THE INFLUENCE OF MIGRATION ON EPIDEMIC PROCESS

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Abstract. We investigate the influence of value of migration coefficient *m* on dynamic of epidemic process. The possibilities of the mathematical modeling of the epidemic process with periodical regimes are presented. The epidemic theory was presented by N.T.J Bailay. He proved the impossibility of modeling of the epidemic process with periodical regimes. We proved the possibility of such modeling. The meaning of the migration coefficients according to the theorem was obtained and biological interpretation of the mathematical results.

1. Introduction

The major traditional problem of deterministic modelling of the epidemic process is connected with impossibility of modelling of the epidemic process dynamics with periodic regimes. Some authors were presented such a problem as a main defect of the deterministic modelling at all (for example professor N. T. J. Bailay, 1957). Real natural epidemic have a periodical character and may be presents by cyclical epidemic process. In this case we need as a whole a deterministic model presenting such a process.

Some factors that can provide the periodical regimes are the mutation process, time-delay process and season influences. All of it has a periodical character that provides such dynamics. There are special periodical functions $\cos \omega t$ or $\sin \omega t$ presenting such a process. Difference and differential equations consist of such periodical functions in this case. Such models take a form:

$$\frac{dx}{dt} = f(x, y, \cos \omega t), \frac{dy}{dt} = f(x, y, \cos \omega t), \qquad (1)$$

where *x=x(t)* is a function of healthy individuals and *y=y(t)* is a function of individuals with disease. Migration may be nonperiodical process. And we are presented how nonperiodical factor can provides the periodical regimes of an epidemic process in nature.

We prove the theorem about the existence of a cyclic type of epidemic if the migration takes place in population and also the necessary conditions for such a process. It is very interesting to discuss the medical and biological aspects of such conditions**.** Now the simulation of such dynamic with different value of *m* was investigated in detail. It was investigated the difference between the influence of value of two types of coefficient m_1 and m_2 on epidemic process.

2. Deterministic modeling of the epidemic process

According to N. T. J. Bailay [1] and Mc. Kendrick A.G. [5] we can use the simplest deterministic model of an epidemic process as follows:

$$\frac{dx}{dt} = -\beta xy, \frac{dy}{dt} = \beta xy - \gamma y, \frac{dz}{dt} = \gamma y \qquad (2)$$

where *x(t)* and *y(t)* were presented at the previous paragraph by us and $z = z(t)$ is a function of isolated individuals (hospitalisation or immunisation of the individuals), β is a coefficient of an infection process speed, γ is a coefficient of an isolation process speed. The calculation of the migration process in a simple model of epidemic process was presented in such a model [2, 4]:

$$\begin{aligned} \frac{dx}{dt} &= -\beta xy + \mu \\ \frac{dx}{dt} &= \beta xy - \gamma y \end{aligned} . \qquad (3)$$

We are modeling of the dynamics of the epidemic processes with registration of the migration of the susceptible individuals and infective individuals. In this case the models of the epidemic process are as follows:

$$\begin{aligned} \dot{x} &= -\beta xy + \gamma_1 y + \mu_1 \\ \dot{y} &= \beta xy - \gamma y + \mu_2 \end{aligned} , \qquad (4)$$

$$
\begin{aligned}
\dot{x} &= -\beta xy + \delta_1 z + \mu_1 \\
\dot{y} &= \beta xy - \gamma y + \mu_2 \quad , \\
\dot{z} &= \gamma y - \delta z
\end{aligned} \tag{5}
$$

where μ_1 and μ_2 are some function of *t* in all cases.

For such models (4) and (5) and for some previous models the numbers *x+y* are not constant for any $\mu_i = const$ condition. So in this case the number of population may increase without limitation or the population can decrease (it depends of the values μ_1, μ_2 and $\gamma - \gamma_1$).

At the special case if $\gamma = \gamma_1$ and $\mu_1 = -\mu_2$ the number *x+y* is constant. N.T. Bailay did not take into account this fact in [1]. There is a special interesting case: if μ_i is a variable value then for $\mu_1 + \mu_2 = \gamma - \gamma_1$ we have constant number of *x+y*. The condition of the existence of the instability positive stationary solution takes of such a form [2]:

$$
-\mu_1 < \mu_2 < \mu_1 + \frac{(\gamma - \gamma_1)}{2\beta}\left[\sqrt{1 + \frac{4\beta\mu_1}{(\gamma - \gamma_1)^2}} - 1\right]. \tag{6}
$$

