INCOME AND RISK TREATMENT ON FIXED BONDS FROM FUZZINESS POINT OF VIEW

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Abstract. The interest rate foresight in a long term temporal horizon has always been a difficult task since its evolution depends on a great number of factors and reasons of macroeconomic disposition which valuation carries jointly a high degree of subjectivity. In the present paper we propose the fitness of techniques of expert's opinions aggregation as the convenient method to give consistent valuations according to a variety of expectations. The aim of this work is to attain the intrinsic value and modified durability as a measure of risk of a fixed bond that incorporates a constant periodical coupon and is to be recovered at its maturity in a temporal horizon of three years. The attainment of both values will give us a better instrument to determine overvaluation or infravaluations in the market. To this aim, we will use the technique of expertizing, utilizing, concretely, a R_m^+ experton by means of real numbers triplets of the interval (0,1) from the rates to be used for the calculation of the theoretical price. The theoretical value obtained will be an approximation of a triangular fuzzy number, the same as the modified durability calculated through the income up to its maturity by means of the maximum presumption theoretical value of the bond.

Fuzzy interest rates estimation by means of expertizing techniques

We consider a temporal horizon of three years where the interest rates are fuzzy and by means of expertizing, using a R_m^+ experton concretely, we pretend to attain the standard rates to be applied to calculate the theoretical value of a fixed bond.

The interval of reference must be large enough as to embrace the practical totality of possibilities. In our case:

$$(e_1, e_2) = (2\%, 6\%)$$

the experts will give their opinions by means of real numbers triplets of the interval (0,1). The opinion of our five experts is the following:

EXPERTS	YEAR 1	YEAR 2	YEAR 3
1	(.6, .7, .8)	(.6 , .8 , .9)	(.5 , .6 , .7)
2	(.4 , .5 , .6)	(.2 , .3 , .4)	(.1 , .2 , .3)
3	(.2 , .3 , .4)	(.5 , .6 , .7)	(.6 , .7 , .8)
4	(.1 , .2 , .3)	(.4 , .5 , .6)	(.5 , .6 , .7)
5	(.3 , .4 , .5)	(.2 , .3 , .4)	(0 , .1 , .2)

The mathematical expectancies attained by the R_m^+ experton, result to be:

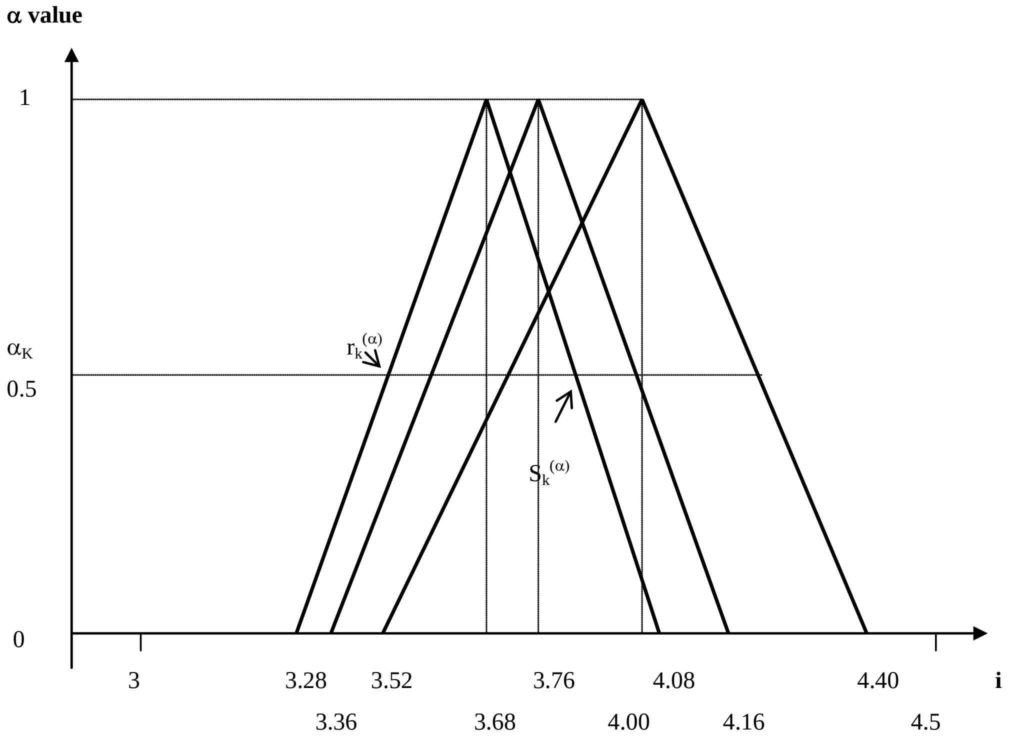
YEARS	MATHEMATICAL EXPECTANCIES	INTERESTS
1	(0.0328 , 0.0368 , 0.0408)	(3.28% , 3.68% ,4.08%)
2	(0.0352 , 0.0400 , 0.0440)	(3.52% , 4% , 4.40%)
3	(0.0336 , 0.0376 , 0.0416)	(3.36% , 3.76% , 4.16)

