

MINIMUM ENERGY CONTROL OF POSITIVE 2D CONTINUOUS-DISCRETE LINEAR SYSTEMS

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The new formulation of the minimum energy control problem for the positive 2D continuous-discrete linear systems is proposed. Necessary and sufficient conditions for the reachability of the systems are established. Conditions for the existence of the solution to the minimum energy control problem and procedures for computation of an input minimizing the given performance index are given. Effectiveness of the procedure is demonstrated on numerical example.

Introduction. A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs [5, 10]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc..

The positive 2D continuous-discrete linear systems have been introduced in [14], positive hybrid linear systems in [11] and the positive fractional 2D hybrid systems in [13]. Different methods of solvability of 2D hybrid linear systems have been discussed in [23] and the solution to singular 2D hybrids linear systems has been derived in [25]. The realization problem for positive 2D hybrid systems has been addressed in [15]. Some problems of dynamics and control of 2D hybrid systems have been considered in [4, 6]. The problems of stability and robust stability of 2D continuous-discrete linear systems have been investigated in [1-3, 9, 24-27] and of positive fractional 2D continuous-discrete linear systems in [12]. Recently the stability and robust stability of general model and of Roesser type model of scalar continuous-discrete linear systems have been analyzed by Busłowicz in [2, 3]. Stability of continuous-discrete 2D linear systems has been considered in [17]. The minimum energy control problem for standard linear systems has been formulated and solved by J. Klamka [19-22] and for 2D linear systems with variable coefficients in [18]. The controllability and minimum energy control problem of fractional discrete-time linear systems has been investigated by Klamka in [22]. The minimum energy control of fractional positive continuous-time linear systems has been addressed in [7] and for descriptor positive discrete-time linear systems in [8].

In this paper a new formulation and solution to the minimum energy control problem for positive 2D continuous-discrete linear systems will be presented.

The paper is organized as follows. In section 2 necessary and sufficient conditions for the positivity of 2D continuous-discrete linear systems are established. The reachability and the problem formulation are given in section 3. Problem solution and a procedure for solving the minimum energy control problem are given in section 4. Concluding remarks are given in section 5.

The following notation will be used: \mathfrak{R} - the set of real numbers, $\mathfrak{R}^{n \times m}$ - the set of $n \times m$ real matrices, $\mathfrak{R}_+^{n \times m}$ - the set of $n \times m$ matrices with nonnegative entries and $\mathfrak{R}_+^n = \mathfrak{R}_+^{n \times 1}$, M_n - the set of $n \times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n \times n$ identity matrix.

1. Positivity of 2D continuous-discrete systems. Consider the 2D continuous-discrete linear system

$$\dot{x}(t, i) = Ax(t, i) + Bu(t, i) \quad (1)$$

where $\dot{x}(t, i) = \frac{\partial x(t, i)}{\partial t}$, $x(t, i) \in \mathfrak{R}^n$, $u(t, i) \in \mathfrak{R}^m$ are the state and input vectors and $A \in \mathfrak{R}^{n \times n}$, $B \in \mathfrak{R}^{n \times m}$ ($n \geq m$) and $t \in \mathfrak{R}_+$ is continuous variable (usually time) and $i \in Z_+$ is discrete variable.

Definition 1. *The system (1) is called (internally) positive if $x(t, i) \in \mathfrak{R}_+^n$, $t \in \mathfrak{R}_+$, $i \in Z_+$ for any boundary conditions $x_{0i} \in \mathfrak{R}_+^n$, $x_{t0} \in \mathfrak{R}_+^n$, $\dot{x}_{t0} \in \mathfrak{R}_+^n$ and all inputs $u(t, i) \in \mathfrak{R}_+^m$, $t \in \mathfrak{R}_+$, $i \in Z_+$.*

Theorem 1. *The system (1) is positive if and only if*

$$A \in M_n \text{ and } B \in \mathfrak{R}_+^{n \times m}. \quad (2)$$

Proof. *Necessity.* Let $u(t, i) = 0$, $t \geq 0$ and $x(0, i) = e_i$ (i -th ($i = 1, \dots, n$) column of the identity matrix I_n). The trajectory does not live the orthant \mathfrak{R}_+^n only if the derivative $\dot{x}(0, i) = Ae_i \geq 0$, what implies $a_{ij} \geq 0$, $i \neq j$. Therefore, the matrix A has to be the Metzler matrix. For the same reasons for $x(0, i) = 0$ we have $\dot{x}(0, i) = Bu(0, i) \geq 0$, what implies $B \in \mathfrak{R}_+^{n \times m}$, since $u(0, i) \in \mathfrak{R}_+^m$ may be arbitrary for $i \in Z_+$.

