One of the perspective fields of SPM development is to obtain information about the subsurface (deep) structure of the materials especially techniques of non-destructive nanotomography. They are based on the fact that the mechanical interaction of the tip with the sample in contact and hard tapping modes leads to a local deformation of the materials. Therefore, the images of topography and different contrasts (lateral forces, phase shift) contain information about the depth of deformation, and are the sensitive to the thickness of «soft» material layer, which cover the «hard» clots. Changing the scan parameters we can «see» these clots, in spite of the fact that they are coated with softer layer. There are some investigations which demonstrated the possibility of subsurface structure imaging, mainly polymeric composites if they used a multi-pass scanning technique with the changing the tip-surface interaction (operation parameters). Here the material of the layers is elastically deformed and does not undergo irreversible changes during the scanning process. Additional aspect in SPM application is the destructive action of the tip on the sample. Under this approach, the indentation methods, scratching, wearing on the nanometer scale is realized.

Thus, using the measurement methods in the field of micro-and nanomechanic on the bases of the scanning probe microscopy is present a wide opportunity to assess the physical and mechanical properties of materials at the nanoscale, and the modeling of precision contact area of the surface. It is necessary to develop adequate models of contact and non-contact interaction in the system tip-sample for receiving more accurate solution.

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FAILURE ANALYSIS OF METALLIC SHELLS, BEAMS AND FRAMES

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Introduction. The concepts (theories, analytical solution procedures, finite element formulations) that have been derived for analysis of large, medium-size or small structures and structural components (like those in civil, aerospace and mechanical engineering) can be readily used to analyze behavior of micro-structures and nano-structures. Of course, (non-trivial) modifications and further developments of now well understood and matured concepts of structural analysis might be necessary. That might relate to constitutive equations and multi-physics, i.e. strongly (or weakly) coupled problems that consider simultaneously (or sequentially) two or more physical problems related to a structure (e. g. thermo-mechanical, electro-thermo-mechanical, etc.). Moreover, some specific problems may arise when dealing with micro-structures and nano-structures. Nevertheless, in our view many aspects of behavior of micro-structures and nano-structures (in different fields of engineering, medicine and natural sciences) can be addressed by the available methodologies of structural analysis.

Modeling of material and structural failures. With the above in mind, we aim to present our experience in numerical (finite element) failure analysis of ductile, metallic (elastoplastic) shell, beam and frame structures. Local material failure in elastoplastic structures produces localized plastic deformation, sometimes called shear band (in solids), (softening) yield line (in plates and shells), and (softening) plastic hinge (in beams). To successfully deal numerically with this kind of localization (or material instability) phenomena, several different (local and non-local) approaches (often called localization limiters, e.g. [1]) have been proposed. We have used in our work two different kinds of localization limiters (one for shell finite elements and another one for beam finite elements) in order to eliminate finite-element-mesh lack of objectivity in dealing with localized softening inelastic behavior.

In what follows, we will shortly present the following: (i) geometrically and materially nonlinear shell computational model that can be used for failure analysis of metallic shells, (ii) geometrically and materially nonlinear beam computational model that can be used for failure analysis of metallic beams and frames, and (iii) multi-level (i.e. shell-beam or meso-macro) computational paradigm to failure analysis of frames. The later takes into account both kind of local instabilities, i.e. geometrical (local buckling) and material (localized plastic deformation), that considerably contribute to softening structural response (i.e. decreasing in loading with simultaneous increasing in displacements).

Computational failure model for metallic shells. Our shell computational model is based on geometrically exact shell finite element formulation. It describes kinematics of one director shell model without any simplifications (i.e. finite rotations of the shell director are described without any singularities, and they are unrestricted in size). We consider small-strain elastoplastic constitutive equations with isotropic and kinematic hardening, defined either in terms of stress-resultants (i.e. by in-plane and shear forces, and moments) or in terms of five components of the stress tensor. We note that it is very difficult to derive consistent elastoplastic stress-resultant constitutive equations for shells. Also, the resulting equations are complex. For example, the consistent shell counterpart of von Miseselasto-plasticity for solids (called Ilyushin-Shapiro elasto-plasticity) has two surface yield criterion. That calls for special integration algorithms for internal variables in the corresponding numerical finite element formulation, as shown e.g. in [2]. Our shell computational model uses the simplest localization limiter that follows the idea of using strain-softening constitutive relations and adjusting the material softening parameters such that the computed plastic dissipation in the softening regime remains the same regardless of the chosen finite element mesh. We note that this kind of modification is much clearer if shell constitutive equations are defined in terms of stresses. With the shell computational model, shortly described above, we can perform failure analysis of shells and other thin-walled structures. Such failure analysis takes into account global (as well as local) buckling and localized material failure (material softening).

