EXPECTED INCOME OF QUEUING STRUCTURE AND ITS APPLICATION IN TRANSPORT LOGISTIC

O.M. Kiturko, M.A. Matalytski
Grodno State University
Grodno, Belarus
e-mail: sytaya_om@mail.ru

Abstract

In article is researched of closed queueing structure of the Markov with one-type messages and the income is conducted. The ordinary differential equation for its expected income is constructed. The offered method of its decision in a case when intensity of the service of messages, number of messages in networks, number of lines of the service in systems, the matrix of probabilities of transitions of messages and the income from transitions between states of the structure depend on time is described. An example when change of parameters has seasonal nature is reviewed. Results of article applied at prediction of the income of the logistic transport system.

1 Introduction

In logistic transport system (LTS) in practice total number of the vehicles moving between various objects and number of loading and unloading crews (workers or, for example, operating loading and unloading tracks at stations) depend on time. Therefore the following closed queueing structure (CQS) can be considered as the LTS model.

Let's consider the closed queueing network consisting of $n+1$ queueing systems S_0, $S_1, \ldots, S_n$, total number of one-type messages in which in an instant t makes $K(t)$. Usually the systemS_0 is understood as environment, and as systems $S_1, S_2, \ldots, S_n$ – concrete queueing systems of network in which the service of messages is made. The queueing networks closed on structure but in which total number of served messages not constantly, and depends on time, are investigated for the first time in special cases in [1] and called the CQS. Let $m_i(t)$– number of service lines in system S_i, $i = \overline{1,n}$, we will put $m_0(t) = K(t)$, and times of messages service of each of service lines are distributed under the exponential law with intensity $\mu_i(t)$, $i, j = \overline{0,n}$. Messages for the service get out according to discipline of FIFO. The message, which service in system S_i ended, with probability $p_{ij}(t)$ passes to system S_j, $i, j = \overline{0,n}$. The matrix of transition probabilities $P(t) = \|p_{ij}(t)\|$, $i, j = \overline{0,n}$, is a matrix of transition probabilities of a nonreducible Markov chain in each moment $0 \leq p_{ij}(t) \leq 1$, $\sum_{j=0}^{n} p_{ij}(t) = 1$. The primal problem of research of the given CQS consists in the asymptotic analysis of the Markov process describing its behavior at a large number of messages.

The state of structure in a moment t is described by a vector

$$k(t) = (k,t) = (k_1, k_2, ..., k_n, t) = (k_1(t), k_2(t), ..., k_n(t))$$

where $k_i(t)$– number of messages in systemS_i in a moment t, $t \in [0, T]$, $i = \overline{0, n}$, which forms n-dimensional Markov process with the continuous time and a finite number of states.

2 Differential equation for the expected income

Let's consider that transitions of messages between systems bring in to CQS the particular income. Let's the task of prediction of its total expected income. Let $V(k,t)$ – the complete expected income which will receive CQS in time t if in an initial moment it is in a state (k,t). It is apparent that $V(k,t) = \sum_{i=0}^{n} V_i(k,t)$, where $V_i(k,t)$ – the expected income which is gained by systemS_i in time t if in an initial moment the CQS is in a state k. In article [2] the concept of distribution density of the expected income of CQS $v(x,t)$, $x = (x_1,\ x_2,\ ...,\ x_n)$is injected , and the following statement is proved.

Theorem. Income distribution density $v(x,t)$ under a state that it is differentiated on $t$and is twice sectionally continuous differentiated on x_i, $i = \overline{1, n}$, satisfies to within terms of order of smallness $\varepsilon^2(t) = \frac{1}{K^2(t)}$ to the following differential equation in partial derivatives:

$$\frac{\partial v(x,t)}{\partial t} = -\sum_{i=1}^{n} A_i(x,t)\frac{\partial v(x,t)}{\partial x_i} - \frac{\varepsilon(t)}{2}\sum_{i,j=1}^{n} B_{ij}(x,t)\frac{\partial^2 v(x,t)}{\partial x_i \partial x_j} -$$

