

A TOPOLOGY CALLED CLAN. ITS IMMERSION IN SOCIAL SCIENCES

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Abstract. In our latter research works we have attempted to introduce ourselves into studies on combinatory pre-topologies and topologies in the hope of finding theoretical and technical elements that are susceptible for allowing us to draw up models and algorithms that are capable of treating economic and management phenomena arising in the field of uncertainty.

The proposal we are presenting on this occasion is aimed at bringing to light the links that join pre-topological and topological spaces with certain models, which under the generic name of clan, have been successfully used in the resolution of problems within the sphere of social sciences. All of this in the hope of being able to offer researchers interested in a panorama that allows for expanding, generalising or creating flexibility for the instruments that are available for treating complex systems. The examples that are included in the text aim at the objective of making their reading easier at the same time illustrating situations that frequently arise in reality. The adaptability of the schemes we show should allow for their use in the extensive field where relations exist between people and social groups.

Topology and the clan theory

In the development of our works on pre-topologies we have resorted to the concept of clans in order to find a pre-topological space which could overcome all the conditioners required in order to be able to speak of isotonic, distributive, etc. pre-topologies. Actually all of this was possible since a clan is, in short, a topological space. Indeed, let us recall that K is a **clan** if, and only if, given a finite set E : $K \in P(E)$

- 1) $E \in K$
- 2) $(A_j \in K) \Rightarrow (\bar{A}_j \in K)$
- 3) $(A_j \in K, A_k \in K) \Rightarrow (A_j \cup A_k \in K)$

From these properties it can easily be deduced that:

- 4) $\emptyset \in K$
- 5) $(A_j \in K, A_k \in K) \Rightarrow (A_j \cap A_k \in K)$

In this way it can be said that a clan is a Boole lattice on the parts of E , or that K also forms a Boole algebra. Therefore, every Boole sub-lattice that includes \emptyset and E is a clan.

On the other hand, if axioms 1), 3), 4) y 5) are compared with the axiomatic of the topology that we reproduce below:

- 1) $\emptyset \in T(E)$ $(A_j \in T(E), A_k \in T(E)) \Rightarrow (A_j \cap A_k \in T(E))$
- 2) $E \in T(E)$
- 3) $(A_j \in T(E), A_k \in T(E)) \Rightarrow (A_j \cap A_k \in T(E))$
- 4) $(A_j \in T(E), A_k \in T(E)) \Rightarrow (A_j \cup A_k \in T(E))$

it can be seen that a clan forms a particular topology where all the opens are closed and that, on the other hand, if the topology contains A_j it also contains \bar{A}_j as a consequence of the second axiom.

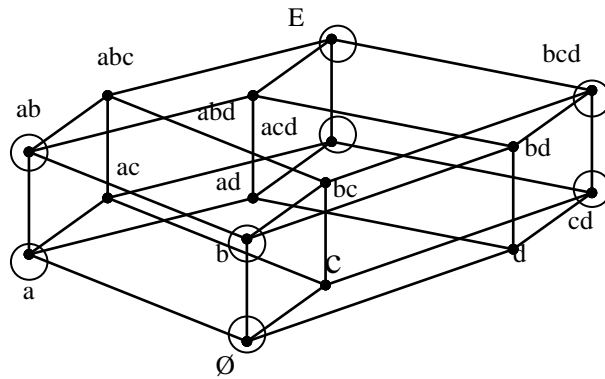
Let us move on to an example. Starting out from referential:

$$E = \{ a, b, c, d \}$$

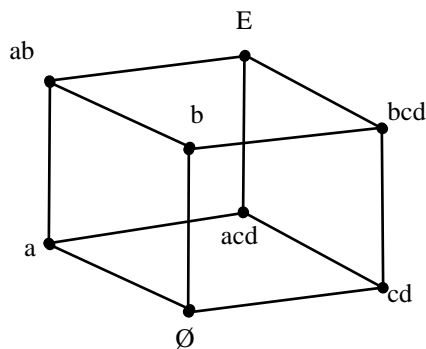
The following sub-set of $P(E)$ is a clan in $P(E)$:

$$\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, E\}$$

In the following figures the clan is located in $P(E)$ and brought to light in a very visible fashion is the Boole sub-lattice formed by the clan. Obviously we have verified "a priori" that the axioms of a clan are complied with.



○ Elements of the clan



Boole sub-lattice that forms the clan

It is known that in ordinary topologies, the elements of $P(E)$ that are not open are arrived at in δ and if $A_j \in P(E)$, the interior of A_j is the largest open element contained in A_j . The same occurs for Γ with the smallest closed element in A_j .

The figure 1 brings to light the interior δ application of K . The adhesive Γ application of K can be found by the formula:

$$\Gamma A_j = \overline{\delta A_j}$$

We reproduce this by figure 2.

We can see that all the open elements are also closed:

$$\begin{aligned} \overline{\delta a} &= \overline{\delta bcd} = \overline{bcd} = a \\ \overline{\delta b} &= \overline{\delta acd} = \overline{acd} = b \\ \overline{\delta ab} &= \overline{\delta cd} = \overline{cd} = ab \\ \overline{\delta cb} &= \overline{\delta ad} = \overline{ad} = cb \\ \overline{\delta acb} &= \overline{\delta b} = \overline{b} = acd \\ \overline{\delta bcd} &= \overline{\delta a} = \overline{a} = bcd \end{aligned}$$

We should remember [1], that a clan can be engendered starting out from a family F , such as the following:

$$\begin{aligned} F &= \{A_1, A_2, \dots, A_r\} \\ A_j &\in P(E), \quad j = 1, 2, \dots, r \\ A_j &\neq \emptyset, \quad j = 1, 2, \dots, r \end{aligned}$$

A clan is engendered from F , by taking these **miniterms or atoms** for all the A_j of F :

$$A_1^* \cap A_2^* \cap \dots \cap A_r^* \text{ where } A_j^* = A_j \text{ or else } \bar{A}_j$$

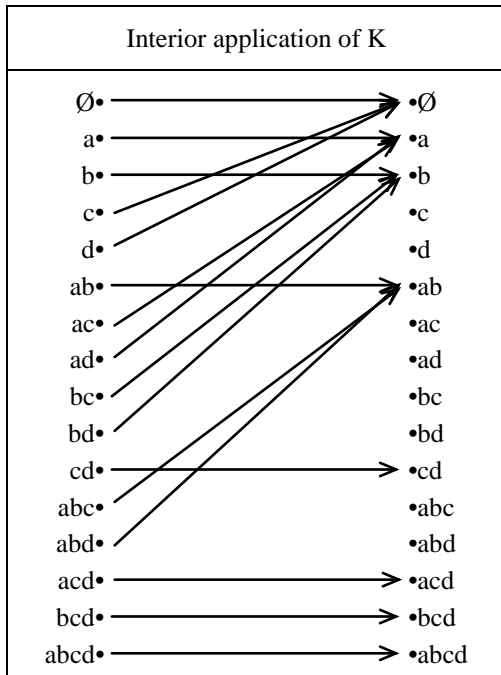


Figure 1

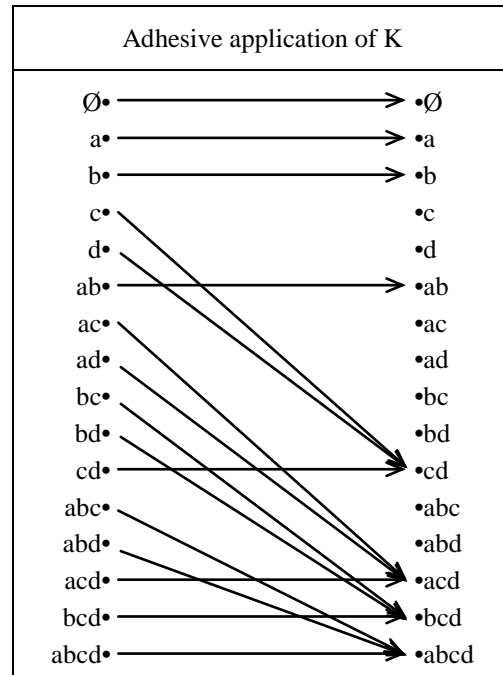


Figure 2

We take these atoms and all the possible links between them, we add \emptyset and arrive at the clan engendered by F. Let us take a look at a very simple example.

Assuming:

$$E = \{a, b, c, d, e, f, g\}$$

and $F = \{ \{a, c, d, e, f\} = A_1, \{b, c, d, g\} = A_2 \}$

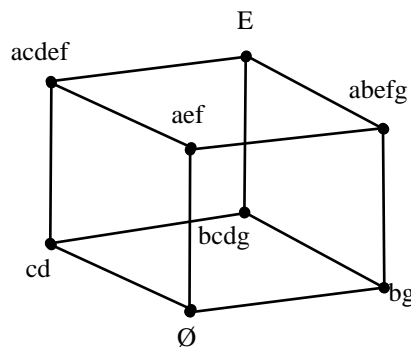
We arrive at:

$$A_1 \cap A_2 = \{c, d\}, A_1 \cap \bar{A}_2 = \{a, e, f\}, \bar{A}_1 \cap A_2 = \{b, g\}, \bar{A}_1 \cap \bar{A}_2 = \emptyset,$$

which are the mini-terms arrived at from F. Therefore we have as the clan engendered by F:

$$K(F) = \emptyset, \{c, d\}, \{a, e, f\}, \{b, g\}, \{a, c, d, e, f\}, \{b, c, d, g\}, \{a, b, e, f, g\}, E$$

In the figure below we have shown the clan engendered by family F:



The use of clans in the management sphere

The notion of a clan is particularly useful for those methods for treating information when the data can be found in the form of a file [2]. A **file** is a finite set E of records, documents, formulae, programmes, specific information of a diverse nature, etc.

A property P that is susceptible of being possessed by at least one record of file E is called a **key**. A file may contain a set of keys:

$$C = \{ P_1, P_2, \dots, P_n \}$$

in such a way that, the whole record may possess at least one property P_j .

We designate by (E, C) the pair formed by a file and the set of keys C.

This is also known by:

$$A_j = f(P_j), \quad j = 1, 2, \dots, n$$

The part $A_j \in P(E)$ possessed by property P_j .

Is also called:

$$F = \{ A_1, A_2, \dots, A_n \} = \{ f(P_1), f(P_2), \dots, f(P_n) \}$$

the family of the non vacant parts of E that possess property P_1 , property P_2 , etc....

A file (E, C) containing a set C of keys can also be designated by the pair (E, F) . In this way we arrive at the concept of topological space.

Starting out from F we can engender a clan that will be designated by $K(E, F)$.

It is obvious that an isomorphism f exists between:

$$K((E, F), \cap, \cup, \bar{}) \text{ and } K((E, C), \Delta, \nabla, \bar{})$$

in which $\cap, \cup, \bar{}$, are the Boolean operations in E and $\Delta, \nabla, \bar{}$, the corresponding operations on the properties.

Let us look at a case with a reduced number of elements of the referential. Let:

$$E = \{ a, b, c, d, e \}$$

And the set of clans:

$$C = \{ P_1, P_2, P_3 \}$$

Let us assume that, according to the nature of the problem, that the following contents are established for the $A_j^*, j = 1, 2, 3$:

$$A_1 = f(P_1) = \{ a, b \}; A_2 = f(P_2) = \{ a, b, c \}; A_3 = f(P_3) = \{ d, e \};$$

$$\bar{A}_1 = f(\bar{P}_1) = \{ c, d, e \}; \bar{A}_2 = f(\bar{P}_2) = \{ d, e \}, \bar{A}_3 = f(\bar{P}_3) = \{ a, b, c \}.$$

The following atoms are arrived at:

$$f(P_1 \Delta P_2 \Delta P_3) = \emptyset; f(P_1 \Delta P_2 \Delta \bar{P}_3) = \{ a, b \}; f(P_1 \Delta \bar{P}_2 \Delta P_3) = \emptyset; f(\bar{P}_1 \Delta P_2 \Delta P_3) = \emptyset;$$

$$f(P_1 \Delta \bar{P}_2 \Delta \bar{P}_3) = \emptyset; f(\bar{P}_1 \Delta P_2 \Delta \bar{P}_3) = \{ c \}; f(\bar{P}_1 \Delta \bar{P}_2 \Delta P_3) = \{ d, e \}; f(\bar{P}_1 \Delta \bar{P}_2 \Delta \bar{P}_3) = \emptyset$$

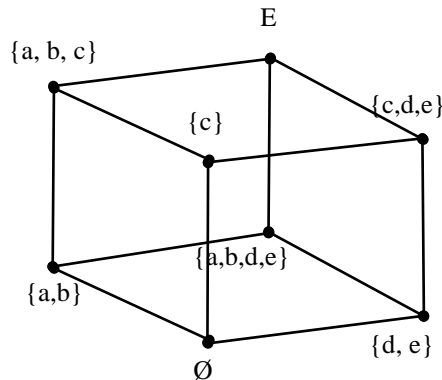
Therefore:

$$C(F) = \{ \{ a, b \}, \{ c \}, \{ d, e \} \}$$

And from here we arrive at the clan we have shown below:

$$K(F) = \{ \emptyset, \{ a, b \}, \{ c \}, \{ d, e \}, \{ a, b, c \}, \{ c, d, e \}, \{ a, b, d, e \}, E \}$$

which can be presented by means of the following Boolean sub-lattice:



These theoretical and technical elements have been repeatedly used for the solution of some of the problems, that were particularly difficult to formalise by other paths. As an example we mention the model

followed for the selection of financial products [3], application in the sphere of investments [4] or the more general work that gave rise to the **Mapclan** model for grouping products [5]

In this type of problem a different approach can be used. We start out from a key and the intention is to arrive at the atom corresponding to it. In order to illustrate what we have just stated, we will start out from a key such as:

$$P = (P_1 \Delta \overline{P_2} \Delta P_3) \nabla (\overline{P_1} \Delta \overline{P_2})$$

Its significance is highly obvious: the intention is to arrive at properties P_1 and P_3 and at the **no** P_2 property, and/or that property P_1 does not exist and neither does the no property P_2 .

It is normal, in scientific and technical media, to transform this key into the form of Boolean variables. In our example this would be:

$$x = x_1 \overline{x_2} x_3 + \overline{x_1} \overline{x_2} = x_1 \overline{x_2} x_3 + \overline{x_1} x_2$$

Although it may be well known for many, we have not been able to resist the temptation to reproduce an example which has become classical in the literature on this subject [6]. This is an attempt to resolve the problem of selecting an automobile. A set of 5 models is considered:

$$E = \{M_1, M_2, M_3, M_4, M_5\}$$

The deciding subject takes into account the following properties:

P_1 : 4 doors	$\overline{P_1}$: 2 doors
P_2 : ≤ 5 HP	$\overline{P_2}$: > 5 HP
P_3 : power steering	$\overline{P_3}$: no power steering
P_4 : de – luxe interior	$\overline{P_4}$: factory interior

An examination of the catalogues for each of the models tells us:

- $f(P_1) = \{M_1, M_2, M_4\}$
- $f(P_2) = \{M_2, M_3, M_4, M_5\}$
- $f(P_3) = \{M_3, M_5\}$
- $f(P_4) = \{M_1, M_2\}$

