SOME SPECIFICATION ASPECTS FOR THREE-FACTOR MODELS OF A COMPANY'S PRODUCTION POTENTIAL TAKING INTO ACCOUNT INTLLECTUAL CAPITAL

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Abstract

We propose a general algorithm that gives a solution of some problems related to specification of 3-factor stochastic models of a company's production potential that take into account main production factors. The proposed formal scheme is based on procedures of statistical hypothesis testing and provides the necessary means to choose a reasonable alternative within the analyzed class of models.

According to [1], a model of production function is a deterministic component of a production potential model. We consider the class of production potential models that is given by relation

$$R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}, \tag{1}$$

where R is the overall production of a company, K is the physical and financial capital input, L is the labor input, I is the intellectual capital input; the random variable (r.v.) V is normally distributed with parameters $(0; \sigma_V^2)$ (i.e. $V \in N(0; \sigma_V^2)$), the r.v. U is distributed according to a truncated at zero normal law with a mean value μ and a variance σ_U^2 (i.e. $U \in N^+(\mu; \sigma_U^2)$). The r.v.'s V and U are considered to be stochastically independent. The parameters $\beta_0, \ldots, \beta_3$ are subject to statistical estimation.

The main goal of the research is to describe a general algorithm of statistical hypothesis testing that provides answers to the following questions related to the model's specification.

(a) Is it reasonable to consider the 3-factor model (1) under given methods of intellectual capital measurement (alternative is the standard 2-factor model)?

(b) In case the answer to the question (a) is positive: is there any inefficiency in use of production factors (alternative: $\sigma_U^2 = 0$)?

(c) If the answer to the question (a) is positive and there's inefficiency in use of inputs ($\sigma_U^2 > 0$): is it reasonable to apply the model (1) with $\mu = 0$ (alternative: $\mu \neq 0$)?

(d) Finally, in case the answers to the questions (a) and (b) are positive and there's possibility to identify indicators $z^{(1)}, z^{(2)}, \ldots, z^{(p)}$ that can influence efficiency in use of the main production factors: how one can proceed with statistical hypothesis testing related to the dependence character of the parameters μ or σ_U^2 of those indicators?

In order to estimate intellectual capital (IC) one can use any known method that meets the requirements of the research. We use the following approach to find out whether it is reasonable to apply one or another way of IC measurement. If the influence of IC on the overall production of a company is positive and statistically significant (under positive and statistically significant estimated coefficients for capital input K and labor input L) we say that the corresponding measurement method is acceptable for practical purposes. Otherwise the considered way of IC measurement is said to be inapplicable for the analysis.

Proceeding with econometric analysis of the model (1) we generally have the data array $E^2 = \left\{R_i, K_i, L_i, I_i, z_i^{(1)}, z_i^{(2)}, \ldots, z_i^{(p)}\right\}_{i=1}^{n}$, where R_i is the total production of the ith company, K_i, L_i, I_i are the values of the main production factors for the ith company, $z_i^{(1)}, \ldots, z_i^{(p)}$are the values of the measurable indicators that characterize efficiency in use of the main production factors for the ith company; n is the number of companies in the sample. If there's no information about any efficiency indicators the specification of the model (1) is carried out on the basis of the reduced data array $E^1 = \{R_i, K_i, L_i, I_i\}_{i=1}^{n}$.

In order to formalize the problems given by items (a) - (d) we should consider the following models:

$M_0: \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V}$, where $V \in N(0; \sigma_V^2)$.

$M_1: \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}$, where $V \in N(0; \sigma_V^2), \ U \in N^{+}(0; \sigma_U^2)$.

$M_2: \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}$, where $V \in N(0; \sigma_V^2), \ U \in N^{+}(\mu; \sigma_U^2)$.

$M_3: \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}$, where $V \in N(0; \sigma_V^2), \ U \in N^{+}(0; \sigma_U^2(z)), \ \ln \sigma_U^2(z) = \theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}$,

$M_4: \ R = \beta_0 K^{\beta_1} L^{\beta_2} I^{\beta_3} e^{V-U}$, where $V \in N(0; \sigma_V^2), \ U \in N^{+}(\mu(z); \sigma_U^2), \ \mu(z) = \delta_0 + \delta_1 z^{(1)} + \ldots + \delta_p z^{(p)}$.