Sufficiency. The solution of the equation (1) is given by

$$x(t, i) = e^{At}x(0, i) + \int_0^t e^{A(t-\tau)}B(\tau, i)d\tau. \quad (3)$$

It is well-known [10, 16] that $e^{At} \in \mathfrak{R}_+^{n \times n}$, $t \in \mathfrak{R}_+$ if and only if $A \in M_n$. From (3) it follows that if the conditions (2) are met and $x(0, i) \in \mathfrak{R}_+^n$, $u(t, i) \in \mathfrak{R}_+^m$, $t \in \mathfrak{R}_+$, $i \in Z_+$ then $x(t, i) \in \mathfrak{R}_+^n$, $t \in \mathfrak{R}_+$, $i \in Z_+$. Hence by Definition 1 the system (1) is positive. \square

2. Reachability and problem formulation. Consider the 2D continuous-discrete linear system (1).

Definition 2. *The positive system (1) is called reachable in segment of line $\{[t_f, 0], [t_f, q]\}$ if for any given final state vector $x_f \in \mathfrak{R}^n$ there exists an input $u(t, i) \in \mathfrak{R}_+^m$, $0 \leq t \leq t_f$, $0 \leq i \leq q$ that steers the state vector $x(t, i)$ of the system from $x(0, i) = 0$, $i = 0, 1, \dots, q$, to $x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q)$.*

Theorem 2. *The positive system (1) is reachable on the segment of line $\{[t_f, 0], [t_f, q]\}$ if and only if the matrix $A \in M_n$ is diagonal and the matrix $B \in \mathfrak{R}_+^{n \times m}$ is monomial.*

Proof. Using (3) for $t = t_f$, $i = 0, 1, \dots, q$ and $x(0, i) = 0$, $i = 0, 1, \dots, q$ we obtain

$$x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q) = \int_0^{t_f} e^{A(t_f - \tau)} \bar{B} \bar{u}(\tau) d\tau \quad (4)$$

where

$$\begin{aligned} \bar{B} &= [B \ B \ \dots \ B] \in \mathfrak{R}_+^{n \times \bar{m}}, \\ \bar{u}(\tau) &= \begin{bmatrix} u(\tau, 0) \\ u(\tau, 1) \\ \vdots \\ u(\tau, q) \end{bmatrix} \in \mathfrak{R}_+^{\bar{m}}, \quad \bar{m} = n(q + 1). \end{aligned} \quad (5)$$

It is well-known [10, 16] that if $A \in M_n$ is diagonal then $e^{At} \in \mathfrak{R}_+^{n \times n}$ is also diagonal and if $B \in \mathfrak{R}_+^{n \times m}$ is monomial then $\bar{B} \bar{B}^T \in \mathfrak{R}_+^{n \times n}$ is also monomial. In this case the matrix

$$R(t_f, q) = \int_0^{t_f} e^{A\tau} \bar{B} \bar{B}^T e^{A^T \tau} d\tau \in \mathfrak{R}_+^{n \times n} \quad (6)$$

is also monomial and $R^{-1}(t_f, q) \in \mathfrak{R}_+^{n \times n}$.

The input

$$\hat{u}(t) = \bar{B}^T e^{A^T(t_f - t)} R^{-1}(t_f, q) x_f \quad (7)$$

steers the state of the system (1) from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to the segment of line $\{[t_f, 0], [t_f, q]\}$. Using (5), (6) and (3) we obtain

$$\begin{aligned} x(t_f, q) &= \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \hat{u}(\tau) d\tau = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \bar{B}^T e^{A^T(t_f-\tau)} d\tau R^{-1}(t_f, q) x_f \\ &= \int_0^{t_f} e^{A\tau} \bar{B} \bar{B}^T e^{A^T\tau} d\tau R^{-1}(t_f, q) x_f = x_f. \end{aligned} \quad (8)$$

Necessity. From the Cayley-Hamilton theorem we have

$$e^{At} = \sum_{k=0}^{n-1} c_k(t) A^k \quad (9)$$

where $c_k(t)$, $k = 0, 1, \dots, n-1$ are some nonzero functions of time depending on the matrix A . Substitution of (9) into

$$\int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \bar{u}(\tau) d\tau \quad (10)$$

yields

$$x_f = [\bar{B} \quad A\bar{B} \quad \dots \quad A^{n-1}\bar{B}] \begin{bmatrix} v_0(t_f) \\ v_1(t_f) \\ \vdots \\ v_{n-1}(t_f) \end{bmatrix} \quad (11)$$

where

$$v_k(t_f) = \int_0^{t_f} c_k(\tau) \bar{u}(t_f - \tau) d\tau, \quad k = 0, 1, \dots, n-1. \quad (12)$$

For given $x_f \in \mathfrak{R}_+^n$ it is possible to compute nonnegative $v_k(t_f)$, $k = 0, 1, \dots, n-1$ if and only if the matrix

$$[B \quad AB \quad \dots \quad A^{n-1}B] \quad (13)$$

has n linearly independent monomial columns and this takes place only if the matrix $[A \ B]$ contains n linearly independent monomial columns [10, 16].