In chart 1 we show graphically the three triangular fuzzy numbers attained through the experts' opinion.

We can express the actualization factor in its fuzzy form as

$$\frac{1}{1+\left(r_k^{(\alpha)}, s_k^{(\alpha)}\right)} = \frac{1}{\left(1+r_k^{(\alpha)}, 1+s_k^{(\alpha)}\right)} = \left(\frac{1}{1+s_k^{(\alpha)}}, \frac{1}{1+r_k^{(\alpha)}}\right)$$

chart 1
triangular fuzzy numbers of interest rates



The theoretical value or the intrinsic one will be:

$$P_0 = [116.002, 117.238, 118.576]$$

This value will be compared to the one in the market in order to take a decision for a possible purchase-sale.
Of course, P_0 is an approximation of a triangular fuzzy number. This approximation is totally reasonable taking in mind that Pearson's coefficient of correlation is practically the unity.

Duration as a measure of risk

Making use for the formulation the terminology utilized for the calculation of the intrinsic value, it will be:

$$D = \frac{\sum_{j=1}^{n} t_j \cdot Q_j / \prod_{k=1}^{j} (1+r_k)}{\sum_{j=1}^{n} Q_j / \prod_{k=1}^{j} (1+r_k)}$$

Where (Q) is the expected flow in the period (t) . That is to say that duration will remain as:

$$D = \frac{P_1}{P_0}$$

Following with the proposed example (n=3):

$$P_1 = t_1 \cdot Q_1 \cdot N_1 + t_2 \cdot Q_2 \cdot N_2 + t_3 \cdot Q_3 \cdot N_3$$

and operating we get:

$$D = (2.695\ ,\ 2.756\ ,\ 2.819\ )$$

With the aim of connecting the concepts of volatility and duration we will use the denominated "modified duration" that results from correcting the duration with the factor $(1 + r)^{-1}$:

$$D_1 = \frac{D}{(1+r)}$$

Being:

D_1 = modified duration.
D = duration.
r = annual income until its maturity.

Having obtained in this case (P_0):

$$P_0 = [\ 116.0015789,\ 117.2374672,\ 118.5760725]$$

Taking as a value of (P_0) the one that corresponds to its maximum presumption that is to say 117,237576,

$$117.2374672 = \frac{Q_1}{(1+r)} + \frac{Q_2}{(1+r)^2} + \frac{Q_3}{(1+r)^3}$$

Being: $Q_1 = Q_2 = 10$, $Q_3 = 110$ and operating, we get the income rate up to its maturity of:

$$r = 3.81079628749\ \%$$

By the other side, and taking in mind that the new duration (D) obtained with the income rate until its maturity, former, will be situated within the triangular fuzzy number first calculated:

$$D = [\ 2.695195439,\ 2.756355865,\ 2.819015072\ ]$$

We will take this triangular number as an approximation to the duration. As for the modified duration, (D_1) we will get it in the following way:

$$D1 = \frac{D}{(1+r)} = D \times \frac{1}{(1+0.0381079628749)} = D \times 0.963290947$$

Operating for each level of (α) we obtain :

$$D1 = (\ 2.59625\ ,\ 2.65517\ ,\ 2.71553\ )$$

The same as in the calculation P_0 , D_1 is an approximation to triangular fuzzy number. Once more, this approximation is totally reasonable since Pearson's coefficient of correlation is practically the unity: (0.999995248 y 0.999994953), or consequently we can affirm that the modified duration (D_1) also is triangular fuzzy number.

Conclusions

Interest rates are always difficult to foresee, moreover when the temporal horizon is established in a long term. The rise and fall evolution depends on a great number of macroeconomic factors which estimation will always carry a high degree of subjectivity.

We consider that the importance of making use of fuzzy mathematics, and the expertizing techniques, concretely, lies in providing coherent estimations according to the great diversity of expectancies.

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