is a source of stress concentrations. Moreover, a rigidity of the inclusion is one of the parameters responsible for crack propagations. There are elastic and rigid inclusions as well as inclusions with a zero rigidity (cracks). We consider the free boundary approach for modeling the phenomenon. In particular, nonlinear boundary conditions of inequality type are considered at the crack faces to prevent a mutual penetration. This approach is much more favorable from the mechanical standpoint as compared to classical linear boundary conditions. Remark that in the case of a rigid inclusion new types of nonlocal boundary conditions appear. This is due to the fact that the inclusion is not fixed, as it is for instance for rigid substrates, but moves rigidly to balance the external forces and the pressure exerted by the surrounding elastic body. Correct mathematical formulations of the problems are proposed. Different problem statements are considered equivalent to each other. We prove a solution existence for the suitable free boundary problems and analyze other properties of the solution for different locations of thin inclusions with respect to the external boundary. Moreover, we analyze