The equalities (1) and (2) show that an upper exhauster as well as a lower one completely characterize the continuous positively homogeneous function p. Therefore if one of the exhausters is known (for instance, an upper one) than it is natural to expect that it is possible to transform it into another (lower) one. A procedure of such transformation is called [4] *an exhauster conversion* or, shortly, *a convertor.*

V.F. Demyanov [4] developed a convertor for exhausters of Lipshitzian positively homogeneous functions. In case of exhausters of continuous positively homogeneous functions which are not Lipshitzian his method of conversion makes it possible to construct only so called a generalized exhauster containing sublinear (or superlinear) functions with values in the extended reals $\overline{\mathbb{R}}$.

In this report we will present the method of exhauster conversion which transforms any initial (upper or lower) exhauster of continuous positively homogeneous functions into a regular (lower or upper) exhauster consisting of only sublinear (or superlinear) functions with values in $\mathbb{R}$.

References

1. *Gorokhovik V.V., Starovoitova M.A.* The characteristic properties of primal exhausters of different classes of positively homogeneous functions. // Proceedings of the Institute of Mathematics. 2011. V. 19. No. 2. P. 12–25.
2. *Demyanov V.F., Rubinov A.M.* Constructive Nonsmooth Analysis. Verlag Peter Lang, Frankfurt, 1995.
3. *Demyanov V.F.* Exhausters of a positively hoomogeneous function // Optimization. 1999. V. 45, No. 1. P. 13–29.
4. *Demyanov V.F.* Exhausters and convexificators — new tools in nonsmooth analysis // In: Demyanov V.F. and Rubinov A.M. (Eds). Quasidifferentiability and Related Topics. Dordrecht: Kluwer Academic Publishers. 2000. P. 85–137.

QUASIDIFFERENTIABLE CALCULUS AND MINIMAL PAIRS OF COMPACT CONVEX SETS

J. Grzybowski[1], D. Pallaschke [2], R. Urbański[1]

[1] Faculty of Mathematics and Computer Science, Adam Mickiewicz University
Umultowska 87, PL-61-614 Poznań, Poland
`{jgrz,rich}@amu.edu.pl`

[2] Institute of Operations, University of Karlsruhe (KIT),
Kaiserstr. 12, D-76128 Karlsruhe, Germany
`diethard.pallaschke@kit.edu`

The quasidifferential calculus developed by V.F. Demyanov and A.M. Rubinov [1] almost 30 years ago provides a complete analogon to the classical calculus of differentiation for a wide class of non-smooth functions.

Although this looks at the first glance as a generalized subgradient calculus for pairs of subdifferentials it turns out that, after a more detailed analysis, the quasidifferential calculus is a kind of Fréchet-differentiations whose gradients are elements of a suitable Minkowski–Rådström–Hörmander space. Since the elements of the Minkowski–Rådström–Hörmander space are not uniquely determined, smallest possible representations of quasidifferentials, i.e. minimal representations are of special interest (see [2]).

To investigate this in a more general frame let $X = (X, \tau)$ be a topological vector space and $\mathcal{B}(X)$ (resp. $\mathcal{K}(X)$) the family of all nonempty bounded closed (resp. compact) convex subsets of X. For nonempty $A, B \subset X: \quad A+B$ denotes the algebraic Minkowski and $A\dot{+}B$ the closure of $A+B$. For $A, B \in \mathcal{K}(X): \quad A\dot{+}B = A+B$. Since $\mathcal{B}(X)$ satisfies the order cancellation law, i.e, for $A, B, C \in \mathcal{B}(X)$ the inclusion $A\dot{+}B \subset B\dot{+}C$ implies $A \subset C$, the set $\mathcal{B}(X)$ endowed with the sum $\dot{+}$ and $\mathcal{K}(X)$ with the Minkowski sum are commutative semigroups with cancellation property. For $A, B, C \in \mathcal{B}(X)$ we put $A \vee B = \text{cl conv}(A \cup B)$ "cl conv" denotes the closed convex hull.

A equivalence relation on $\mathcal{B}^2(X) = \mathcal{B}(X) \times \mathcal{B}(X)$ is given by $(A, B) \sim (C, D)$ iff $A\dot{+}D = B\dot{+}C$ and a partial ordering by the relation: $(A, B) \leq (C, D)$ iff $A \subset C$ and $B \subset D$. With $[A, B]$ the eqivalence class of (A, B) is denoted.