Note that for nonnegative $v_k(t_f)$, $k = 0, 1, \dots, n-1$ it is possible to find a nonnegative input $\bar{u}(t) \in \mathfrak{R}_+^m$ only if the matrix $B \in \mathfrak{R}_+^{n \times m}$ is monomial and the matrix $A \in M_n$ is diagonal. \square

If the positive system (1) is reachable on the segment of line $\{[t_f, 0], [t_f, q]\}$, then usually there exists many different inputs $\bar{u}(\tau) \in \mathfrak{R}_+^m$

that steers the state of the system from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to $x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q) \in \mathfrak{R}_+^n$. Among these inputs we are looking for an input $\hat{u}(t) \in \mathfrak{R}_+^{\bar{m}}$ for $t \in [0, t_f]$ that minimizes the performance index

$$I(u) = \int_0^{t_f} u^T(\tau) Q u(\tau) d\tau \quad (14)$$

where $Q \in \mathfrak{R}_+^{\bar{m} \times \bar{m}}$ is a symmetric positive defined matrix and $Q^{-1} \in \mathfrak{R}_+^{\bar{m} \times \bar{m}}$.

The minimum energy control problem for can be stated as follows: Given the matrices $A \in M_n$, $B \in \mathfrak{R}_+^{n \times m}$, $Q \in \mathfrak{R}_+^{\bar{m} \times \bar{m}}$ and $x_f \in \mathfrak{R}_+^n$, find an input $\bar{u}(t) \in \mathfrak{R}_+^{\bar{m}}$ for $t \in [0, t_f]$ that steers the state vector of the system from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to $x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q) \in \mathfrak{R}_+^n$ and minimizes the performance index (14).

3. Problem solution. To solve the problem we define the matrix

$$W = W(t_f, Q) = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} Q^{-1} \bar{B}^T e^{A^T(t_f-\tau)} d\tau \quad (15)$$

where \bar{B} is defined by (5).

By Theorem 2 the matrix (15) is monomial and $W^{-1} \in \mathfrak{R}_+^{n \times n}$ if and only if the positive system (1) is reachable at the segment of line $\{[t_f, 0], [t_f, q]\}$.

In this case we may define the input

$$\hat{u}(t) = Q^{-1} \bar{B}^T e^{A^T(t_f-t)} W^{-1} x_f \text{ for } t \in [0, t_f]. \quad (16)$$

Note that $\hat{u}(t) \in \mathfrak{R}_+^{\bar{m}}$ for $t \in [0, t_f]$ if

$$Q^{-1} \in \mathfrak{R}_+^{\bar{m} \times \bar{m}} \text{ and } W^{-1} \in \mathfrak{R}_+^{n \times n}. \quad (17)$$

Theorem 3. Let $\bar{u}(t) \in \mathfrak{R}_+^{\bar{m}}$ for $t \in [0, t_f]$ be an input that steers the state of the positive system (1) from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to $x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q) \in \mathfrak{R}_+^n$. Then the input (16) also steers the state of the system from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to $x_f = x(t_f, 0) + x(t_f, 1) + \dots + x(t_f, q) \in \mathfrak{R}_+^n$ and minimizes the performance index (14), i.e. $I(\hat{u}) \leq I(\bar{u})$. The minimal value of the performance index (14) is equal to

$$I(\hat{u}) = x_f^T W^{-1} x_f. \quad (18)$$

Proof. If the conditions (17) are met then $\hat{u}(t) \in \mathfrak{R}_+^{\bar{m}}$ for $t \in [0, t_f]$. We shall show that the input steers the state of the system from $x(0, i) = 0$,

$i = 0, 1, \dots, q$ to $x_f \in \mathfrak{R}_+^n$. Substitution of (16) into (4) for $t = t_f$ yields

$$x(t_f) = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \hat{u}(\tau) d\tau = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} Q^{-1} \bar{B}^T e^{A^T(t_f-\tau)} d\tau W_f^{-1} x_f = x_f$$

since (15) holds. By assumption the inputs $\bar{u}(t)$ and $\hat{u}(t)$, $t \in [0, t_f]$ steers the state of the system from $x(0, i) = 0$, $i = 0, 1, \dots, q$ to $x_f \in \mathfrak{R}_+^n$. Hence

$$x_f = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \bar{u}(\tau) d\tau = \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} \hat{u}(\tau) d\tau \quad (19)$$

or

$$\int_0^{t_f} e^{A(t_f-\tau)} \bar{B} [\bar{u}(\tau) - \hat{u}(\tau)] d\tau = 0. \quad (20)$$

