THE FUZZY DECISION OF TRANSPORTATION PROBLEM

P. Sevastianov, L. Dymova, M. Dolata

Institute of Comp. & Information Sci., Czestochowa University of Technology, Dabrowskiego 73, 42-200 Czestochowa, Poland, e-mail: *sevast@icis.pcz.czest.pl*

Abstract. The paper presents the further development of transportation problem solution in the case of fuzzy coefficients. The direct fuzzy extension of usual simplex method is used to realize the elaborated numerical fuzzy optimization algorithm with fuzzy constraints. It must be emphasized that the fuzzy numerical method proposed is based on the practical embodiment of the probabilistic approach to the comparison of fuzzy values. The problem is formulated in the more general form of the distributor's benefit maximization. The results are compared with those obtained with use Monte-Carlo method.

1. Introduction

The task of distributor's decisions optimization can be reformulated as the generalization of classical transportation problem. Conventional transportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. In classical approach, transporting costs from *M* wholesalers to the *N* consumers are minimized.

In 1979, Isermann [5] introduced algorithm for solving this problem, which provides effective solutions. The Ringuest and Rinks [9] proposed two iterative algorithms for solving linear, multicriterial transportation problem. Similar solution proposed in [1]. In work [4], this problem was solved in case of interval uncertainty of transporting costs. In works by S. Chanas and D. Kuchta [2, 3], an approach based on interval and fuzzy coefficients had been elaborated. The further development of this approach presented in work [10]. All the above mentioned works introduce the restrictions in a form of membership function. This allows to transform the initial fuzzy linear programming problem into the net of usual linear programming tasks by use of well defined analytic procedures. However in practice the membership functions, which describes uncertain parameters of used models can have the considerable complicated forms. In such cases, the numerical approach is needed.

The main technical problem when constructing the numerical fuzzy optimization algorithm is to compare the fuzzy values. To decide this problem, we use the approach proposed in [6,7] and well described in [8], which is based on α-level representation of fuzzy numbers and probability estimation of the fact that given interval is greater than/equal another interval. The method allows to compare the interval and real number and to take into account (implicitly) the widths of intervals ordered. The proposed approach allows us to accomplish the direct fuzzy extension of classical numerical simplex method with its implementation using tools of object-oriented programming.

2. The method's description

In the proposed approach we not only minimize the transportation costs but in addition we maximize the distributor's profits under the same conditions. The distributor deals with *M* wholesalers and *N* consumers (see Fig.1).

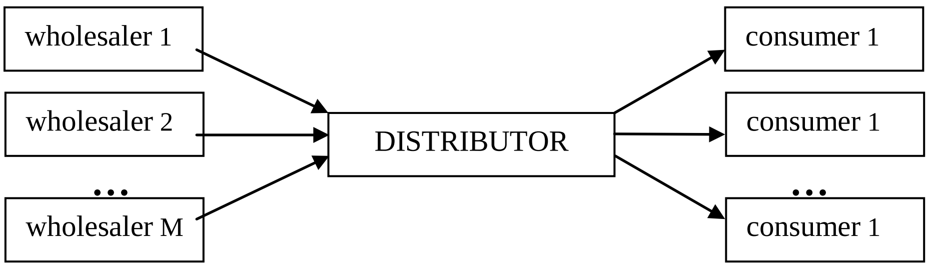

Fig. 1 The scheme of distributor's activity

Let a_i, $i=1$ to M, be the maximal quantities of goods that can be proposed by wholesalers and b_j, $j=1$ to N, be the maximal good requirements of consumers. In accordance with the signed contracts distributor must buy at least p_i good units at price of t_i monetary units for unit of good from each i*th* wholesaler and to sell at least q_j good units at price of s_j monetary units for unit of good to each j*th* consumer. The total transportation cost of delivering good unit from i*th* wholesaler to j*th* consumer is denoted as c_{ij}.

There are reduced prices k_i for distributor if he/she buy the greater quantities of good then stipulated in contract quantities p_i and also the reduced prices r_j for consumers if they buy the good quantities grater then contracted q_j . The problem is to find the optimal good quantities x_{ij} (*i=1,...,M; j=1,...,N*) delivering from i*th* wholesaler to j*th* consumer maximizing the distributor's total benefit *D* under restrictions. Assuming that all above mentioned parameters are fuzzy ones, resulting optimization task has been formulated as:

$$\hat{D} = \sum_{i=1}^{M} \sum_{j=1}^{N} (\hat{z}_{ij} * \hat{x}_{ij}) \rightarrow \max , \tag{1}$$