A pair $(A, B) \in \mathcal{B}^2(X)$ is called *minimal* if there exists no pair $(C, D) \in [A, B]$ with $(C, D) < (A, B)$. For any $(A, B) \in \mathcal{K}^2(X)$ exists a minimal pair $(A_0, B_0) \in [A, B]$, but this is not true for $\mathcal{B}^2(X)$. There exists a class $[A, B] \in \mathcal{B}^2(c_0)$ which contains no minimal element, where c_0 is the Banach space of all real sequences which converge to zero. For the 2-dimensional case, equivalent minimal pairs of compact convex sets are uniquely determined up to translation. For the 3-dimensional case, this is not true.

Let $A, B, S \in \mathcal{B}(X)$, then we say that S *separates* the sets A and B if for every $a \in A$ and $b \in B$ we have $[a, b] \cap S \neq \emptyset$.

The following statements are equivalent:
i) $A \cup B$ is convex, *ii*) $A \cap B$ separates A and B, *iii*) $A \vee B$ is a summand of $A\dot{+}B$.

The condition that for $A, B, S \in \mathcal{B}(X)$ the inclusion $A+B \subset (A\vee B)\dot{+}S$ implies that S separates the sets A and B is called the *separation law.* It is equivalent to the order cancellation law.

For finitely sets $(A_i)_{i\in I}$ a generalization of the separation law holds and it can be shown, that a separating set can be constructed from Demyanov-differences of the sets A_i.

We consider conditional minimality: A pair $(A,B) \in \mathcal{K}^2(X)$ is called *convex* if $A \cup B$ is a convex set and a convex pair $(A,B) \in \mathcal{K}^2(X)$ is called *minimal convex* if for any convex pair $(C,D) \in [A,B]$ the relation $(C,D) \leq (A,B)$ implies that $(A,B) = (C,D)$.

It is possible to consider the problem pairs of convex sets in the more general frame of a commutative semigroup S which is ordered by a relation $\leq$ and which satifies the condition: if $as \leq bs$ for some $s \in S$, then $a \leq b$. Then $(a,b) \in S^2 = S \times S$ corresponds to a *fraction* $a/b \in S^2$ and *minimality* to a relative prime representation of $a/b \in S^2$.

References

1. *Demyanov V.F., Rubinov A.M.* Quasidifferential calculus.Optimization Software Inc., Publications Division, New York, 1986.
2. *Pallaschke D., Urbański R.* Pairs of Compact Convex Sets —Fractional Arithmetic with Convex Sets. Mathematics and its Applications, Vol. **548**, Kluwer Acad. Publ. Dordrecht, 2002.

DEMYANOV DIFFERENCE IN INFINITE DIMENSIONAL SPACES

J. Grzybowski[1], D. Pallaschke[2], R. Urbański[1]

[1] Faculty of Mathematics and Computer Science, Adam Mickiewicz University
Umultowska 87, PL-61-614 Poznań, Poland
{jgrz,rich}@amu.edu.pl

[2] Institute of Operations Research, University of Karlsruhe
Kaiserstr. 12, D-76128 Karlsruhe, Germany
diethard.pallaschke@kit.edu

We generalize the Demyanov difference to the case of real Hausdorff topological vector spaces.

For $A, B \subset X$ we define *upper difference* $\mathcal{E}_{A,B}$ as the family $\mathcal{E}_{A,B} = \{C \in \mathcal{C}(X) | A \subset \overline{B + C}\}$, where $\mathcal{C}(X)$ is the family of all nonempty closed convex subsets of the topological vector space X. We denote the family of inclusion minimal elements of $\mathcal{E}_{A,B}$ by $m\mathcal{E}_{A,B}$. We define a new subtraction by $A \overset{D}{-} B = \overline{\text{conv}} \bigcup m\mathcal{E}_{A,B}$. We show that $A \overset{D}{-} B$ is a generalization of Demyanov difference.

We prove some clasical properties of the Demyanov difference.

For a locally convex vector space X and compact sets $A, B, C \in \mathcal{C}(X)$ the Demyanov-Difference has the following properties: