APPLICATION OF MULTILEVEL HM-NETWORK AT DESIGNING OF WAREHOUSES SQUARES IN LOGISTICS TRANSPORT SYSTEMS

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The article deals with the technique allowing to estimate and forecast expected incomes, logistics transport systems subjects warehouse squares. The technique is based on application of HM (Howard–Matalytski) – queueing networks.

Keywords: HM – queueing networks, logistics transport systems, warehouse squares.

1. INTRODUCTION

The logistics structure is a system which consists of various functional areas, such as: stores, information, stacking and warehouse handling, transporting of products and other areas [1]. Thus the main task is cost minimization and profit maximization for producers', customers', warehouses' etc.

Working out LS models is an important problem. The number and the arrangement of producing units (the enterprises, firms, etc.), an amount and the arrangement of warehouses, transport models, connection and information systems are to be taken into consideration.

2. APPLICATION OF NETWORKS AT WAREHOUSE DESIGNING

We'd like to mention that the HM-network can be used not only for LTS subjects expected incomes forecasting, but also for designing of warehouse squares, determination of transport warehouse workers amount in brigades that are engaged in cargo loading and unloading. We will illustrate it in the following example. Let's assume that LTS subjects are n warehouses $S_1, S_2, \ldots, S_n$ between which cargo transportation is carried out. For cars loading and unloading in a warehouse S_i m_i brigades are created, $i = \overline{1, n}$. For simplicity we suggest that the same brigade is engaged in car unloading and in loading it afterwards with new production for further transportation so that the car's stay idle time was minimum. Transporting of the goods from one subject to another brings the latter some casual income and consequently the income of the first subject is reduced to this random variable (RV), however, or it can be vice versa, it doesn't matter for the model. Let's consider the dynamics of the network system S_i

incomes modification (warehouse S_i of LTS). Let's designate by $V_i(t)$ system S_i income at the moment t, and by $v_{i0} = V_i(0)$ its income at the initial moment. Then it is possible to present its income at the moment $t + \Delta t$ as the following:

$$V_i(t + \Delta t) = V_i(t) + \Delta V_i(t, \Delta t), \tag{1}$$

where $\Delta V_i(t, \Delta t)$ – the modification of income QS S_i inn time interval $[t, t + \Delta t)$. To determinate this magnitude let's write out probable events which can happen in time Δt, and incomes modification of S_i systems connected with these events.

1) With the probability $\lambda(t)p_{0i}\Delta t + o(\Delta t)$ the request to system S_i will be received (to a warehouse S_i car) which will bring some income, occupying space r_{0i}, where r_{0i} – RV with expectation $M\{r_{0i}\} = a_{0i}$, $i = \overline{1, n}$.

2) With the probability $\mu_i(k_i(t))u(k_i(t))p_{i0}\Delta t + o(\Delta t)$ the request of S_i will go to the outer world, thus income of QS S_i will be reduced to the magnitude, occupying the of space R_{i0}, where R_{i0} – RV from expectation $M\{R_{i0}\} = b_{i0}$, $i = \overline{1, n}$.

3) With the probability $\mu_j(k_j(t))u(k_j(t))p_{ji}\Delta t + o(\Delta t)$ the request S_j system will pass to the S_i system, thus S_i system income will increase by magnitude, occupying the space of r_{ji}, and the S_j income (i.e. the occupied area of the warehouse S_j) will decrease by this magnitude, where r_{ji} – RV from expectation $M\{r_{ji}\} = a_{ji}$, $i, j = \overline{1, n}$, $i \neq j$.

4) With the probability $\mu_i(k_i(t))u(k_i(t))p_{ij}\Delta t + o(\Delta t)$ a request from system S_i transits to system S_j, thus the income of S_i will decrease by magnitude, occupying space R_{ij}, and the income of S_j (i.e. the occupied square of the warehouse S_j) will increase by this magnitude, where R_{ij} – RV from expectation $M\{R_{ij}\} = b_{ij}$, $i, j = \overline{1, n}$, $i \neq j$.