To find out whether a method of IC measurement is acceptable for practical use one should test the following hypothesis:

H_0 : *there exists* $i (i \in \{1, 2, 3\})$ *such that*: $\beta_i \leq 0$ i.e. there exists a production factor that is not statistically significant or that provides negative influence on the total production;

$\mathrm{H}_0^A : \beta_i > 0, \ \ i = 1, 2, 3$, i.e. all the considered production factors are statistically significant and provide positive impact on the overall production.

Testing procedure of the hypothesis H_0 against the alternative H_0^A is based on the fact that the statistics $\hat{t}_i = \hat{\beta}_i / s_{\hat{\beta}_i}$ (where $\hat{\beta}_i$ is an estimate of the coefficient β_i and $s_{\hat{\beta}_i}$ is an estimate of the standard deviation in β_i estimation) is distributed according to $t(n-k)$ - law.

To get an answer to the question (b) one should test the following statistical hypothesis within the frame of the model M_1:

$\mathrm{H}_1 : \sigma_U^2 = 0$ (no inefficiency in use of production factors) with the alternative $\mathrm{H}_1^A : \sigma_U^2 > 0$ (inefficiency is observed).

Testing of the hypothesis H_1 (against the alternative H_1^A) is done on the basis of asymptotic characteristics of the likelihood ratio statistics (see corresponding results in [3, 4, 6]).

The choice of a proper model between M_1 and M_2 is formalized be the hypothesis:

$\mathrm{H}_{1,2}: \quad \mu = 0$ for the model M_2 (inefficiency in the models M_1 and M_2 cannot be distinguished),

$H^A_{1,2}$: $\mu \neq 0$ for the model M_2 (inefficiency in the models M_1 and M_2 can be distinguished).

The corrected Akaike criterion is used in testing of the hypothesis $H_{1,2}$ against the alternative $H^A_{1,2}$ (see [5], [2]).

To find out whether the indicators $z^{(1)}, \ldots, z^{(p)}$ have a real effect on the variance σ^2_{U} in the model M_3 we test the hypotheses:

$H_{3.1}$: $\forall j = 1, \ldots, p$: $\theta_j = 0$ (influence of all the efficiency indicators in the model M_3 are not statistically significant),

$H^A_{3.1}$: $\exists j = 1, \ldots, p$: $\theta_j \neq 0$ (there exists at least one statistically significant efficiency factor in the model).

The problem is solved by means of general theory of linear hypotheses testing using the corresponding F statistics (see [2]).

To form the a posteriori set of efficiency indicators for the model M_3 one should test the hypotheses:

$H_{3.2}$: $\exists j = 1, \ldots, p$: $\theta_j = 0$ (there are some (at least one) non-significant efficiency factors in the model M_3),

$H^A_{3.2}$: $\forall j = 1, \ldots, p$: $\theta_j \neq 0$ (all the efficiency indicators in the model M_3 are statistically significant).

Testing procedure of the hypothesis $H_{3.2}$ (against the alternative $H^A_{3.2}$) is based on the fact that the statistics $\hat{z}_j = \hat{\theta}^2_j \Big/ s^2_{\hat{\theta}_j}$ is distributed according to $\chi^2(1)$-law.

Analysis of the model M_4 should clarify whether the efficiency indicators $z^{(1)}, \ldots, z^{(p)}$ really influence the value μ in the distribution $N^+(\mu; \sigma^2_U)$. The following hypothesis should be tested:

$H_{4.1}$: $\forall j = 1, \ldots, p$: $\delta_j = 0$ (all the efficiency factors in the model M_4 are not statistically significant) against the alternative

$H^A_{4.1}$: $\exists j = 1, \ldots, p$: $\delta_j \neq 0$ (there's at least one statistically significant efficiency indicator in the model M_4).

To form the a posteriori set of efficiency indicators for the model M_3 one is advised to test the hypotheses:

$H_{4.2}$: $\exists j = 1, \ldots, p$: $\delta_j = 0$ (there are some (at least one) non-significant efficiency factors in the modelM_4),

$H^A_{4.2}$: $\forall j = 1, \ldots, p$: $\delta_j \neq 0$ (all the efficiency indicators in the model M_4 are statistically significant).