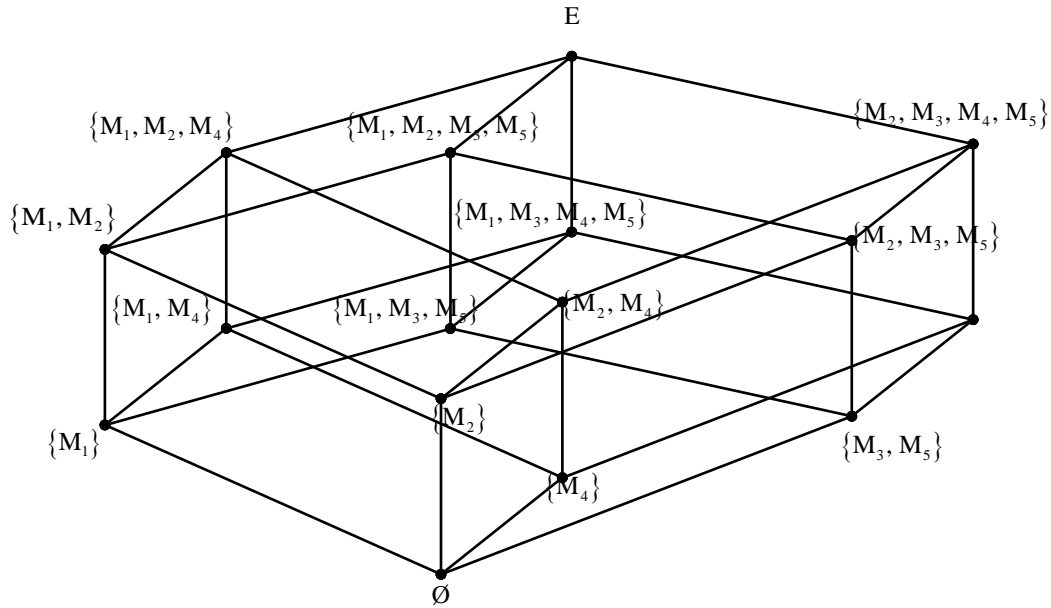
With the object of “organising” this information it is normal to use the matrix that makes the correspondence between models of cars and properties

	P_1	P_2	P_3	P_4	$\overline{P_1}$	$\overline{P_2}$	$\overline{P_3}$	$\overline{P_4}$
M_1	1			1		1	1	
M_2	1	1		1			1	
M_3		1	1		1			1
M_4	1	1					1	1
M_5		1	1		1			1

The mini-terms are:

- $f(1234) = \emptyset, f(12\overline{34}) = \emptyset, f(12\overline{3}4) = \{M_2\}, f(12\overline{3}\overline{4}) = \{M_4\}$
- $f(\overline{1}234) = \emptyset, f(\overline{1}2\overline{34}) = \emptyset, f(\overline{1}2\overline{3}4) = \{M_1\}, f(\overline{1}2\overline{3}\overline{4}) = \emptyset$
- $f(\overline{1}\overline{2}34) = \emptyset, f(\overline{1}\overline{2}3\overline{4}) = \{M_3, M_5\}, f(\overline{1}\overline{2}\overline{3}4) = \emptyset, f(\overline{1}\overline{2}\overline{3}\overline{4}) = \emptyset$
- $f(\overline{1}\overline{2}3\overline{4}) = \emptyset, f(\overline{1}\overline{2}\overline{3}4) = \emptyset, f(\overline{1}\overline{2}34) = \emptyset, f(\overline{1}\overline{2}3\overline{4}) = \emptyset$

Therefore the **non vacant** mini-terms or atoms are: $\{M_1\}, \{M_2\}, \{M_3\}, \{M_4\}, \{M_3, M_5\}$. As we have repeatedly pointed out, these atoms, are possible unions and the vacant one, form a clan, that is to say a topology. This topology can be represented by means of a Boole lattice such as:



We feel that it is not necessary to insist on the fact that this lattice, at the same time, is a Boole lattice of the elements of the “power set” $P(E)$.

We are now going to see how the information contained in the clan can be used. For this, let us assume that we must find the models or models that possess the property or properties:

$$P = \overline{P_1} \Delta P_2 \Delta P_3 \Delta \overline{P_4}$$

By observing the mini-terms it will be seen that $\{M_3, M_5\}$ satisfy this requirement:

$$\overline{P_1} \Delta P_2 \Delta P_3 \Delta \overline{P_4} = \{M_3, M_5\} \cap \{M_2, M_3, M_4, M_5\} \cap \dots \cap \{M_3, M_5\} \cap \{M_3, M_4, M_5\} = \{M_3, M_5\}$$

Obviously this scheme cannot be limited to the clans that coincide with the mini-terms, but that any condition with “and”, and with “and/or” can be established. Therefore let us assume that only the following is required:

$$(P_1 \Delta P_2) \nabla \overline{P_4}$$

We arrive at:

$$\begin{aligned} (P_1 \Delta P_2) \nabla \overline{P_4} &= (\{M_1, M_2, M_4\} \cap \{M_2, M_3, M_4, M_5\}) \cup \dots \cup \{M_3, M_4, M_5\} = \\ &= \{M_2, M_4\} \cup \{M_3, M_4, M_5\} = \{M_2, M_3, M_4, M_5\} \end{aligned}$$

Therefore, all the models satisfy the required conditions, except M_1 .

The example we have used, and many other we could present, brings to light the fact that if we have available 1) the file broken down into atoms and 2) a procedure that allows us to specify the atoms, any key that we desire can be arrived at.

The use of clans for the solution of problems and decision in the sphere of finance, has been quite frequent in latter years. But, perhaps, where the greater attention of researchers and businessmen has been warranted was due to the extension of the clan method with the use of fuzzy variables, that is in $[0, 1]$.

We now move on to a generalisation with the use of fuzzy logic.

Clans in uncertainty

The axiomatic fact we have mentioned for the definition of a clan is also maintained when an extension to fuzzy sub-sets takes place, but the sub-sets that we consider would either be fuzzy sub-sets of a referential E , or else they are the sub-sets themselves which are set up as the elements of the referential. This will give rise to two approaches to the uncertain topological spaces.

In this work we will place all our attention on the first of these, leaving the second of our proposals for future development.

Let us look then, very briefly, how the first of these paths is conceived. We start out from a non fuzzy sub-set of fuzzy sub-sets $A_j^{(i)} \in L^E$, $L = [0, 1]$, $i = 1, 2, \dots, r$. These fuzzy sub-sets will be defined as follows.

Assuming given a referential:

$$E: \{x_1, x_2, \dots, x_n\}$$

And also another referential:

$$C = \{P_1, P_2, \dots, P_m\}$$

Between them there exists a relation such that if $x_i \in E$, when it has a value of the characteristic function for P_j with level μ , the following is written:

$$\mu_{A_j}(x) = \mu, \quad \mu \in [0,1]$$

That is:

$$A_j = f(P_j)$$

We then define a fuzzy sub-set $\cup \subset C$ called a **threshold sub-set**, in which:

$$\mu_u(P_j) = \lambda_j \in [0,1]$$

Before continuing and with the object of clarifying what we have just stated, we are going to resort to an example, as follows:

$$E = \{ a, b, c, d, e, f, g \}; C = \{ P_1, P_2, P_3 \}$$

Let us assume that, due to the characteristics of the problem under study, we have the following fuzzy sub-sets:

$$\begin{array}{l}
 A_1 = f(P_1) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline .3 & 1 & .8 & 0 & .2 & 0 & .4 \\ \hline \end{array} \\
 A_2 = f(P_2) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline .7 & 0 & .7 & .2 & .1 & .8 & 1 \\ \hline \end{array} \\
 A_3 = f(P_3) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline 1 & .8 & .7 & .7 & .3 & .9 & .2 \\ \hline \end{array} \\
 \cup = \begin{array}{|c|c|c|} \hline P_1 & P_2 & P_3 \\ \hline .6 & .3 & .8 \\ \hline \end{array}
 \end{array}$$

From here, by means of the α - cuts of the threshold sub-set U, we arrive at:

$$\begin{array}{l}
 A_1^{0.6} = f^{0.6}(P_1) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline \end{array} = \{b, c\} \\
 A_2^{0.3} = f^{0.3}(P_2) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ \hline \end{array} = \{a, c, f, g\} \\
 A_3^{0.8} = f^{0.8}(P_3) = \begin{array}{|c|c|c|c|c|c|c|} \hline a & b & c & d & e & f & g \\ \hline 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline \end{array} = \{a, b, f\}
 \end{array}$$

We have taken the type of α - cut:

$$< \alpha \Rightarrow 0; \geq \alpha \Rightarrow 1$$

We are now in a position to arrive at the mini-terms as follows:

$$\begin{array}{l}
 f(1,2,3) = \emptyset, f(1,2,\bar{3}) = \{c\}, f(1,\bar{2},3) = \{b\}, f(1,\bar{2},\bar{3}) = \emptyset, \\
 f(\bar{1},2,3) = \{a, f\}, f(\bar{1},2,\bar{3}) = \{g\}, f(\bar{1},\bar{2},3) = \emptyset, f(\bar{1},\bar{2},\bar{3}) = \{d, e\}
 \end{array}$$

The immediate result is then to arrive at the corresponding clan. Assuming that the following key has been established:

$$P = (P_1 \Delta P_2) \nabla (\bar{P}_1 \Delta \bar{P}_3)$$

corresponding to it is:

$$\begin{array}{l}
 f(P) = f((P_1 \Delta P_2) \nabla (\bar{P}_1 \Delta \bar{P}_3)) = (A_1^{0.6} \cap A_2^{0.3}) \cup (\bar{A}_1^{0.6} \cap \bar{A}_3^{0.8}) = \\
 (\{b, c\} \cap \{a, c, f, g\}) \cup (\{a, d, e, f, g\} \cap \{c, d, e, g\}) = \{c\} \cup \{d, e, g\} = \{c, d, e, g\}
 \end{array}$$

The use of topology in finance

Having stated the above, obviously of a high theoretical content, we feel it would illustrative to reproduce an application in the sphere of finance.

For this we are going to assume [6] that for the analysis of the financial-economic situation of a business or institution it is considered that certain aspects exist that could give rise to difficulties or “illnesses”:

M_1 = A lack of cash; M_2 = Difficulty in obtaining outside financing; M_3 = Problems in raising in-house financing; M_4 = Possibility of arriving at a situation of suspension of payment; M_5 = Imbalance between the economic and financial structure; M_6 = Continued losses.

The experts estimate that these difficulties can be detected by means of the value given to these problems (symptoms of difficulty) $S_1, S_2, S_3, S_4, S_5, S_6$.

	M_1	M_2	M_3	M_4	M_5	M_6
For S_1 :	[.6, 1]	[.6, 1]	[.8, 1]	[.8, 1]	[.5, 1]	[.6, 1]
For S_2 :	< 10000	< 5000	< 0	< 1000	<20000 >30000	< 8000
For S_3 :	≤ 1.2	≤ 1.3	≤ 1.2	≤ 1	≤ 1.6	≤ 0.8
For S_4 :	< 0.4	< 0.3	< 0.2	< 0.1	< 0.4	<0.2
For S_5 :	< 0.7	< 1.5	< 1.4	< 0.8	< 1.5	<1
For S_6 :	≤ 0.6	≤ 0.5	≤ 0.5	≤ 0	≤ 0.6	≤ 0

What we have then is what could be called a “table of financial pathologies”, that is valid for a determined type of activity and for a determined moment in time.

