SUBSECTION 2.2. MECHANICS OF MEMBRANES AND FILMS AND ITS APPLICATION TO BIOPHYSICS AND MEDICINE

NANO-SCALE SHELL THEORY-BASED ESTIMATION OF THE ELASTIC CHARACTERISTIC OF BIOLOGICAL CELLS

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Mechanical properties are fundamental properties of cells and tissues. They characterize a number of cytophysiological and cytopathological processes. The cell mechanical parameters are possible to use as certain markers of the pathology [1, 2]. The study of the elastic properties allows to obtain new knowledge about biological cells and also is of clinical interest. That’s why the development of theoretical models and experimental methods of the cell mechanics are actual task. Early we proposed the method of the local elastic modulus estimation at nanoscale using a spherical indenter [3]. And now the goal is to create a theoretical model of this process using the nano-scale shell model proposed before for nano-scale thin-walled objects [4].

On the bases of the shell theory [5] we consider the elastic properties of the cells and compare them with the experimental results which was received using atomic force microscopy (AFM) date. In experiments the AFM function of force spectroscopy was used. It is possible to obtain force curve by recording the cantilever deflection while the tip is in contact with the body. The force curve contains the information about long- and short-range interactions and represents a basis for estimation of sample elastic (Young’s) modulus. Also it is possible to evaluate the adhesion between the AFM tip and the cell. We noticed that membrane interacted with the tip and stretched for some time and decided to solve the task about spherical shell under the action of normal force directed outwards [6]. To describe the interaction between the AFM tip and the cell under stretching deformation, we have constructed new solution that can be used to calculate the elastic modulus on the basis of the force spectroscopy:

\[
\begin{align*}
\w &= \w^* + \w^{**}, \\
\w^* &= \frac{P l^2}{2\pi D} \left(-\k E (x) - k_R \left(1 + \nu\right) \left(\frac{\pi}{2} Y_0 \left(c\sqrt{2k_R}\right) + \k E(x)\right) + \frac{1}{2} c \k E'(x)\right) + \\
&\quad + \left(\eta - \varepsilon\right) \k E(x) + \frac{\eta}{4} c \k E'(x), \\
\w^{**} &= \frac{P \left(\varepsilon_0 a\right)^2}{2\pi D} \left(\k E(x) + k E(x) \left(\eta \left(2 - \varepsilon\right) - \varepsilon - k_R \left(3 + \nu\right)\right)\right).
\end{align*}
\]
\[ D = \frac{Et^3}{12(1-v^2)}, \quad l = \sqrt{\frac{rt}{12(1-v^2)}}, \quad \varepsilon = \frac{vt^2}{10(1-v^2)l^2}, \quad \eta = \frac{t^2}{5(1-v)l^2}, \quad k_R = \left( \frac{1}{r} \right)^2, \]

where \( w \) is stretching of the material, which is equal the difference between the coordinate tip contact point and the coordinate of separation when the AFM tip withdraw from the surface (moving at material stretching), \( P \) is applied load, \( r \) is radius of the shell \( t \) is thickness of the shell, \( C \) is the contact area \( Y_0 \) is Bessel function of the second type and of the zeroth order, \( kei(x) \) and \( ker(x) \) are Thompson (Kelvin) functions, \( E \) is Young’s modulus of the sample, \( v \) is Poisson’s ratio of the sample, \( \epsilon_0 \) is the material constant of nonlocality, and \( a \) is the internal characteristic length of the material.

It is assumed that the stresses and displacements are very small at some distance from the force application point, the tangent moving along the middle surface is considerably smaller then the movement in the normal direction, the shell is infinite in all directions (to eliminate the influence of the boundary effects), the shell material is elastic and conforms to the nonlocal theory of elasticity [4, 7].

From the point of view of the indentation, the radius of a circle corresponds to the contact area. Contact is provided by the forces of adhesion (we consider the withdrawal of the tip out of the contact). Thus, in order to present the equation of the known decision, which could be used to calculate the modulus of elasticity, we used the theory of Johnson – Kendall – Roberts. This theory is taking into account the effect of cohesion. Then some transformations were made and as the result, the values of the elastic modulus and contact area were obtained from the system of equations. The values of load and displacement were chosen on the basis of experimental data. Shell radius was assumed to be 2 \( \mu m \) (average thickness of the red blood cell membrane) [8]. The shell thickness initially wondered equal to 10 nm, which corresponds to the average thickness of the cell membrane.

Thus it was shown that in practice the nonlocal shell theory may be useful if it is necessary to evaluate the elastic properties of the cell membrane.

References

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**PULSATILE FLOWS IN DISTENSIBLE TUBES: A MEMBRANE MODEL WITH FLUID-STRUCTURE INTERACTION**

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**Introduction.** Fluid flows in distensible tubes exhibit different types of instabilities due to the fluid-structure interaction and the energy transfer at the interface [1]. Different fluid- and wall-based instable modes can be found in straight and curved tubes [2]. In numerous technical and biomedical systems providing fluid delivery and distribution through the distensible ducts the instabilities may lead to the flow limitation phenomena, high frequency oscillations of the wall, noise generation and destruction of the system. In the living organisms the flows of biological fluids through the vessels, ducts and cavities are also accompanied by development of instabilities that may play either constructive or destructive role [3].

Biofluids are usually considered as Newtonian or non-Newtonian liquids, while there are still no accepted agreements for the solid wall modeling. The viscoelastic thick wall model was found to be useful in application to large and medium arteries, veins and lymphatic vessels [4]. Different shell models have been proposed for the small blood vessels, ducts of glands and hollow organs [5]. Viscoelastic behavior is proper to the normal blood vessel walls, while the bending rigidity decreases rapidly with age-related wall loosening or development of aneurism. The 2D rigid tube with a membrane insertion has been proposed and studied as a model of the aneurism [6,7]. Here the membrane model describing fluid-structure interaction in the fluid flow through the flexible duct is developed. Some preliminary results for the axisymmetric case have been reported in [8].

**Problem formulation.** The pulsatile flow of the incompressible inviscid fluid in the thin-walled tube treated as a membrane is studied. At an arbitrary time moment the position of the tube is described by the vector \( \mathbf{r}(t) = r(t, \theta_0, x_0) \mathbf{e}_r(\theta) + x(t, \theta_0, x_0) \mathbf{e}_x \) (fig. 1), where \( \theta = \theta(t, \theta_0, x_0) \). At \( t = 0 \) one can have \( \mathbf{r}(t_0) = \tilde{r}_0 \mathbf{e}_r(\theta_0) + \tilde{x}_0 \mathbf{e}_x \). The wall displacement is \( \mathbf{u} = \mathbf{r}(t) - \mathbf{r}(t_0) \).

The governing equation for the membrane is considered in the form

\[
\rho_m \frac{\partial^2 \mathbf{u}}{\partial t^2} = \left( p_i - p_e \right) \mathbf{n} + \mathbf{f}_e, \quad \frac{\partial^2 \mathbf{u}}{\partial t^2} = \left( \mathbf{r} - r \dot{\theta}^2 \right) \mathbf{e}_r(\theta) + \left( r \dot{r} \dot{\theta} + r \dot{\theta} \right) \mathbf{e}_x(\theta) + \mathbf{u},
\]

where \( \rho_m \) is the surface mass density of the membrane, \( p_i \) and \( p_e \) are the hydrostatic pressures inside and outside the tube, \( \mathbf{f}_e \) is the elastic forces in the membrane per unit surface, \( \mathbf{n} \) is the normal vector to the membrane surface. The elastic force