$$-nK(t)\varepsilon'(t)v(x,t) + r(t) + K(t)\sum_{i,j=0}^{n} \mu_j(t)p_{ji}(t)\min(l_j(t), x_j)r_{ji}(t),\ v(x,t_0) = v_0,$$

where

$$A_i(x,t) = \sum_{j=1}^{n} \mu_j(t)p_{ji}^*(t)\min(l_j(t), x_j) + \mu_0(t)p_{0i}(t)\left(1 - \sum_{i=1}^{n} x_i\right), \tag{1}$$

$$p_{ji}^*(t) = \begin{cases} p_{ji}(t) - 1,\ j = i, \\ p_{ji}(t),\ j \neq i, \end{cases} \quad l_j(t) = \frac{m_j(t)}{K(t)}, j = \overline{1, n},$$

$B_{ij}(x,t)$ – sectionally continuous function concerning x,

$$B_{ii}(x,t) = \sum_{j=0}^{n} \mu_j(t)q_{ji}^*(t)\min(l_j(t), x_j), B_{ij}(x,t) = \mu_i(t)p_{ij}(t)\min(l_i(t), x_i), \tag{2}$$

$$q_{ji}^*(t) = \begin{cases} -1 - p_{ji}(t),\ j = i, \\ -p_{ji}(t),\ j \neq i. \end{cases}$$

Considering (2), expression $\frac{\varepsilon(t)}{2}\sum_{i,j=1}^{n} B_{ij}(x,t)\frac{\partial^2 v(x,t)}{\partial x_i \partial x_j}$ can be referred to $O(\varepsilon^2(t))$. Therefore we will consider the following equation:

$$\frac{\partial v(k,t)}{\partial t} = -\sum_{i=1}^{n} A_i(x,t)\frac{\partial v(x,t)}{\partial x_i} - nK(t)\varepsilon'(t)v(x,t)+$$

$$+K(t)\sum_{i,j=0}^{n} \mu_j(t)p_{ji}(t)\min(l_j(t),x_j)r_{ji}(t) + r(t).$$

Having integrated both parts of this equation on $x = (x_1, x_2, ..., x_n)$ in area $G = \{x = (x_1, x_2, ..., x_n) : \ x_i \geq 0, \ \ \sum_{i=1}^{n} x_i \leq 1\}$ and having divided both members of equation into the volume of area G, equal $m(G)$, we will receive:

$$\frac{1}{m(G)}\iint\limits_{G}...\int \frac{\partial v(x,t)}{\partial t}dx = -\frac{1}{m(G)}\sum_{i=1}^{n}\iint\limits_{G}...\int A_i(x,t)\frac{\partial v(x,t)}{\partial x_i}dx-$$

$$-\frac{nK(t)\varepsilon'(t)}{m(G)}\iint\limits_{G}...\int v(x,t)dx+\frac{K(t)}{m(G)}\sum_{\substack{i=0,\\ j=1}}^{n} \mu_j(t)p_{ji}(t)r_{ji}(t)\iint\limits_{G}...\int \min(l_j(t),x_j)dx+$$

$$+\frac{K(t)}{m(G)}\sum_{i=0}^{n}\mu_0(t)p_{0i}(t)r_{0i}(t)\iint\limits_{G}...\int\left(1-\sum_{j=1}^{n}x_j\right)dx + \frac{r(t)}{m(G)}\iint\limits_{G}...\int dx. \qquad (3)$$

Considering it and certain boundary states, it's possible to show that (3) it's reduced to

$$\frac{d}{dt}\overline{v_G}(t) = \left[\sum_{i=1}^{n}\frac{\partial A_i(x,t)}{\partial x_i} - nK(t)\varepsilon'(t)\right]\overline{v_G}(t)+$$

$$+\frac{K(t)}{m(G)}\sum_{j=1}^{n}\sum_{i=0}^{n}\mu_j(t)p_{ji}(t)r_{ji}(t)\iint\limits_{G}...\int \min(l_j(t),x_j)dx+$$

$$+\frac{K(t)}{m(G)}\sum_{i=0}^{n}\mu_0(t)p_{0i}(t)r_{0i}(t)\iint\limits_{G}...\int\left(1-\sum_{j=1}^{n}x_j\right)dx + \frac{r(t)}{m(G)}\iint\limits_{G}...\int dx. \qquad (4)$$