The “doctor”, or financial analyst who visits the business or institution must estimate in what situation it will be at some future moment in time, that is what values are foreseen will be attained by each one of the symptoms $S_i, i = 1, 2, \dots, 6$. After the corresponding studies the following valuations are established, some in crisp numbers, others in confidence intervals, and others in triplets:

$$S_1 = [0.6, 0.7] \quad S_2 = 7000. \quad S_3 = [1.1, 1.2]$$

$$S_4 = 0.3 \quad S_5 = (1. 1.2. 1.3). \quad S_6 = 0.6$$

In this way we arrive at the following Boolean matrix:

	M_1	M_2	M_3	M_4	M_5	M_6
S_1 :	1	1			1	1
S_2 :	1				1	1
S_3 :	1	1	1		1	
S_4 :	1				1	
S_5 :		1	1		1	
S_6 :	1				1	

A simple glance at the above allows us to reach certain initial conclusions which, although they are very superficial, could be interesting.

This business, at the date that has been set, will have a certain imbalance between the economic and financial structures, and very possibly a lack of cash (all the symptoms in M_5 and all except S_5 in M_1). We now move on to arrive at the mini-terms or atoms by means of this Boolean matrix:

	S_1	S_2	S_3	S_4	S_5	S_6	\bar{S}_1	\bar{S}_2	\bar{S}_3	\bar{S}_4	\bar{S}_5	\bar{S}_6
M_2	1	1	1	1		1					1	
M_2	1		1		1			1		1		1
M_3			1		1		1	1		1		1
M_4							1	1	1	1	1	1
M_5	1	1	1	1	1	1						

M_6	1	1							1	1	1	1
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In this case $2^6 = 64$ mini-terms exist.

$$f(123456) = \{M_5\}, f(1234\bar{5}6) = \emptyset, f(1234\bar{5}\bar{6}) = \{M_1\}, f(\bar{1}\bar{2}\bar{3}\bar{4}\bar{5}\bar{6}) = \emptyset, f(\bar{1}\bar{2}\bar{3}\bar{4}5\bar{6}) = \emptyset, f(\bar{1}\bar{2}\bar{3}\bar{4}56) = \{M_4\}$$

From the non-vacant mini-terms we will construct the corresponding clan in the way we already know. Finally let us assume a key such as:

$$(S_1\Delta S_3\Delta \bar{S}_4)\nabla(\bar{S}_2\Delta S_5\Delta \bar{S}_6)$$

We arrive at:

$$(\{M_1, M_2, M_5, M_6\} \cap \{M_1, M_2, M_3, M_5\} \cap \{M_2, M_3, M_4, M_6\}) \cup (\{M_2, M_3, M_4\} \cap \{M_2, M_3, M_5\} \cap \dots \cap \{M_2, M_3, M_4\}) = \{M_2\} \cup \{M_2, M_3\} = \{M_2, M_3\}$$

From this key we can deduce, by the very nature of the symptoms that have been established, that the business will have difficulties in obtaining outside financing and problems in raising in-house financing.

We feel that what has been stated is sufficient for illustrating the possibilities of this aspect of the clan theory in financial analysis.

Conclusions

The scheme we have presented brings to light the fact that the conception of a clan as a topology allows for the extension, in a very significant manner, of the possibilities of use of topological spaces in the sphere of social sciences in general and in financial economy in particular. Obviously the two case we have presented cannot be considered as an exception relative to the use of these elements of non numerical mathematics in the field of economy and business and institutional management, but just a very tiny sample of the possibilities that are opened up with the development of these management instruments.

With this work we have attempted to draw attention to one of the paths that can be followed for generalising treatments in the determinist sphere by placing them in the sphere of uncertainty. In it can be seen that the greater information provided by fuzzy sub-sets relative to Boolean sub-sets, allows for better decisions, which leads us to reiterate, once again, the aphorism so often repeated by us in the sense that **information is power**. In our field of study, **power** has the meaning of a substantial improvement in the capacity for adopting correct decisions.

As we have stated in our work, **the door is not closed**. Another path will turn up with a different interpretation of the “fuzzyfying” process. We hope that we can take it up again in the near future.

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