Given a number $\varepsilon \geq 0$, denote

$$\mathrm{Er}\,\{\mathrm{Ptb}\,(f,\bar{x},\varepsilon)\}(\bar{x}) := \inf_{g\in \mathrm{Ptb}\,(f,\bar{x},\varepsilon)} \mathrm{Er}\, g(\bar{x}).$$

This number characterizes the error bound property for the whole family of convex ε-perturbations of f near $\bar{x}$. Obviously, $\mathrm{Er}\,\{\mathrm{Ptb}\,(f,\bar{x},\varepsilon)\}(\bar{x}) \leq \mathrm{Er}\, f(\bar{x})$.

Corollary 2. *The following properties are equivalent:*

(i) $\mathrm{Er}\,\{\mathrm{Ptb}\,(f,\bar{x},\varepsilon)\}(\bar{x}) > 0$ *for some* $\varepsilon > 0$*;*
(ii) $0 \notin \mathrm{bd}\,\partial f(\bar{x})$.

For a detailed study of stability of local and global error bounds for convex functions and convex semi-infinite constraint systems of the form

$$f_t(x) \leq 0 \quad \text{for all} \quad t \in T,$$

where T is a compact Hausdorff space, $f_t : X \to \mathbb{R}$, $t \in T$, are given continuous convex functions such that $t \mapsto f_t(x)$ is continuous on T for each $x \in X$, we refer the reader to [1, 2].

Acknowledgements. The research was partially supported by the Australian Research Council, project DP110102011; XLIM, University of Limoges and NAFOSTED.

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DAD SYSTEMS OF CONTROL AND OBSERVATION

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Scientific-technical progress, in particular, the widespread use of microprocessors in industry, subjects the development of new control and observation systems and the support of existing ones to new, higher requirements related to the necessity of a more adequate description of these systems and the use of their specific properties, which often leads to hybrid dynamical systems.

However, note that there is no common viewpoint to the notion of "hybrid systems".

From our viewpoint, being hybrid means, in general, being inhomogeneous in the nature of the considered process or in its investigation methods. The notion "hybrid systems" can be used for systems that describe processes or objects with essentially distinct characteristics, for example, containing continuous and discrete variables (signals) in the basic dynamics, deterministic and random variables or inputs, and so on, which, in the end, defines the character (nature) of hybrid systems.

In the lecture, we consider differential-algebraic time-delay (DAD) systems to which, in particular, some standard types of discrete-continuous and systems with retarded argument of neutral type can be reduced. Such systems can be qualified as hybrid difference-differential systems or quite regular DAD systems which, in turn, a special case of descriptor (singular, implicit) systems with after-effect.

We deal with linear DAD systems consisting of differential and difference equations. We study the stability of solutions of such systems and derive necessary and sufficient conditions for their asymptotic and exponential stability. In the scalar case, these conditions are refined and expressed via the original coefficients of the system in parametric form, which permits one to keep track of how the perturbations in the coefficients affect the solutions and to find the limiting value of the delay for which stability is preserved.

A determining equation system is introduced and a number of algebraic properties of the determining equation solutions is established, in particular, the well-known Cayley-Hamilton matrix theorem is generalized to the solutions of determining equation. As a result, an effective parametric reachability and observability rank criteria are given.

We pay attention to the simplest DAD control and observation system of normal form:

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + B_1u(t), \tag{1}$$

$$x_2(t) = A_{21}x_1(t) + A_{22}x_2(t-h) + B_2u(t), \; t \geq 0, \tag{2}$$

with the initial conditions

$$x_1(0) = x_{10} \in R^{n_1}, \; x_2(\tau) = \psi(\tau), \; \tau \in [-h, 0), \tag{3}$$

and the output

$$y(t) = C_1x_1(t) + C_2x_2(t), \; t \geq 0, \tag{4}$$

where $A_{11} \in R^{n_1 \times n_1}$, $A_{12} \in R^{n_1 \times n_2}$, $B_1 \in R^{n_1 \times r}$, $A_{21} \in R^{n_2 \times n_1}$, $A_{22} \in R^{n_2 \times n_2}$, $B_2 \in R^{n_2 \times r}$, $C_1 \in R^{m \times n_1}$, $C_2 \in R^{m \times n_2}$,$\psi \in PC([-h, 0], R^{n_2})$; the external action $u(t)$ for $t \geq 0$ is a piecewise continuous r-vector function (admissible control); the symbol $PC(J, \mathfrak{M})$ denotes the set of piecewise continuous $\mathfrak{M}$-valued functions in the interval J.

We regard an absolutely continuous function $x_1(\cdot)$ and a piecewise continuous function $x_2(\cdot)$ as a solution of System (1)–(3) if it satisfies the initial conditions (3), it satisfies the equation (2) for $t \geq 0$ and Equation (1) almost everywhere (a. e.) for $t \geq 0$. If Equation (1) is satisfied for all $t \geq 0$ with right-hand value at $t = 0$ then we consider the solution $x_1(\cdot)$, $x_2(\cdot)$ as a strong solution of the system.