Now we will determine the evidence proposition about the value of immigration coefficient μ_1 and emigration (evacuation) coefficient μ_2. The inflow of the healthy individuals according to μ_1 can not be prolonged unlimitedly. Therefore the number of the population will increase for this case unlimitedly (it is unreal). So the inflow of healthy individuals must depend on the number of individuals. We presented [2] the elementary proposition about the value of μ_1:

$$
\mu_1 = \begin{cases} \mu_{10} & \text{if } x + y \le C - \varepsilon_1 \\ = 0 & \text{if } x + y \le C \\ \text{any monotonous ly decreasing function} & \\ \text{of } x + y & \text{if } C - \varepsilon_1 \le x + y \le C. \end{cases} \tag{7}
$$

The next proposition will be as follows: the emigration (evacuation) is impossible if the infective individuals are absent ($y = 0$). As a result of such a process the number of population will be negative (it is impossible). For real process there is some small inflow of the infective individuals. So the infective individuals in the population can not disappear completely if at some moment all individuals will be healthy (it means that $y = 0$). For example, there are some sources of the infection: rodents, birds etc.). So there is an elementary dependence [2] of μ_2 on function *y* as follows:

$$
\mu_2 = \begin{cases} \mu_{20} < 0 & \text{if } y \ge \varepsilon_2 \\ 0 & \text{if } y = 0 \\ \text{any decreasing function of } y & \text{if } 0 \le y \le \varepsilon_2. \end{cases} \tag{8}
$$

It was proved [2] the basic theorem about existing of Puancare cycle for deterministic model of the epidemic process.

Theorem. The system (4) presenting dynamics of diffusion of the diseases in a population with migration has a periodically solution if conditions (6), (7), (8) take place

So such theorem wades the possibility of deterministic modeling of security diseases. Such possibility relatives by N. T. J. Bailay.

According to N. T. J. Bailay [1] for the general case of the epidemic processes (it may be qualified as a model of the epidemic process with chronic immunity) and for the case of hard diseases ($\delta_1 = 0$) we can take the analogous result. Really the model (5) if $\delta_1 = 0$ differs from the model (4) not at all. The difference is based on recovery coefficient γ_1=0. But in previous discussion we did not superimpose limitations on γ_1. So we will have the oscillations of the numbers x and y too. The values of the function $z(t)$ of the system (6) will increase if δ = 0 or will be oscillate about some value if $\delta > 0$.

So the migration of the individuals can be one of the sources of the oscillation of the number x, y and z if the diffusion of infectious and uninfectious diseases in a population takes place [7].

3. The influence of migration on epidemic process

Let's analyses the influence of migration on dynamic of the real epidemic process in detail. Each of the coefficients μ_i has some interval of the oscillation. Now we will determine the influence of migration on dynamic of the real epidemic process about the value of immigration coefficient μ_1. Limits of the coefficients speed of migration of the healthy individuals are from 2,15 to 4,5. If coefficient μ_1>4,5 or μ_1<2,15 then the epidemic process have not oscillatory regime. In this conditions numbers of healthy individuals and individuals with disease will became constants. If the value of the coefficient speed of migration of the healthy individuals is near 2,15 (fig. 1) amplitude of oscillate has some fluctuation. So the oscillations are fading slowly. Decrement of the fade is small. If we increase of μ_1 value the dynamic of process slowly changing (see fig. 2) with decreasing of decrement.

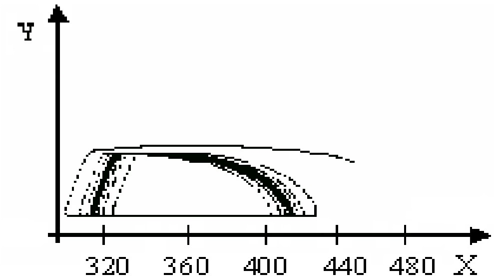

Fig. 1. *The dependence of the number of disease individuals y on the number of the healthy individuals x. Parameters of the model (5): x0 =469, y0 = 12; δ = 0.65; δ_1 = 0.61; β= 0.0018; μ_1 = 2.15; μ_2 = -1.69.*

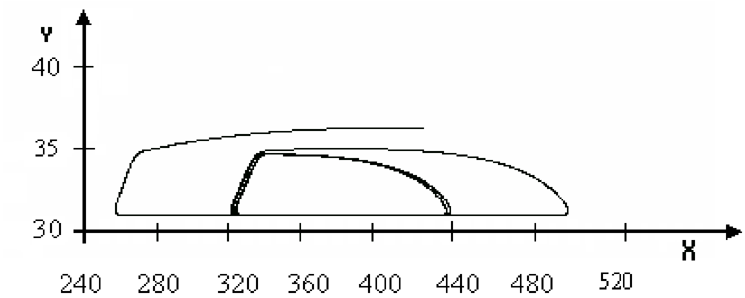

Fig. 2. *The dependence of the number of disease individuals y on the number of the healthy individuals x. Parameters of the model (5): x_0 =469, y_0 = 12; δ = 0.65; δ_1 = 0.61; β= 0.0018; μ_1 = 2.95; μ_2 = -1.69.*

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