The dependence character between efficiency in use of the main production factors and the indicators $z^{(1)}, \ldots, z^{(p)}$ can be clarified by testing the following hypotheses

$H_{2,3}$: $\mu \neq 0$, $\sigma^2_U = const$ (the variance of the component U should not be expressed via the efficiency indicators),

$H^A_{2,3}$: $\mu = 0$, $\sigma^2_U = e^{\theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}}$ (the variance of the component U should be decomposed by the efficiency indicators under assumption that the mathematical expectation μ is equal to 0).

Finally, if one assumes that both parameters of the r.v. U (μ and σ^2_U) might depend on the indicators $z^{(1)}, \ldots, z^{(p)}$ we recommend to test the hypothesis:

$H^A_{3,4}$: $\mu = 0$, $\sigma^2_U = e^{\theta_0 + \theta_1 z^{(1)} + \ldots + \theta_p z^{(p)}}$ (the variance σ^2_U (but not the mathematical expectation μ) should be decomposed by the efficiency indicators), against the alternative

$H^A_{3,4}:\quad \mu = \delta_0 + \delta_1 z^{(1)} + \ldots + \delta_p z^{(p)},\ \sigma^2_U = const$ (the mathematical expectation μ (but not the variance σ^2_U) should be decomposed by the efficiency indicators).

To describe the expanded general methodological algorithm that helps to choose a proper model for the class (1) and takes into account availability of the information regarding efficiency indicators we use the following notation:

Input 1 — start of the analysis with the data array $E^1 = \{R_i, K_i, L_i, I_i\}_{i=1}^{n}$,

Mi — calculation of the estimates in the model M_i,

Mi+ — the estimates are successfully obtained,

Mi- — the estimates are not obtained (due to the sample specifics, problems of non-identifiability etc.),

Hi — application of the hypothesis testing procedure,

Hi+ — the hypothesis is accepted,

Hi- — the hypothesis is discarded in favor of the alternative H^A_i.

Mi — the model M_i is finally chosen,

ICNS — conclusion that the used estimate of intellectual capital (IC) is not statistically significant,

Input 2 — start of the analysis with the data array $E^2 = \left\{R_i, K_i, L_i, I_i, z_i^{(1)}, z_i^{(2)}, \ldots, z_i^{(p)}\right\}_{i=1}^{n}$,

ENSF(zi) — exclusion of the ith non-significant efficiency indicator that has the maximum p-value in the testing of the hypotheses $H_{3.2}$ and $H_{4.2}$,

ECF(zi) — exclusion of the ith efficiency indicator that has the maximum absolute correlation coefficient with intellectual capital indicator,

Check — check whether there still exist non-excluded efficiency indicators,

Check+ — in the analyzed model there still exist non-excluded efficiency indicators,

Check- — in the analyzed model all the efficiency indicators are excluded.

As shown at figure 1, the algorithm starts with the model that has the biggest number of variables and provides the widest opportunities for analysis, i.e. with the model M_4. In case there's at least one statistically significant efficiency indicator in the models M_4 or M_3 one should choose one of these models as the final result (given that the basic principles of the provided methodology are not violated). If in both models M_3 and M_4 all the efficiency indicators are not statistically significant or at least one of production factors is not statistically significant the analysis is reduced to the models of production potential that do not take into account the dependence of efficiency in use of production factors of any additional indicators.

In [2] we consider the modeling of production potential for a sample of US companies that operate in the sector "Biotechnology and Drugs". The sequence of procedures given below leads to a conclusion that one should use the 3-factor model M_3 to estimate the production potential:

$$\left\{ \begin{array}{l} E^2; M_4; M_4^-;\ ECF(z^1);\ Check\,;\ Check^{\,+}; M_4; M_4^-; ECF(z^2); \\ Check; Check^{\,-}; E^2; M_3; M_3^+;\ H_0; H_0^-;\ H_{3.1};\ H_{3.1}^-; H_{3.2};\ H_{3.2}^+; \\ ENSF(z^1); Check\,;\ Check^{\,+}; M_3; M_3^+;\ H_0; H_0^-; H_{3.1};\ H_{3.1}^-; H_{3.2}; \\ H_{3.2}^-; E^1; M_2; M_2^-; \hat{M}_3 \end{array} \right\}$$

The detailed description of initial data and the results of calculations done according to the described scheme is given in [2].