5) With the probability $1 - (\lambda(t)p_{0i} + \mu_i(k_i(t))u(k_i(t)) + \sum\limits_{\substack{j=1,\\ j \neq i}}^{n} \mu_j(k_j(t))u(k_j(t))p_{ji}) \times$ $\times \Delta t + o(t)$ at time interval $[t, t + \Delta t)$ the S_i system state modification will not happen (i.e. the occupied square of the warehouse S_i will not vary), $i = \overline{1, n}$.

Besides, for each small time interval Δt system S_i (subject S_i) increases the income by magnitude $r_i\Delta t$ by the means of percent on the money which is in its bank, where r_i – RV from expectation $M\{r_i\} = c_i$, $i = \overline{1, n}$. We will consider also that RV r_{ji}, R_{ij}, r_{0i}, R_{i0} are independent in relation to RV r_i, $i, j = \overline{1, n}$.

It is obvious that $r_{ji} = R_{ji}$ with probability 1, i.e.

$$a_{ji} = b_{ji}, \quad i, j = \overline{1, n}. \tag{2}$$

Then from the mentioned above we can conclude the following, at the fixed realization of process $k(t)$, it is possible to note:

$$M\{\Delta V_i(t, \Delta t)\} = \sum_k P(k(t) = k)\, M\{\Delta V_i(t, \Delta t)/k(t)\} =$$
$$= \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \dots \sum_{k_n=0}^{\infty} P(k(t) = (k_1(t), k_2(t), \dots, k_n(t))) \times$$

$$\times M\left\{\Delta V_i(t,\,\Delta t)/k(t)=(k_1(t),\,k_2(t),\,...,\,k_n(t))\right\}=$$

$$=[\lambda(t)p_{0i}a_{0i}+c_i-(p_{i0}b_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ij}b_{ij})\sum_k P\left(k(t)=k\right)\mu_i(k_i(t))u(k_i(t))+$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ji}a_{ji}\sum_k P\left(k(t)=k\right)\mu_j(k_j(t))u(k_j(t))]\Delta t+o(\Delta t).$$

Averaging on $k(t)$ taking into account a normalization state $\sum_k P\left(k(t)=k\right)=1$, for modification of subject S_i expected income we receive:

$$M\left\{\Delta V_i(t,\,\Delta t)\right\}=\sum_k P\left(k(t)=k\right)M\left\{\Delta V_i(t,\,\Delta t)/k(t)\right\}=$$

$$=\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}...\sum_{k_n=0}^{\infty}P\left(k(t)=(k_1(t),\,k_2(t),\,...,\,k_n(t))\right)\times$$

$$\times M\left\{\Delta V_i(t,\,\Delta t)/k(t)=(k_1(t),\,k_2(t),\,...,\,k_n(t))\right\}=$$

$$=[\lambda(t)p_{0i}a_{0i}+c_i-(p_{i0}b_{i0}+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ij}b_{ij})\sum_k P\left(k(t)=k\right)\mu_i(k_i(t))u(k_i(t))+$$

$$+\sum_{\substack{j=1\\ j\neq i}}^{n}p_{ji}a_{ji}\sum_k P\left(k(t)=k\right)\mu_j(k_j(t))u(k_j(t))]\Delta t+o(\Delta t).$$

Let's consider that time intervals of servicing a request in the system S_i (intervals of one car "unloadings – loadings" at the warehouse S_i) are distributed upon the demonstrative law with parameter μ_i, $i=\overline{1,\,n}$, and $\lambda(t)=\lambda$. In this case

$$\mu_i(k_i(t))=\begin{cases}\mu_i k_i(t), & k_i(t)\leq m_i,\\ \mu_i m_i, & k_i(t)>m_i,\end{cases}\qquad \mu_i(k_i(t))u(k_i(t))=\mu_i\min(k_i(t),\,m_i),\quad i=\overline{1,\,n}.$$

