Equivalence principle and experimental tests of gravitational spin effects

Alexander J. Silenko*

Institute of Nuclear Problems, Belarusian State University, Minsk 220080, Belarus

Oleg V. Teryaev†

Bogoliubov Laboratory of Theoretical Physics,

Joint Institute for Nuclear Research, Dubna 141980, Russia

(Dated: June 20, 2012)

Abstract

We study the possibility of experimental testing the manifestations of equivalence principle in spin-gravity interactions. We reconsider the earlier experimental data and get the first experimental bound on anomalous gravitomagnetic moment. The spin coupling to the Earth's rotation may also be explored at the extensions of neutron EDM and g−2 experiments. The spin coupling to the terrestrial gravity produces a considerable effect which may be discovered at the planned deuteron EDM experiment. The Earth's rotation should also be taken into account in optical experiments on a search for axionlike particles.

PACS numbers: 04.20.Cv, 04.25.Nx, 04.80.Cc

*Electronic address: silenko@inp.minsk.by

†Electronic address: teryaev@thsun1.jinr.ru

I. INTRODUCTION

Equivalence principle is known to be one of the basic postulates of the modern physics, constituting the cornerstone of General Relativity. Its simplest and well-known counterpart corresponds to the equality of inertial and gravitational mass and is tested with good accuracy. The equivalence principle is also manifested in the interaction of spin with gravity, as it was first shown in the seminal paper of I. Yu. Kobzarev and L. B. Okun [1]. It means the absence of both the anomalous gravitomagnetic moment (AGM) and the gravitoelectric dipole moment which are gravitational analogs of the anomalous magnetic moment and the electric dipole moment (EDM), respectively. It may be derived as a low energy theorem due to the conservation of momentum and orbital angular momentum [2].

Relations obtained by Kobzarev and Okun predict equal frequencies of precession of quantum (spin) and classical (orbital) angular momenta and the preservation of helicity of Dirac particles in gravitomagnetic fields (i.e., the fields defined by the components g_{i0} of the metric tensor, see Ref. [3] and references therein).

This holds in any reference frame in which such components of metric tensor appear. In particular, one should mention the inertial frame (where the gravitomagnetic field may be created by the rotation of massive body) and the rotating noninertial frame. These properties of spin-gravity interaction were explored in the number of theoretical papers and suggestions for experiments (see Refs. [4, 5, 6] and references therein). There are also some evidences supporting the conjecture [7] that the absence of the AGM is valid separately for quarks and gluons in the nucleon, which may be related to the phenomena of confinement and spontaneous chiral symmetry breaking.

The experimental tests of the Kobzarev-Okun relations are lacking and are therefore of much importance. The problem of existence of the dipole spin-gravity coupling in a static gravitational field has been discussed for a long time (see Refs. [4, 5, 8] and references therein). Evidently, this coupling given in the form of $\boldsymbol{S}\cdot\boldsymbol{g}$ ($\boldsymbol{g}$ is the acceleration) contradicts to the theory [4, 6] and violates both the CP invariance and the relation predicting the absence of the gravitoelectric dipole moment [1]. Therefore, the negative result of realized experiments imposes some restrictions not only on the spin-gravity coupling but also on the gravitoelectric dipole moment.

In this article we carefully reanalyze the results of spin experiments with Hg atoms and get the first experimental bound on their AGMs. We also suggest to extend some experiments with spinning particles for testing the absence of the AGMs and calculate related gravitational effects.

II. SPIN COUPLING TO GRAVITY AND ROTATION

Spin rotation due to the action of the Earth's gravity is [4, 6]

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{\Omega}_g \times \boldsymbol{S}, \quad \boldsymbol{\Omega}_g = -\frac{2\gamma + 1}{\gamma(\gamma+1)mc^2}\boldsymbol{g} \times \boldsymbol{p}, \tag{1}$$

where $\boldsymbol{S}$ is the spin vector and γ is the Lorentz factor. In deriving this equation, we neglected small corrections due to the derivatives from the metric tensor including those depending on curvature. The maximum value of Ω_g is $2g/c$.