Computational failure model for metallic beams and frames. To perform failure analysis of beams and frames, we derived planar, stress resultant, beam finite element formulation. It takes into account geometrical nonlinearity only approximately by von Karman approximations of non-linear strains. In order to accommodate localization limiter based on deformation discontinuity, we enriched standard Euler-Bernoulli beam kinematics by a strong-discontinuity jump in cross-section rotation. Small-strain elasto-plastic constitutive relations with hardening are defined in terms of beam moment and axial force. When a cross-section of a beam reaches its failure capacity, the softening plastic hinge appears at that place. Rigid softening plastic constitutive law is applied at the softening plastic hinge, relating the hinge-moment and the rotation-jump. With such beam formulation, failure of metallic frames can be analyzed. However, one has to note that possibility of local buckling of a frame structural element (i.e. local buckling of a flange) and localized material failure (i.e. failure of material of part of cross-section) can only be considered through the applied moment-curvature and moment versus jump-in-rotation curves. This observation led us to apply a two-scale (shell computational model as a meso scale and beam computational model as a macro scale) failure analysis of frames, shortly described below, see [3] for details.

Two-scale (shell-beam) approach to failure analysis of metallic frame structure. The basic idea goes as follows. Take a representative unit of a frame structural element, and perform its failure analysis (at some pre-described level of axial force) by the computational shell model that is shortly described above. Result of such analysis is a moment-rotation curve. By using this curve one can construct beam constitutive relation data [2]: moment-curvature and softening-plastic-hinge moment versus jump-in-rotation curves. Those curves naturally incorporate local geometric and material failures, since the shell computational model (the meso-scale model) is able to capture them. The obtained curves are further used for failure analysis of the entire frame performed by the beam computational model (the macro-scale model) shortly described above. Such two-level (two-scale) analysis combines the better of two worlds. On one side, there is effectiveness and robustness of the (macro-scale) beam computational model that is used for analysis of entire structure. On another side, there is a refined representation of localized instability effects (both geometric and material) by the (meso-scale) shell computational model. The latter is captured and stored within the (macro-scale) beam model in a manner which is compatible with enhanced beam kinematics with embedded strong-discontinuity in rotation. The applied multi-scale procedure is weakly coupled, since computations are carried out sequentially (results of the shell model computations are stored to be used within the beam model). One of main features of described approach is that detection and development of softening plastic hinges in the frame is fully automatic (and spreads gradually in accordance with stress redistribution in the course of the nonlinear analysis).

Conclusions. Examples of failure analysis of metallic shell, beam and frame structures, as well as examples of two-scale (shell-beam) failure analysis of metallic frame structure will be shown at the conference. We believe that shell and beam computational models and failure analysis procedures, shortly described above (see [2], [3] for further details), can be also successfully applied for micro- and nano-structures of shell, beam or frame type that are made of different metallic (or other elasto-plastic) materials.

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FINITE AXISYMMETRIC DEFORMATION OF AN INFLATABLE ANISOTROPIC TOROIDAL MEMBRANE MADE OF NEO-HOOKEAN MATERIAL

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Introduction. Textile composites and pneumatic structures have become increasingly popular for a variety of applications in a civil engineering, architecture, aerospace engineering, etc. [1]. A toroidal membrane of the circular cross section, composed of an isotropic elastic material and inflated by the uniform internal pressure has been considered in [2]. Equilibrium states of the membrane composed of an anisotropic (reinforced of fibers) material were found in [3–4].

Basic equations. It is supposed that the toroidal membrane is made of a cylindrical textile composite pipe which contains two systems of threads located on parallels and meridians. The lengths of not deformed threads are equal accordingly L and l. We assume that the fibers are disposed frequently enough. After averaging we get the dimensionless differential equations, describing the axisymmetric deformation of the anisotropic toroidal membrane

$$\frac{d\theta}{d\varphi}=\frac{\mu}{t_1}\left(\lambda_1\lambda_2 Q-\tilde{t}_2\cos(\theta)\right),\ \frac{dt_1}{d\varphi}=-\mu\tilde{t}_2\sin(\theta),$$
$$\frac{d\lambda_2}{d\varphi}=-\mu\lambda_1\sin(\theta),\ \frac{dz}{d\varphi}=\mu\lambda_1\cos(\theta),\ \tilde{t}_2=\max\{t_2,0\},\ 0\le\varphi\le 2\pi. \tag{1}$$

For the incompressible neo-Hookean material