Let's assume also that the expression averaging $\mu_i(k_i(t))u(k_i(t))$ gives $\mu_i\min(N_i(t),\,m_i)$, i.e.

$$M\min(k_i(t),\,m_i)=\min(N_i(t),\,m_i), \tag{3}$$

where $N_i(t)$ is the average number of requests (expecting and served) in S_i at the moment of time t, $i=\overline{1,\,n}$. Taking into account this supposition we receive the following approximate relation:

$$M\left\{\Delta V_i(t,\,\Delta t)\right\} = [\lambda(t)p_{0i}a_{0i} + c_i - \mu_i \min(N_i(t),\, m_i)(p_{i0}b_{i0} + \sum_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} p_{ij}b_{ij}) +$$

$$+ \sum_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} \mu_j \min(N_j(t),\, m_j)p_{ji}a_{ji}]\Delta t + o(\Delta t). \quad (4)$$

As an elementary stream of requests arrives in a network with intensity λ, i.e. the probability of a requests inflow in S_i for time Δt looks like $P_a(\Delta t) = \dfrac{(\lambda p_{0i}\Delta t)^a}{a!} e^{-\lambda p_{0i}\Delta t}$, $a = 0,\, 1,\, 2,\, ...$, the average number of the requests which have arrived from the outside to S_i for time Δt is equal to $\lambda p_{0i}\Delta t$. We will find the average number of the occupied service lines in S_i at the moment t, $i = \overline{1,\, n}$, by $\rho_i(t)$. Then $\mu_i\rho_i(t)\Delta t$ is the average number of the requests which have abandoned S_i for time Δt, and $\sum\limits_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} \mu_j\rho_j(t)p_{ji}\Delta t$ is the average number of the requests which have arrived to S_i from other subjects for time Δt. Therefore

$$N_i(t+\Delta t) - N_i(t) = \lambda p_{0i}\Delta t + \sum_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} \mu_j\rho_j(t)p_{ji}\Delta t - \mu_i\rho_i(t)\Delta t, \quad i = \overline{1,\, n},$$

Consequently, at the $\Delta t \to 0$ we receive the UDE system for $N_i(t)$:

$$\frac{dN_i(t)}{dt} = \sum_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} \mu_j\rho_j(t)p_{ji} - \mu_i\rho_i(t) + \lambda p_{0i}, \quad i = \overline{1,\, n}. \quad (5)$$

The precise magnitude $\rho_i(t)$ is impossible to discover and consequently it is approximated by the expression

$$\rho_i(t) = \begin{cases} N_i(t),\ N_i(t) \leq m_i, \\ m_i,\ N_i(t) > m_i, \end{cases} = \min(N_i(t),\ m_i)\ .$$

Then the set of equations (5) will become

$$\frac{dN_i(t)}{dt} = \sum_{\substack{j\,=\,1\\ j\,\neq\, i}}^{n} \mu_j p_{ji} \min(N_j(t),\, m_j) - \mu_i \min(N_i(t),\, m_i) + \lambda p_{0i}, \quad i = \overline{1,\, n}. \quad (6)$$

It is a linear UDE system with the discontinious right members. It is necessary to solve it by space phase partition into a series of areas and solve each of them. The system (6) can be solved, for example, by the means of computer mathematics Maple 8 system.

Let's introduce a sign $v_i(t) = M\{V_i(t)\}$, $i = \overline{1,\, n}$. Then, from (1), (4) we receive passing to a limit at $\Delta t \to 0$, we will receive inhomogeneous linear UDE of the first order

$$\frac{dv_i(t)}{dt} = -\mu_i \min(N_i(t),\, m_i)(p_{i0}b_{i0} + \sum_{j=1}^{n} p_{ij}b_{ij}) +$$
$$+ \sum_{\substack{j=1 \\ j \neq i}}^{n} \mu_j \min(N_j(t),\, m_j) p_{ji} a_{ji} + \lambda p_{0i} a_{0i} + c_i, \quad i = \overline{1,\, n}. \quad (7)$$

Having set entry conditions $v_i(0) = v_{i0}$, $i = \overline{1,\, n}$, it is possible to discover expected incomes of network systems (average magnitudes of squares occupied in warehouses).