We have carried out the relativistic generalization of the pioneering results [9] on the effect of Earth's rotation on particle spin. We have performed the *exact* Foldy-Wouthuysen transformation (see Ref. [10]) of Dirac Hamiltonian [9]. For the particle in the rotating frame, this Hamiltonian takes the form ($\hbar = c = 1$)

$$\mathcal{H} = \beta m + \mathcal{E} + \mathcal{O}, \tag{2}$$

where

$$\mathcal{E} = -\boldsymbol{\omega} \cdot \boldsymbol{J}, \quad \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}, \quad \mathcal{O} = \boldsymbol{\alpha} \cdot \boldsymbol{p}. \tag{3}$$

$\boldsymbol{\omega}$ is the angular frequency of the Earth's rotation, $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$ and $\boldsymbol{S} = \boldsymbol{\Sigma}/2$ are the angular momentum and spin operators, $\boldsymbol{p} = -i\nabla$ is the momentum operator, $\mathcal{E}$ and $\mathcal{O}$ mean even and odd terms that commute and anticommute with the matrix β, respectively. We use commonly accepted definitions of Dirac matrices [11].

The Hamiltonian in the Foldy-Wouthuysen representation is given by

$$\mathcal{H}_{FW} = \beta\sqrt{m^2 + \boldsymbol{p}^2} - \boldsymbol{\omega} \cdot \boldsymbol{J}. \tag{4}$$

and the lower spinor in this representation is zero.

It is very important that exact Hamiltonian (4) does not contain any quantum corrections to the classical Hamiltonian derived by Mashhoon [12] and that is also agrees with the earlier result by Gorbatsevich [13]. The equal coupling of rotation to orbital and spin momenta (which is not true for a magnetic field) is a manifestation of the absence of the AGMs.

The particle motion is characterized by the operators of velocity and acceleration:

$$v^i\equiv\frac{dx^i}{dx^0}=i[\mathcal{H},x^i],\quad x^0\equiv t,$$
$$w^i\equiv\frac{dv^i}{dx^0}=i[\mathcal{H},v^i]=-\left[\mathcal{H},[\mathcal{H},x^i]\right]. \tag{5}$$

In the considered case, these operators are equal to

$$\boldsymbol{v}=\beta\frac{\boldsymbol{p}}{\epsilon}-\boldsymbol{\omega}\times\boldsymbol{r},\quad \epsilon=\sqrt{m^2+\boldsymbol{p}^2},$$
$$\boldsymbol{w}=2\beta\frac{\boldsymbol{p}\times\boldsymbol{\omega}}{\epsilon}+\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r})$$
$$=2\boldsymbol{v}\times\boldsymbol{\omega}-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{r}). \tag{6}$$

Eq. (6) also results in the quantum formula for the force acting on the relativistic particle which coincides with the classical formula [14] for the sum of the Coriolis and centrifugal forces. Thus, the classical and quantum approaches are in full agreement.

III. EXPERIMENTS WITH ATOMS AND COLD NEUTRONS

AGM would manifest itself in the coupling to any (nonflat) metric, in particular, in the case of rotating frames. In this section, we discuss the emerging opportunity to test the absence of the AGMs provided by experiments with atoms and cold neutrons. The experiments are performed with two kinds of atoms (or with neutrons and atoms) designated by indices 1 and 2.

Let us reconsider the earlier results [15] as restrictions on the AGM rather than on the dipole spin-gravity coupling. Recall that latter violates not only the Kobzarev-Okun relation for the gravitoelectric dipole moment but also CP invariance and may be neglected. The spin-dependent Hamiltonian for atoms in S states may be obtained by the modification of the coefficient of the term defining the spin-rotation coupling and has the form

$$\mathcal{H}=-g\mu_N\boldsymbol{B}\cdot\boldsymbol{S}-\zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S},\quad \zeta=1+\chi, \tag{7}$$

where g is the nuclear g factor, μ_N is the nuclear magneton, and χ is the AGM. The measured ratio of energy differences in neighboring Zeeman levels, $R=|\nu_2|/|\nu_1|$, depends on the AGMs. The difference of these ratios for two opposite directions of magnetic field is given by

$$R_+-R_-=\pm\frac{2f\cos\theta}{|\nu_1|}\left(\zeta_2-\mathcal{G}\zeta_1\right),\quad \mathcal{G}=\frac{g_2}{g_1}, \tag{8}$$

where θ is the angle between the directions of magnetic field and the Earth's rotation axis, $f = \omega/(2\pi) = 11.6$ μHz is the Earth's rotation frequency, and $|\nu_1|$ is the Zeeman frequency for atoms of the first kind. The experimental conditions of [15] for ^{199}Hg and ^{201}Hg atoms correspond to $\theta \approx 0$, $\mathcal{G} = -0.369139$. Reconsidering the bound for $R_+ - R_-$ obtained in that Ref., we drop the contribution of CP-violating gravitoelectric dipole moment, but account for the possibility for nonzero AGM, which makes a difference between (8) and their Eq. (4). As a result, their data lead to the following restriction:

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.}).$$

To our best knowledge, this is the first experimental bound on the AGM, and consequently the first test of the Kobzarev-Okun relations. The sensitivities of similar experiments fulfilled with deuterium [16] and beryllium [17] atoms are not sufficient to obtain significant restrictions.

Another experiment is fulfilled at Institute Laue-Langevin (ILL) with ultracold neutrons placed in electric and magnetic fields [18] and aimed to search for their EDM. There is a recent claim [19] that spin-rotation coupling should be already taken into account when analyzing the data obtained. To address the problem of testing the absence of the AGMs, the data for the opposite directions of magnetic field should be considered separately, while averaging over the directions of electric field should be performed. The correction for the Earth's rotation is rather large and corresponds to the EDM of 1.7×10^{-24} e·cm when $E = 10$ kV/cm. The expected sensitivity of this experiment to the AGM is also of order of 10^{-2}. It is also possible [20] to use the magnetic resonance methods for atomic and molecular beams.

Spin coupling to the Earth's rotation may in principle also be investigated in the GRANIT (GRAvitational Neutron Induced Transitions) [21, 22] experiment, where quantum states of cold neutrons in the terrestrial gravitational field were observed.

Ultracold neutrons can also be used in interferometer experiments with rotating spin-flippers [23] and implemented at the existing and developed interferometers at ILL and Tokai [24]. It seems reasonable to have two (rather than one as suggested in [23]) rotating spin-flippers. Signals should be absent if they are rotated in the same directions. In the case of rotation in opposite directions, signals should be twice larger in comparison when only one flipper rotates.

IV. COMPARISON OF SPIN-ROTATION COUPLING FOR ELECTRONS AND POSITRONS

The above mentioned experiments do not solve the important problem of the equivalence of gravitational effects for particles and antiparticles which may be tested in the storage rings. The corresponding equation of spin motion in a cylindrical coordinate system [25] with an addition of the gravitational correction is given by

$$\frac{d\boldsymbol{S}}{dt} = \boldsymbol{\omega}_a \times \boldsymbol{S}, \quad \boldsymbol{\omega}_a = \boldsymbol{\Omega} + \boldsymbol{\Omega}_{EDM} + \boldsymbol{\omega} + \boldsymbol{\Omega}_g, \tag{9}$$

where $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}_{EDM}$ are angular velocities of spin rotation caused by magnetic and electric dipole moments, respectively. The (pseudo)vectors $\boldsymbol{\Omega}$ and $\boldsymbol{\Omega}_{EDM}$ are oppositely directed for particles and antiparticles. The gravitational corrections to the angular velocity of spin rotation in Cartesian and cylindrical coordinates are $-\boldsymbol{\omega} + \boldsymbol{\Omega}_g$ and $\boldsymbol{\omega} + \boldsymbol{\Omega}_g$, respectively.

It is the quantity ω_a which is measured in storage ring and Penning trap experiments. The Earth's rotation can simulate the CPT violation because it brings a fictitious difference between g factors of electron and positron. The measurements of electron and positron g factors in the Penning trap at the level of accuracy of order of 0.1 Hz [26, 27] were not sensitive to the Earth's rotation.

To make the gravitational corrections observable, it is desirable to use a relatively weak magnetic field in order to decrease the spin rotation frequency as much as possible. Since this frequency is proportional to the cyclotron one, the particle trajectory should be extended and gravitational experiments should be performed in storage rings. is equal

The best condition for the comparison of the spin-rotation coupling for electrons and positrons is perhaps provided by the use of a muon g−2 ring (namely, the 7.11 m ring of the Brookhaven National Laboratory). The electron/positron beam polarization may be measured with the methods described in Refs. [31, 32]. The frequency of spin rotation (g−2 frequency) actually measured with the accuracy of 0.16 Hz (0.7 ppm) [29] is almost the same for muons and electrons/positrons. The best sensitivity of experiment with electrons and positrons can be achieved with electric focusing and the "magic" Lorentz factor ensuring a dramatic reduction of the influence of the electric field on the spin rotation and resulting in a small width of resonance line [28, 29].

The sensitivity of the proposed experiment is not affected by the systematical error in measurement of the magnetic field because it is the same for electrons and positrons and therefore is canceled in the difference $\omega_a(e^+) - \omega_a(e^-)$. To compare the sensitivities of the proposed experiment and the muon g−2 experiment, one should take into consideration only systematical errors due to the electric field and the fitting procedure. The systematical errors caused by the horizontal and vertical coherent betatron oscillations (0.07 and 0.04 ppm, respectively, for the muons [29]) are much less important for the electrons and positrons because the decay time of these oscillations ($\sim 100\ \mu$s [29]) is very small in comparison with the beam circulation time. These systematical errors can additionally be reduced due to the fact that the electric focusing is 207 times stronger for the electrons/positrons than for the muons. The shift of the precession frequency due to the electric field depends on the momentum spread and is given by (Eqs. (17) and (21) in Ref. [29])

$$\frac{\delta\omega_a}{\omega_a} = -2\beta^2 \frac{n}{1-n} \left(\frac{p-p_0}{p_0}\right)^2, \tag{10}$$

where n is the field index and $\beta = v/c$. The momentum spread, $(p-p_0)/p_0$, equal to 0.5% for the muons [29] can be considerably less (right up to 10^{-6} [30]) for the electron and positron beams providing a great reduction of the systematical error. In the muon g−2 experiment, this systematical error was ∼0.01 Hz [29], i.e., about 10% of the electric field correction and about 1% of the linewidth $\sqrt{<(\delta\omega_a)^2>}$. We suppose that a relation between these quantities cannot be very different in the proposed experiment. If the momentum spread of the electrons/positrons is 5×10^{-5} (two orders of magnitude less than for the muons) and $n \leq 0.2$, the linewidth is reduced 10^4 times in comparison with the muon g−2 experiment. Even if the related systematical error would be $6\% \div 8\%$ of the linewidth, the resulting error of frequency determination is about 10 μHz. Besides the comparison of gravitational spin-rotation coupling for particles and antiparticles, the restriction on the CPT violation would also be improved.

V. EFFECT OF EARTH'S GRAVITY ON SPIN DYNAMICS IN STORAGE RINGS

To measure the effect of the Earth's gravity on spin dynamics, one needs to detect the spin rotation about a horizontal axis. The detection can be provided if the particle spin is governed by a uniform upward magnetic field and a resonant longitudinal electric one ($\boldsymbol{E}\|\boldsymbol{v}$). This field configuration corresponds to the resonant deuteron electric-dipole-moment (dEDM) experiment [33].