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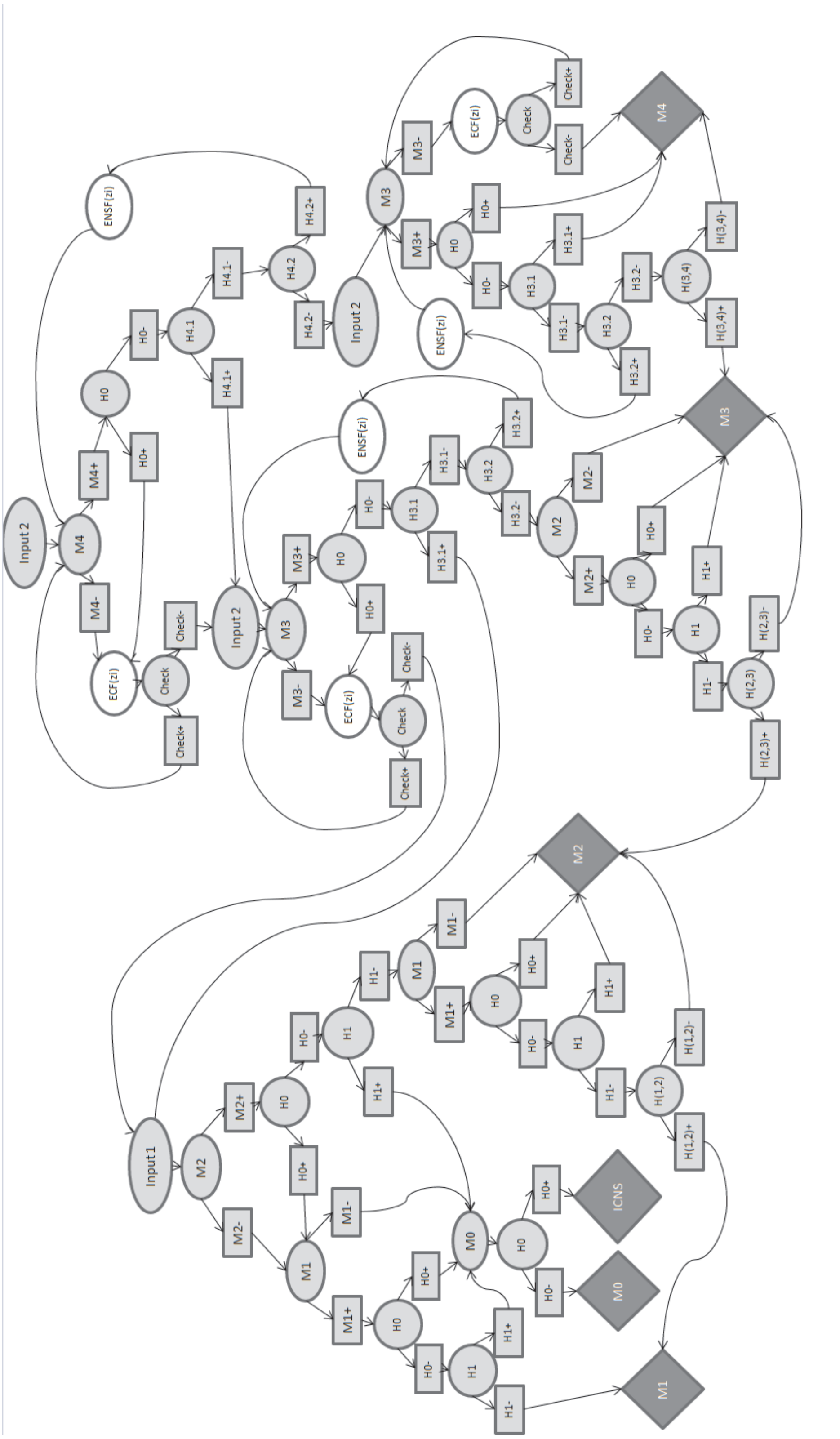

Figure 1: General methodological algorithm of specification problem solution