THE COMPARATIVE ANALYSIS OF VECTOR SPACE FILTRATIONS

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Let $(\Lambda, \leqslant)$ be a totally ordered set, $\theta \subseteq \Lambda$, $M = \{M_d\}_{d\in\theta}$, and X a vector space over the field $\mathbb{K}$.

Definition 1. A map $\lambda : X \to \Lambda$ is called Lyapunov norm [1] or Lyapunov — Bogdanov (L.-B.) functional [2] if the following conditions hold: $\lambda(cx) \leqslant \lambda(x)$, $\lambda(x+y) \leqslant \leqslant \max\{\lambda(x), \lambda(y)\}$ for every $x, y \in X$, $c \in \mathbb{K}$.

By $LB(X, \Lambda)$ denote the set of L.-B. functionals from X to Λ. By $R(\lambda)$ denote the range of functional λ. Note that $\lambda(0) = \min_{x\in X} \lambda(x)$ for every $\lambda \in LB(X, \Lambda)$.

Definition 2. We say that M is prefiltration of X if the following conditions hold:

i) M_d is a subspace of X for every $d \in \theta$;

ii) $M_{d_1} \subset M_{d_2}$ for every $d_1, d_2 \in \theta$ such that $d_1 < d_2$;

iii) $M_d \setminus \widehat{M}_d \neq \varnothing$ for every $d \in \theta$, where $\widehat{M}_d = \bigcup_{i<d} M_i$ if $d \in \theta \setminus \min(\theta)$ and $\widehat{M}_d = \varnothing$ if $d = \min(\theta)$.

Definition 3 [2]. Prefiltration M of X is the $\mathcal{B}$-filtration of X if $X = \bigcup_{d\in\theta} M_d$.

Definition 4. We say that the prefiltration M of X is the $\mathcal{G}$-filtration of X if the set $\{d \in \theta \mid x \in M_d\}$ contains the least element for every $x \in X$.

Statement 1. *$\mathcal{G}$-filtration of X is a $\mathcal{B}$-filtration of X.*

Statement 2. *$\mathcal{B}$-filtration M of X is $\mathcal{G}$-filtration of X if $(\theta, \leqslant)$ is a well ordered set.*

Now we introduce the following notions associated with functional $\lambda : X \to \Lambda$ [2]: $\widetilde{X}_{\lambda d} := \{x \in X \mid \lambda(x) \leqslant d\}$ is a Lebesque set of the functional λ, $\widetilde{\mathbb{X}}_\lambda := \{\widetilde{X}_{\lambda d}\}_{d\in R(\lambda)}$ is the condensed spectral family of the functional λ.

Theorem 1. *$\widetilde{\mathbb{X}}_\lambda$ is a $\mathcal{G}$-filtration of X for every $\lambda \in LB(X, \Lambda)$.*

Theorem 2. *The map $\lambda : X \to \theta$ such that $\lambda(x) = d$ for every $x \in M_d \setminus \widehat{M}_d$, $d \in \theta$ is an L.-B. functional if M is $\mathcal{G}$-filtration of X.*

Theorem 3. *Let $\mathbb{I}$ be a set of positive irrational numbers, $\theta := [0, +\infty) \bigcap \mathbb{Q}$, and X a vector space such that $F := \{\lambda \in LB(X, \theta) \mid R(\lambda) = \theta\} \neq \varnothing$. Then the following assertions are valid for every $\lambda \in F$:*

i) the family $\{\widetilde{X}_{\lambda d}\}_{d\in R(\lambda)\setminus\{q\}}$ is a $\mathcal{B}$-filtration of X for every $q \in R(\lambda)$;

ii) the family $\{\widetilde{X}_{\lambda d}\}_{d\in R(\lambda)\setminus\{q\}}$ is not $\mathcal{G}$-filtration of X for every $q \in R(\lambda)$;

iii) $\bigcap_{d\in G(i)} \widetilde{X}_{\lambda d} \notin \widetilde{\mathbb{X}}_\lambda$ for every $i \in \mathbb{I}$, where $G(i) = \{q \in \theta \mid i \leqslant q\}$.

The following corollary ensues directly from the theorem 3.

Corollary 1. *There exists a $\mathcal{B}$-filtration M of some vector space X such that it is not $\mathcal{G}$-filtration of X and its indexing set $(\theta, \leqslant)$ contains the least element.*

In paper [2] it is shown that every $\mathcal{B}$-filtration of X is the condensed spectral family of some L.-B. functional on X. But it is wrong. It follows from the theorem 1 and the corollary 1. However every $\mathcal{G}$-filtration of X is the condensed spectral family of some L.-B. functional on X. It follows from the theorem 2.

The following corollary ensues directly from the theorems 1 and 3.

Corollary 2. *There exists a $\mathcal{G}$-filtration M of some vector space X such that every its element is important for $\mathcal{G}$-filtration property of X by the family M.*

References. 1. Bogdanov Yu. S.// Matematicheskii Sbornik. 1959. Vol. 49. P. 225–231. **2.** Borukhov V.T. *Application of Lyapunov — Bogdanov functionals for ultrametrization of abelian groups with operators.* Actual problems of mathematics. Grodno: GrSU, 2008. P. 38–48.