$$\sum_{j=1}^{N} \hat{x}_{ij} \le \hat{a}_i , \ (i = 1,...,M) \qquad \sum_{i=1}^{M} \hat{x}_{ij} \le \hat{b}_j , \ (j = 1,...,N) \tag{2}$$

$$\sum_{j=1}^{N} \hat{x}_{ij} \ge \hat{p}_i , \ (i = 1,...,M) \ \ \sum_{i=1}^{M} \hat{x}_{ij} \ge \hat{q}_j , \ (j = 1,...,N) \tag{3}$$

where $\hat{z}_{ij} = \hat{r}_j - \hat{k}_i - \hat{c}_{ij}$ (*i=1,...,M; j=1,...,N*) and $\hat{D}, \hat{z}_{ij}, \hat{a}, \hat{b}, \hat{q}, \hat{p}$ are fuzzy values.

An approach based on the α-cuts presentation of fuzzy numbers is used. So, if $\tilde{A}$ is a fuzzy number, then $\tilde{A} = \cup \alpha A_\alpha$, where A_α is the crisp interval $\{x: \mu_A (x) \ge \alpha \}$, αA_α is the fuzzy interval $\{(x,\alpha): x \in A_\alpha \}$. It was proved that if $\tilde{A}$ and $\tilde{B}$ are fuzzy numbers (intervals), then all the operations on them may be presented as operations on the set of crisp intervals corresponding to their α-cuts : $(A@B)_\alpha = A_\alpha @ B_\alpha$. So, the α-cut presentation for fuzzy numbers (intervals) and operations on them can be accepted as the basic concept for fuzzy modeling of the real-world processes. Since in the case of α-cut presentation the fuzzy arithmetic is based on crisp interval arithmetic rules, the basic definitions of applied interval analysis must be presented too. There are some definitions of interval arithmetic in literature, but in practical applications the so-called «naive» form proved the best. According to it, if $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are crisp intervals, then

$$Z = A@B = \{ z = x@y, \ \forall x \in A, \forall y \in B \}.$$

As the direct consequence of this basic definition the next expressions were obtained:

$$A+B=[a_1+b_1, b_2+b_2], \qquad A-B=[a_1-b_2, a_2-b_1],$$

$$A \cdot B=[\min(a_1 \cdot b_1, a_2 \cdot b_2, a_1 \cdot b_2, a_2 \cdot b_1), \max(a_1 \cdot b_1, a_2 \cdot b_2, a_1 \cdot b_2, a_2 \cdot b_1)], \ A/B=[a_1, a_2] \cdot [1/b_2, 1/b_1].$$

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To decide the problem (1)-(3), the numerical method based on the α-cut representation of fuzzy numbers and probabilistic approach to the interval and fuzzy interval comparison has been elaborated. The direct fuzzy extension of usual simplex method is used. The use of object-programming tools makes it possible to get the results of fuzzy optimization, i.e. $\hat{x}_{ij}$, in the form of fuzzy numbers as well. To estimate the effectiveness of method proposed, the results of fuzzy optimization were compared with those obtained from (1)-(3) when all the uncertain parameters were considered as normally distributed random values. Of course, in the last case all the parameters in (1)-(3) were considered as real numbers.

To make the results we got using the fuzzy and probability approaches comparable, the simple special method for transformation frequency distributions into fuzzy numbers without lost of useful information was used to achieve the comparability of uncertain initial data in fuzzy and random cases. In practice, we often have a problem with different precisions of representation the uncertain data we use. For instance, one part of parameters used can be represented in the trapezoid fuzzy numbers form on basis of the expert's opinions and at the same time, the other part of them can have the form of the histogram or frequency distributions of considerable complicated form we got as a result of statistical analyses. In these cases, the methodologically correct approach is to transform all the uncertain data available to the form of smallest certain level we met in our task. Thus, we have to transform the data represented in form of frequency distributions or histogram to the membership functions of fuzzy numbers.

To present the initial data in fuzzy number form, at first we should apply an algorithm, which builds the membership function on basis of frequency distribution, if such exists, or directly using histogram. In the simplest case of normal frequency distributions, they can be exhaustively described be their averages m and standard deviations σ. In the more complicated situations it seems better to use directly the histograms. That is why, we use the numerical algorithm which allows us to transform the frequency distribution or histogram to trapezoidal fuzzy number.

The elaborated method of fuzzy programming problem (1)-(3) solution is realized performing all fuzzy numbers as the sets of α-cuts. In fact, it reduces fuzzy problem into the set of crisp interval optimization tasks. The final solution has been obtained numerically with using probabilistic approach to interval comparison. The interval arithmetic rules needed were realized with a help of object-oriented programming tools. The standard Monte-Carlo procedure was used for the realization of probability approach to the description of uncertain parameters of the optimization task (1)-(3). In fact, for each randomly selected set of real valued parameters of task (1)-(3) we solve the usual linear programming problem.