By transposition of (20) and postmultiplication by $W^{-1} x_f$ we obtain

$$\int_0^{t_f} [\bar{u}(\tau) - \hat{u}(\tau)]^T \bar{B}^T e^{A^T(t_f-\tau)} d\tau W^{-1} x_f = 0. \quad (21)$$

Substitution of (16) into (21) yields

$$\int_0^{t_f} [\bar{u}(\tau) - \hat{u}(\tau)]^T Q \hat{u}(\tau) d\tau = 0 \quad (22)$$

since

$$Q \hat{u}(\tau) = \bar{B}^T e^{A^T(t_f-\tau)} W^{-1} x_f = x_f. \quad (23)$$

Using (22) it is easy to verify that

$$\begin{aligned} \int_0^{t_f} \bar{u}(\tau)^T Q \bar{u}(\tau) d\tau &= \\ &= \int_0^{t_f} \hat{u}(\tau)^T Q \hat{u}(\tau) d\tau + \int_0^{t_f} [\bar{u}(\tau) - \hat{u}(\tau)]^T Q [\bar{u}(\tau) - \hat{u}(\tau)] d\tau. \end{aligned} \quad (24)$$

From (24) it follows that $I(\hat{u}) < I(\bar{u})$ since the second term in the right-hand side of the inequality is nonnegative.

To find the minimal value of the performance index (14) we substitute (16) into (14) and we obtain

$$\begin{aligned}
I(\hat{u}) &= \int_0^{t_f} \hat{u}^T(\tau) Q \hat{u}(\tau) d\tau = \\
&= x_f^T W^{-1} \int_0^{t_f} e^{A(t_f-\tau)} \bar{B} Q^{-1} \bar{B}^T e^{A^T(t_f-\tau)} d\tau W^{-1} x_f = x_f^T W^{-1} x_f
\end{aligned} \tag{25}$$

since (15) holds. \square

From the above considerations we have the following procedure for solving the minimum energy control problem.

Procedure 1.

- Step 1. Using for example the Sylvester formula compute the matrix e^{At} .
Step 2. Knowing the matrices A , B , Q , t_f and using (15) compute the matrix W .
Step 3. Using (16) compute $\hat{u}(t)$.
Step 4. Using (18) compute the minimal value of the performance index $I(\hat{u})$.

Concluding remarks. A new formulation and solution to the minimum energy control problem have been proposed. New necessary and sufficient conditions for the positivity of a class of 2D continuous-discrete linear systems have been established (Theorem 1). A new notion of the reachability to the segment of line has been introduced and necessary and sufficient conditions for the reachability have been presented (Theorem 2). Conditions for the existence of the solution to the minimum energy control problem (Theorem 3) and procedures for computation of an input minimizing the given performance index have been proposed.

An open problem is an extension of the minimum energy control problem to standard and positive 2D continuous-discrete linear systems described by the equation [10, 16]

$$\begin{aligned}
\dot{x}(t, i + 1) &= A_0 x(t, i) + A_1 \dot{x}(t, i) + A_2 x(t, i + 1) + \\
&\quad + B_0 u(t, i) + B_1 \dot{u}(t, i) + B_2 u(t, i + 1)
\end{aligned}$$

where $\dot{x}(t, i) = \frac{\partial x(t, i)}{\partial t}$, $x(t, i) \in \mathfrak{R}^n$, $u(t, i) \in \mathfrak{R}^m$ are the state and input vectors and $A_k \in \mathfrak{R}^{n \times n}$, $B_k \in \mathfrak{R}^{n \times m}$, $k = 0, 1, 2$.

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CONVEX SIP PROBLEMS WITH FINITELY REPRESENTABLE COMPACT INDEX SETS

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Introduction. Semi-Infinite Programming (SIP) deals with extremal problems that involve infinitely many constraints in a finite dimensional space. Due to the numerous theoretical and practical applications, today semi-infinite optimization is a topic of a special interest. Since the most efficient methods for solving optimization problems are usually based on optimality conditions that permit not only to test the optimality of a given feasible solution, but also to find the better direction to optimality, the study of these conditions is essential. Usually the optimality conditions are formulated under certain assumptions that are called Constraint Qualifications (CQ) [1, 2]. On the other hand, the optimality conditions that do not use too strong additional assumptions are of special interest since they are more universal and have more applications.

1. Problem statement. Consider a convex Semi-Infinite Programming problem in the form

$$(P) : \quad \min_{x \in \mathbb{R}^n} c(x) \quad \text{s.t.} \quad f(x, t) \leq 0 \quad \forall t \in T,$$

with a finitely representable compact index set $T = \{t \in \mathbb{R}^s : g_k(t) \leq 0, k \in K\}$, $|K| < \infty$.