SOME NON-CLASSIC MODELS OF BEAMS, PLATES AND SHELLS

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Introduction. We refer to the Kirchhoff-Love (KL) model of beams, plates and shells as to a classic model. A short issue of some non-classic models is given here. Among them there are the Timoshenko-Reissner (TR) models, the plates and shells made of a material with the general anisotropy, the multi-layered plates, the plates and shells lying on the elastic foundation. We are bounded with the static problems, with the vibration and buckling problems. The problems of the non-stationary waves propagation [1, 2] are not discussed here. Also such important problems as the 2D models of the higher than 10 orders [3, 4], the 2D Cosserat shells [5], the shells made of a momentous elastic material [6], and many others remain out of consideration.

The TR model. The first step of the KL theory generalization is the TR theory including a transversal shear. The differential order of the TR theory is equal to 10, and the boundary conditions formulation in this theory is simpler compared with the KL theory. But sometimes the TR theory is asymptotically incorrect [7, 8] in the following sense. Accept that at the asymptotically correct 2D shell theory the wave length in the tangential directions of the internal stress strain state (SSS) is much larger than the shell thickness. Consider a plate or a shell made of isotropic or orthotropic material and suppose that the transversal shear elastic modulus is of the same asymptotic order as the rest modules. Then among solutions of the TR theory equations there are solutions of the boundary layer type which quickly decrease away from the shell edges. But in all cases it is impossible to describe correctly the boundary layer by the 2D theory [9]. The exactness of the internal SSS given by the TR theory is the same as the exactness of the KL theory. Therefore for such materials the TR theory is useless.

The small transversal shear modulus. The opposite answer is in the case when the transversal shear modulus is much smaller than the rest elastic modules. In this case the TR theory is asymptotically correct and it gives results which are essentially more exact than the results of the KL theory. In all cases the exactness is estimated by comparison with the solutions of the 3D problems which have the exact or the asymptotically exact solutions. Note that if the transversal shear modulus is very small then the 2D theories are unacceptable [8]. As an example the classical problem of a circular cylindrical shell axial compression is studied and the difference between isotropic and anisotropic materials is discussed [10].

The general anisotropy of material. Consider plates and shells which made of materials with the general anisotropy describing by 21 elastic modules [11, 12]. To deliver the 2D model it is necessary to accept the generalized TR cinematic hypotheses. Only in this case we get the model which is asymptotically correct in the zero approximation with respect to the small thickness parameter. The obtained system of 10^{th} order is comparatively new and it is not sufficiently investigated. For plates this system does not divide in two parts describing the tangential and the bending deformations. For shells the typical SSS are constructed, namely the membranous state, the pure bending state, the edge effect. For the mixed state the system of Donnell's type is delivered. By using this system some static, dynamic and buckling problems are solved. The main problem which is to be solved for this model consist in excluding the boundary layer and to reduce this system to the system of 8^{th} order (in the case when all elastic modules are of the same asymptotical orders). The difficulty is to correctly formulate the boundary conditions for this system. Also the case when the asymptotical orders of elastic modules are different is to be investigated. As an examples the multi-layered plates and the plates and shells reinforced by fibers are studied. The averaging of elastic properties leads to the anisotropic material.

The elastic foundation. The plates and shells lying on the elastic foundation are studied. The exactness of Winkler's, Pasternak's, and Ilgamov's models is discussed. The typical vibration and buckling modes for shells on the foundation are the local modes [13]. Dependence of the critical load on the Gaussian curvature of the shell midsurface is investigated.

The compressed plate on foundation. The buckling modes of an infinite compressed plate lying on the elastic half-space is investigated. Firstly [14] it is established that the set of modes $w(x,y) = w_0 \sin px \sin qy$ with $p^2 + q^2 = r^2 = const$ corresponds to the critical compression. By using the post-critical non-linear analysis it is found that the minimum energy corresponds to the chessboard-like mode ($p = q$). In [15] the issue of mechanical and biological phenomena accompanied with the chessboard-like buckling modes is presented. The exact analysis is given in [16]. It is established that in the linear approximation the any solution of the Helmholtz equation $\Delta w + r^2 = 0$ corresponds to the critical load. Examples of the possible buckling modes are presented. Note that the modes of free vibrations of an infinite plate on the foundation are the same as the buckling modes. Nonlinear analysis of the super-critical behavior of a compressed plate shows again that to the chessboard-like mode the minimum energy corresponds.

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ОПРЕДЕЛЕНИЕ ФИЗИКО-МЕХАНИЧЕСКИХ СВОЙСТВ БИОЛОГИЧЕСКИХ ТКАНЕЙ НА ОСНОВЕ МОДЕЛЕЙ ВЯЗКОУПРУГОСТИ ДРОБНОГО ПОРЯДКА

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Углубление и расширение знаний в механизме распространения механических волн в вязкоупругих материалах и структурах имеет большое значение для дальнейшего развития разнообразных прикладных технологий. Такие технологии имеют большой спектр приложений, как в медицине, так и в биомеханике [1, 2]. В качестве примера можно привести совершенствование и развитие теоретических основ методов медицинской томографии (динамической эластографии), совершенствование методик определения состояния биотканей. Использование в методиках построения изображений биологических тканей подходов, основанных на неинвазивных измерениях движения волн сдвига в мягких биологических тканях, позволяет получать уникальную пространственную локализованную информацию о свойствах материала. Методики, использующие такую информацию, позволяют проследить во времени развитие патологий и изменений биомеханических свойств биотканей. Кроме того, технологии, основанные на изучении распространения механических волн,