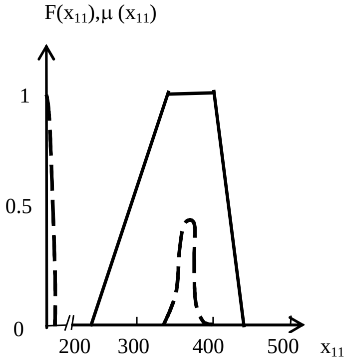
3. Numerical example

To compare the results of fuzzy programming with those obtained when using the Monte-Carlo method, all the uncertain parameters previously were performed by Gaussian frequency distributions. The averages of them are presented in Table 1. For simplicity, all the standard deviations σ were accepted as equal to 10 i.m.

Table 1. Average values of Gaussian distributions of uncertain parameters.

a_1=460	b_1=410	p_1=440	q_1=390	t_1=600	s_1=1000	k_1=590	r_1=990
a_2= 460	b_2=510	p_2=440	q_2=490	t_2=491	s_2=1130	k_2=480	r_2=1100
a_3= 610	b_3=610	p_3=590	q_3=590	t_3=581	s_3=1197	k_3=570	r_3=1180
c_{11}=100	c_{12}=30	c_{13}=100					
c_{21}=110	c_{22}=36	c_{23}=405					
c_{31}=120	c_{32}=148	c_{33}=11					

The results we got with using of fuzzy optimization method and Monte-Carlo method (usual linear programming with real valued but random parameters) are presented in Fig. 2-Fig. 4 for the case M=N=3, where the final frequency distributions F are drown by dotted lines, fuzzy numbers μ are drown by continuous lines.



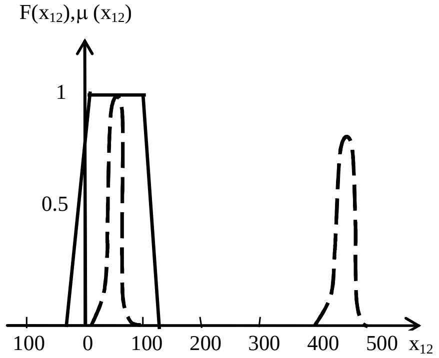


Fig. 2. Frequency distribution F *Fig.3. Frequency distribution F and*

and fuzzy number μ for optimized x_{11} fuzzy number μ for optimized x_{12}

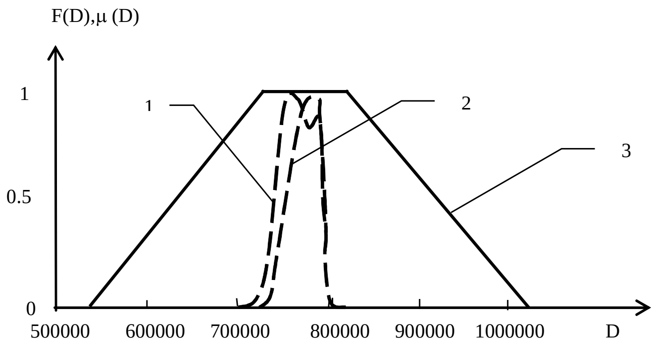


Fig. 4 Frequency distribution F and fuzzy number μ for optimized benefit D: 1 - Monte-Carlo method for 10 000 random steps; 2 - Monte-Carlo method for 100 000 000 random steps; 3 - Fuzzy approach

It is easy to see that fuzzy approach give us some more wider fuzzy intervals then method Monte–Carlo. It is interesting that using probabilistic method we can get even two-extreme results whereas fuzzy approach always give us the results without ambiguity. It is worth noting that probabilistic method demands too mach of random steps (about 100 000 000) to obtain the smooth frequency distribution of resulting benefit *D*. Thus, it seems rather senseless to use this method in practice.

4. Summary

The direct numerical method for solving of fuzzy transportation problem is elaborated. The method is based on α-level representation of fuzzy numbers and probability estimation of the fact that given interval is greater/equal then another interval (this idea was firstly proposed by S. Chanas). The proposed approach makes it possible to accomplish the direct fuzzy extension of usual simplex method.

The results of case studies with using of fuzzy optimization method and Monte-Carlo method (usual linear programming with real valued but random parameters) show that the fuzzy approach have considerable advantages in comparison with Monte-Carlo method, especially prom the computational point of view.

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