From (1) we see that coefficients $A_i(x,t)$ represent piecewise linear functions on x_j, $j = \overline{1,n}$, that is (4) is differential equation with a piecewise constant right member. Let's designate a set of indexes of components of a vector $x = (x_1, x_2, ..., x_n)$ through $\Omega = \{1, \ 2, \ ..., \ n\}$. Let's break Ω into two non-overlapping sets $\Omega_0(\tau)$, $\Omega_1(\tau)$ such that $\Omega_0(\tau) = \{j : l_j(t) < x_j \leq 1\}$, $\Omega_1(\tau) = \{j : 0 \leq x_j \leq l_j(t)\}$, τ – splitting number. At fixed t number of splittings of this kind is equal 2^n, $\tau = \overline{1, 2^n}$. Each splitting will set in a set G not being crossed areas G_τ such that

$$G_\tau = \left\{ x(t) : l_i(t) < x_i \le 1, i \in \Omega_0(\tau);\ 0 \le x_j \le l_j(t), j \in \Omega_1(\tau);\ \sum_{i=1}^{n} x_i \le 1 \right\},$$

$$\tau = 1,\ 2,\ ...,\ 2^n, \bigcup_{\tau=1}^{2^n} G_\tau = G.$$

Now in each of area of splitting of a phase space we can write down an apparent look (4) and at particular starting states to find the average expected income for each of areas G_τ.

Let's set, for example, splitting: $\Omega_0(1) = \{1,\ 2,\ ...,\ n\}$, $\Omega_1(1) = \{\varnothing\}$, $\tau = 1$, that corresponds to existence of queues $S_1,\ S_2,\ \ldots,\ S_n$. Then, solving the equation (4) at starting states $\overline{v_{G_1}}(0) = S$, it is possible to determine the average expected income changing of a reference state of area $G_1 = \left\{x(t) : l_i(t) < x_i \le 1, i = \overline{1,n},\ \sum_{i=1}^{n} x_i \le 1\right\}$. The equation (4) thus looks like

$$\frac{d}{dt}\overline{v_{G_1}}(t) = \left[\mu_0(t)\sum_{i=1}^{n} p_{0i}(t) - nK(t)\varepsilon'(t)\right]\overline{v_{G_1}}(t) + \frac{1}{m(G_1)}\left[\sum_{j=1}^{n}\sum_{i=0}^{n}\mu_j(t)p_{ji}(t)m_j(t)r_{ji}(t) + \right.$$

$$\left. + K(t)\sum_{i=0}^{n}\mu_0(t)p_{0i}(t)r_{0i}(t)\iint_{G_1}\ldots\int\left(1 - \sum_{j=1}^{n}x_j\right)dx\right] + r(t). \qquad (5)$$

The received outcomes were applied at forecasting of the income of transport enterprise (TE) which functions as follows. The TE (system S_3) at the disposal of which a large number of cars (messages) is available, sends it for realization of a number of particular transportations between the various cities (environment, system S_0). After that they come back to the TE, before having passed in two points (systems S_1 and S_2) technical inspection which can also include car repairs. Number of functioning lines of a service in points S_1 and S_2 in a moment t are equal respectively $m_1(t)$ and $m_2(t)$. In system S_3 loading of cars before the flight in which are engaged $m_3(t)$ loading crews (service lines) is carried out. The similar situation can arise when cars come back from environment and unload in two warehouses TE (systems S_1 and S_2); in this case $m_1(t)$ and $m_2(t)$ – number of crews of unloading accordingly in a warehouse S_1 and S_2. In both cases model of functioning of transportations is CQS.

References

[1] Matalytski M., Rusilko T. (2007). *Mathematical analysis of stochastic models for claim processing in insurance companies*. GrSU, Grodno.

[2] Kiturko O., Matalytski M., Rusilko T. (2013). Asymptotic analysis of the total expected income of closed queuqing structure with the one-type messages and application. *Wiestnik GrUP*. Vol. **2**