When magnetic focusing is used, the gravitational force acting on particles, $\boldsymbol{F}_g = (2\gamma^2 - 1)m\boldsymbol{g}/\gamma$, defines the nonzero radial magnetic field which causes the spin turn with the average angular velocity

$$\boldsymbol{\omega}_m = \frac{(1+a\gamma)(2\gamma^2-1)}{mc^2\gamma(\gamma^2-1)}\boldsymbol{g}\times\boldsymbol{p}. \tag{11}$$

The resulting angular velocity $\boldsymbol{\omega}_a$ has the vertical and radial components and is given by

$$\boldsymbol{\omega}_a = \Omega_z\boldsymbol{e}_z + \boldsymbol{\Omega}_{EDM} + \boldsymbol{\Omega}_g + \boldsymbol{\omega}_m, \tag{12}$$

while the average radial component of angular velocity of Earth's rotation is zero. If we disregard terms describing systematical errors, Eq. (12) takes the form (here and below $\hbar = c = 1$)

$$\begin{aligned}\boldsymbol{\omega}_a = -\frac{ea}{m}B_z\boldsymbol{e}_z - \frac{d}{S}\left(\frac{1}{\gamma}\boldsymbol{E} + \boldsymbol{\beta}\times\boldsymbol{B}\right)\\ +\frac{\left[1+a(2\gamma^2-1)\right]\gamma}{\gamma^2-1}\boldsymbol{g}\times\boldsymbol{\beta},\end{aligned} \tag{13}$$

where d is the EDM and S is the spin quantum number.

In Eq. (13), the quantities $\boldsymbol{E}$ and $\boldsymbol{\beta}$ oscillate at a near-resonant frequency (see Ref. [34]). The resulting buildup of the vertical polarization calculated by the method elaborated in Ref. [34] is equal to

$$\begin{aligned}P_z = -\frac{1}{2}P_0\Delta\beta_m\sin\left(\psi-\varphi_m\right)\Big\{\frac{d}{S}B_0\left(1+a\gamma^2\right)\\ +g|\sin\Phi|\frac{\gamma^3}{\gamma^2-1}\left[1-a(2\gamma^2-3)\right]\Big\}t,\end{aligned} \tag{14}$$

where $\Delta\beta_m \equiv \Delta v_m/c$ and φ_m characterize the resonant modulation of the beam velocity [33], ψ is the azimuthal angle of spin direction (with respect to the $\boldsymbol{e}_\rho$ axis) at zero time, Φ is the geographic latitude, and P_0 is the polarization of the incident beam. In the planned dEDM experiment, the Earth's gravity would bring the effect identical to that given by the deuteron EDM of $d = 2\times10^{-29}$ e·cm. This effect is rather important, because the expected sensitivity of the dEDM experiment [33] is of the same order.

It is rather difficult to incorporate the AGM χ into Eqs. (13),(14) in the general case because the initial Dirac equation does not contain the AGM. However, the AGM can be inserted into Eq. (1) in the nonrelativistic approximation when one keeps quantities of order of β and neglects those of order of β^2. In this approximation, the angular velocity of spin rotation, $\boldsymbol{\Omega}_g$, should be proportional to the total gravitomagnetic moment and may be obtained by the modification of the respective coefficient:

$$\boldsymbol{\Omega}_g = -\frac{3\zeta}{2c}\boldsymbol{g}\times\boldsymbol{\beta}.$$

As a result, the quantity $\chi\boldsymbol{\Omega}_g/\zeta$ should be added to the right-hand side of Eq. (13). Other terms in Eq. (12) are not affected by the AGM and Eq. (13) takes the form ($c = 1$)

$$\begin{aligned}\boldsymbol{\omega}_a = &-\frac{ea}{m}B_z\boldsymbol{e}_z - \frac{d}{S}\left(\frac{1}{\gamma}\boldsymbol{E} + \boldsymbol{\beta}\times\boldsymbol{B}\right)\\ &+\left\{\frac{\left[1 + a(2\gamma^2 - 1)\right]\gamma}{\gamma^2 - 1} - \frac{3\chi}{2}\right\}\boldsymbol{g}\times\boldsymbol{\beta}.\end{aligned} \tag{15}$$

In the planned deuteron EDM experiment, $\gamma = 1.28$ [33] and such a consideration of the AGM is applicable. However, one should take into account that magnetic focusing does not affect a particle at rest and the equality $\beta = 0$ results in the divergence of the quantity $\boldsymbol{\omega}_m$.