Computing the solution $x_1(t) = x_1(t, x_{10}, \psi, u)$, $x_2(t) = x_2(t, x_{10}, \psi, u)$, $t \geq 0$, of the system (1)–(3) by "step by step" one can prove that it exists, is unique, and its growth rate does not exceed an exponential one for any admissible control having no higher than the exponential rate of growth. This permits to apply the Laplace transform to the system.

Introduce matrix-valued functions $X^*_{i1}(\cdot)$, $X^*_{i2}(\cdot)$, and $Z^*_i(\cdot)$ as the solutions of the following *adjoint* system:

$$\dot{X}^*_{i1}(t) = X^*_{i1}(t)A_{11} + X^*_{i2}(t)A_{21},\ t \in (jh, (j+1)h),\ j = 0, 1, ...; \tag{5}$$

$$X^*_{i2}(t) = X^*_{i1}(t)A_{12} + X^*_{i2}(t-h)A_{22},\ t \geq 0, \tag{6}$$

$$X^*_{i1}(kh+0) - X^*_{i1}(kh-0) = Z^*_i[k]A_{21}, \tag{7}$$

$$Z^*_i[k] = Z^*_i[k-1]A_{22},\ k = 1, ...; \tag{8}$$

$$X^*_{i2}(\tau) = 0,\ \tau < 0;\ i = 1, 2; \tag{9}$$

with initial conditions of the form:

$$X^*_{11}(0) = X^*_{11}(-0) = X^*_{11}(+0) = I_{n_1},\ Z^*_1[0] = 0; \tag{10}$$

$$X^*_{21}(0) = X^*_{21}(-0) = A_{21},\ Z^*_2[0] = I_{n_2}. \tag{11}$$

Here and throughout the following, the symbol I_k stands for the identity k by k matrix.

The matrix-functions $X^*_{i1}(t)$, $t \leq 0$, $i = 1, 2$; are assumed to be left continuous. It is not difficult to check that $X^*_{11}(t)$ and $X^*_{12}(t) - X^*_{12}(t-h)A_{22}$ are continuous for $t \geq 0$.

Then the solution $x_1(t) = x_1(t, x_{10}, \psi)$, $x_2(t) = x_2(t, x_{10}, \psi)$ of System (1), (2), (3) can be computed by the formulas:

$$x_1(t) = X^*_{11}(t)x_{10} + \int_0^h X^*_{12}(t-\tau)A_{22}\psi(\tau - h)d\tau + \tag{12}$$

$$+\int\limits_0^t (X_{11}^*(t-\tau)B_1 + X_{12}^*(t-\tau)B_2)u(\tau)d\tau,$$

$$x_2(t) = X_{21}^*(t+0)x_{10} + \int\limits_0^h X_{22}^*(t-\tau)A_{22}\psi(\tau-h)d\tau + Z_2^*[T_t]A_{22}\psi(t-T_t h-h)+$$

$$+\int\limits_0^t (X_{21}^*(t-\tau)B_1 + X_{22}^*(t-\tau)B_2)u(\tau)d\tau + \sum_{k=0}^{T_t} Z_2^*[k]B_2 u(t-kh), \quad (13)$$

where $T_t = [\frac{t}{h}]$ is the integer part of $\frac{t}{h}$ that can be given in the universal form as Variation-of-Constants Formula (generalized *Couchy formula*):

$$x_i(t) = X_{i1}^*(t+0)x_{10} + \int\limits_0^h X_{i2}^*(t-\tau)A_{22}\psi(\tau-h)d\tau + Z_i^*[T_t]A_{22}\psi(t-T_t h-h)+$$

$$+\int\limits_0^t (X_{i1}^*(t-\tau)B_1 + X_{i2}^*(t-\tau)B_2)u(\tau)d\tau + \sum_{k=0}^{T_t} Z_i^*[k]B_2 u(kh),$$

$$t > 0,\ i = 1, 2. \quad (14)$$

Using the formula (14), we obtain effective parametric criteria for stability, reachability and observability of the considered DAD system.

ON AN EQUATION IN DYNAMICS OF ECOLOGICAL PROCESSES WITH DELAY

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The use of functional-differential equations while modeling the population dynamics processes has begun apparently with V. Volterra's papers. At first differential equations with one concentrated delay were considered. Then there appeared papers on several delays, with variable, integro-differential equations. Lately, partial equations with delay argument are considered.

In the paper we consider mathematical model of dynamics described by partial differential equation with one constant delay

$$u_t(x,t) = a_1^2 u_{xx}(x,t) + a_2^2 u_{xx}(x,t-\tau)+$$