VI. OPTICAL EFFECTS CAUSED BY THE EARTH'S ROTATION

Photon polarization is significantly influenced by the Earth's rotation. When frequencies of left-circularly and right-circularly polarized electromagnetic waves coincide in an inertial frame, they differ in a rotating frame [35]. This effect has been observed (see Ref. [36] and references therein). The plane of polarization of a linearly polarized electromagnetic wave rotates in a stationary (but nonstatic) spacetime (Skrotskii effect [37]). This effect results in an optical rotation of electromagnetic wave in vacuum caused by the Earth's rotation and defined by $d\phi/dl = \boldsymbol{\omega}\cdot\boldsymbol{l}_0/c$ [38], where $\boldsymbol{l}_0$ is the unit vector pointing in the wave direction and $\omega/c = 2.43\times10^{-10}$ rad/km. This relatively large optical rotation has not been taken into consideration in the Brookhaven, Fermilab, Rochester, Trieste (BFRT) [39] and PVLAS [40] experiments on a search for axionlike particles.

In the PVLAS experiment, the light direction is vertical, $\boldsymbol{\omega}\cdot\boldsymbol{l}_0 = \omega\sin\Phi$, and the effect of the Earth's rotation is $d\phi/dl = 1.73\times10^{-10}$ rad/km. This value corresponds to the optical rotation $\alpha_E = 1.1\times10^{-12}$ rad/pass and therefore is of the same order as the much discussed effect observed by the PVLAS collaboration: $\alpha = (3.9\pm0.5)\times10^{-12}$ rad/pass [40]. Evidently, the Skrotskii effect can be discovered in the framework of the PVLAS experiment and it can be used for checking the sensitivity. It can also be measured in a similar experiment performed without magnetic field.

The effect of the Earth's rotation did not become apparent in the BFRT experiment because all effects independent of the angle between the plane of polarization and the magnetic field direction were eliminated [39].

VII. CONCLUSIONS

There is a number of possibilities to measure the coupling of spin to rotation and gravity and therefore to verify the Kobzarev-Okun relations. We suggest the reinterpretation of earlier experiment with atomic spin [15] leading to the first check of the AGMs at few per cent level of accuracy. The straightforward extensions of experiments with (ultra)cold neutrons can also provide the important test of the absence of the AGMs. Possible gravitational experiment in the g$-$2 ring enables to compare the spin-rotation coupling for particles (electrons) and antiparticles (positrons). The proposed extension of the deuteron EDM experiment gives an exciting opportunity to detect the spin-gravity coupling existing only for moving particles. The Earth's rotation should be taken into account in optical experiments on a search for axionlike particles, where observed effect [40] is of the same order as that of the Earth's rotation.

Acknowledgements

This work was supported in part by the Belarusian Republican Foundation for Fundamental Research (Grant No. F06D-002), the Deutsche Forschungsgemeinschaft (Grant No. 436 RUS 113/881/0), the Russian Foundation for Basic Research (Grant No. 03-02-16816), and the Russian Federation Ministry of Education and Science (Grant No. MIREA 2.2.2.2.6546). We have been informed that another group intends to make an estimate of the level of perturbation on several precision experiments involving spin [41]. We are grateful to C.J.G. Onderwater for this information and helpful correspondence. We are also indebted to G. Papini for interest in our work and bringing to our attention Refs. [42, 43, 44]. O.T. is thankful to V. Nesvizhevsky, K. Protasov, H. Rauch and H. Shimizu for useful discussions.

---

[1] I. Yu. Kobzarev, L. B. Okun, Zh. Eksp. Teor. Fiz. **43**, 1904 (1962) [Sov. Phys. JETP **16**, 1343 (1963)].

[2] I. Yu. Kobzarev and V. I. Zakharov, Ann. of Phys. **37**, 1 (1966).

[3] O. V. Teryaev, arXiv:hep-ph/9904376 (1999).

[4] A.J. Silenko and O.V. Teryaev, Phys. Rev. D **71**, 064016 (2005).

[5] B. Mashhoon, Lect. Notes Phys. **702**, 112 (2006).

[6] A. A. Pomeransky and I. B. Khriplovich, Zh. Eksp. Teor. Fiz. **113**, 1537 (1998) [J. Exp. Theor. Phys. **86**, 839 (1998)].

[7] O. V. Teryaev, Czech. J. Phys. **53**, A47 (2003); arXiv:hep-ph/0612205.

[8] Yu. N. Obukhov, Phys. Rev. Lett. **86**, 192 (2001).

[9] F. W. Hehl and W. T. Ni, Phys. Rev. D **42**, 2045 (1990).

[10] A. J. Silenko, J. Math. Phys. **44**, 2952 (2003).

[11] L. L. Foldy, S. A. Wouthuysen, *Phys. Rev.* **78**, 29 (1950).

[12] B. Mashhoon, Phys. Rev. Lett. **61**, 2639 (1988).

[13] A. Gorbatsevich, Exp. Tech. Phys. **27**, 529 (1979).

[14] R. T. Gangadhara, Astron. Astrophys. **314**, 853 (1996).

[15] B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson, Phys. Rev. Lett. **68**, 135 (1992).

[16] D. J. Wineland and N. F. Ramsey, Phys. Rev. A **5**, 821 (1972).

[17] D. J. Wineland, J. J. Bollinger, D. J. Heinzen, W. M. Itano, M. G. Raizen, Phys. Rev. Lett. **67**, 1735 (1991).

[18] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006).

[19] S. K. Lamoreaux and R. Golub, Phys. Rev. Lett. **98**, 149101 (2007).

[20] J. Anandan, Phys. Rev. Lett. **68**, 3809 (1992).

[21] V. V. Nesvizhevsky *et al.*, Phys. Rev. D **67**, 102002 (2003).

[22] V. V. Nesvizhevsky *et al.*, Eur. Phys. J. C **40**, 479 (2005).

[23] B. Mashhoon, R. Neutze, M. Hannam and G. E. Stedman, Phys. Lett. A **249**, 161 (1998); B. Mashhoon and H. Kaiser, Physica B **385**, 1381 (2006).

[24] H. Rauch, private communication (2006), H. Shimizu, private communication (2006).

[25] A. J. Silenko, Phys. Rev. ST Accel. Beams **9**, 034003 (2006).

[26] R. S. Van Dyck, Jr., P. B. Schwinberg, and H. G. Dehmelt, *Phys. Rev. Lett.* **59**, 26 (1987).

[27] H. Dehmelt, R. Mittleman, R. S. Van Dyck, Jr., and P. Schwinberg, Phys. Rev. Lett. **83**, 4694 (1999).

[28] F. J. M. Farley and Y. K. Semertzidis, Prog. Part. Nucl. Phys. **52**, 1 (2004).

[29] Muon g-2 Collaboration (G. W. Bennett *et al.*), Phys. Rev. D **73**, 072003 (2006).

[30] A. O. Sidorin, private communication (2006).

[31] C. K. Sinclair, AIP Conf. Proc. **451**, 23 (1998).

[32] K. Laihem, S. Riemann, A. Schälicke, A. Ushakov, R. Dollan, T. Lohse, in Proceedings of the 10th European Particle Accelerator Conference, Edinburgh, 2006, p. 2475.

[33] Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).

[34] A. J. Silenko, arXiv:hep-ph/0604095.

[35] B. Mashhoon, Phys. Lett. A **198**, 9 (1995).

[36] N. Ashby, Liv. Rev. Rel. **6**, 1 (2003).

[37] G. V. Scrotskii, Dokl. Akad. Nauk SSSR **114**, 73 (1957) [Sov. Phys. – Dokl. **2**, 226 (1957)].

[38] S. Kopeikin and B. Mashhoon, Phys. Rev. D **65**, 064025 (2002).

[39] R. Cameron *et al.*, Phys. Rev. D **47**, 3707 (1993).

[40] E. Zavattini *et al.* (PVLAS Collaboration), Phys. Rev. Lett. **96**, 110406 (2006).

[41] C.J.G. Onderwater, private communication (2006).

[42] G. Papini, G. Lambiase, Phys. Lett. A **294**, 175 (2002).

[43] G. Papini, Phys. Rev. D **65**, 077901 (2002).

[44] G. Lambiase and G. Papini, Phys. Rev. D **70**, 097901 (2004).