Knowing the expressions for $N_i(t)$ and the average warehouse squares occupied with cargoes, it is possible to design the warehouse squares of subjects S_i, $i = \overline{1,\, n}$. Let's consider the following modeling example.

Example 1. We will consider the closed network presented on fig. 1, consisting of $n = 15$ one-linear QS, where $K = 70$ is the number of requests in the network. The requests service intensities in the network system lines are equal to: $\mu_1 = \mu_7 = \mu_{12} = 3.5$, $\mu_2 = \mu_5 = \mu_{11} = 2.8$, $\mu_3 = \mu_6 = \mu_{10} = \mu_{14} = 2.1$, $\mu_4 = \mu_8 = 2.2$, $\mu_9 = \mu_{15} = 4.1$, $\mu_{13} = 2.9$ and probabilities of request transitions between QS networks is $p_{15\,i} = \frac{1}{14}$, $p_{i\,15} = 1$, $i = \overline{1, 14}$, let's define also $p_{ii} = -1$, $i = \overline{1,\, 15}$, remaining $p_{ij} = 0$, $i, j = \overline{1,\, 15}$.

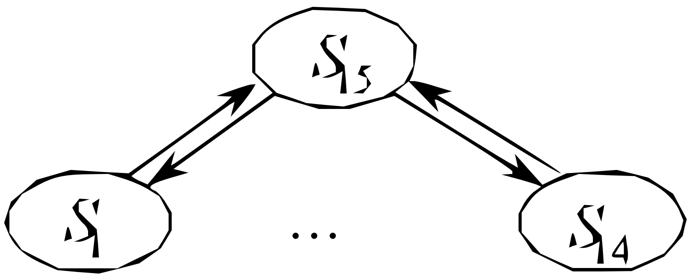

Fig. 1. The network scheme for example 1

Let's assume that the network functions so that on the average there are no queues observed in the peripheral QS, and the central QS functions in the conditions high workload. Then system DDE for the average number of requests in the network systems (5) will be represented as follows:

$$\frac{dN_i(t)}{dt} = \sum_{j=1}^{14} \mu_j p_{ji} N_j(t) + \mu_{15} p_{15i}, \quad i = \overline{1,\, 15}. \quad (8)$$

Let's set values of income expectations from transitions between network states in the following way:

$$c_i = 9 \sin \frac{\pi}{5(i-1)}, \quad i = \overline{1,\, 16},$$
$$a_{15\ i} = 1.6, \ i = \overline{1,\, 6}, \quad a_{15\, i} = 1.1, \ i = \overline{7,\, 13}, \quad a_{15\ 14} = 1.35,$$
$$a_{i\ 15} = (7,\, 11,\, 6,\, 19,\, 28,\, 31,\, 8,\, 17,\, 24,\, 9,\, 12,\, 17,\, 8,\, 13,\, 9), \quad i = \overline{1,\, 14}.$$

Then expected incomes of network systems discovered by the means of the package Mathematica 5.1 provided that at the initial instant of time $v_i(0) = 5$, $i = \overline{1,\, 14}$, $v_{15}(0) = 50$, and entry conditions $N_i(0) = 5$, $i = 1, 7, 9$, $N_i(0) = 3$, $i = 2,\, 3,\, 12$, $N_i(0) = 4$, $i = 4,\, 13,\, 14$, $N_i(0) = 6$, $i = 5,\ 8,\ 11$, $N_i(0) = 2$, $i = 6,\ 10$, $N_{15}(0) = 18$, behave as it is shown in fig. 2.

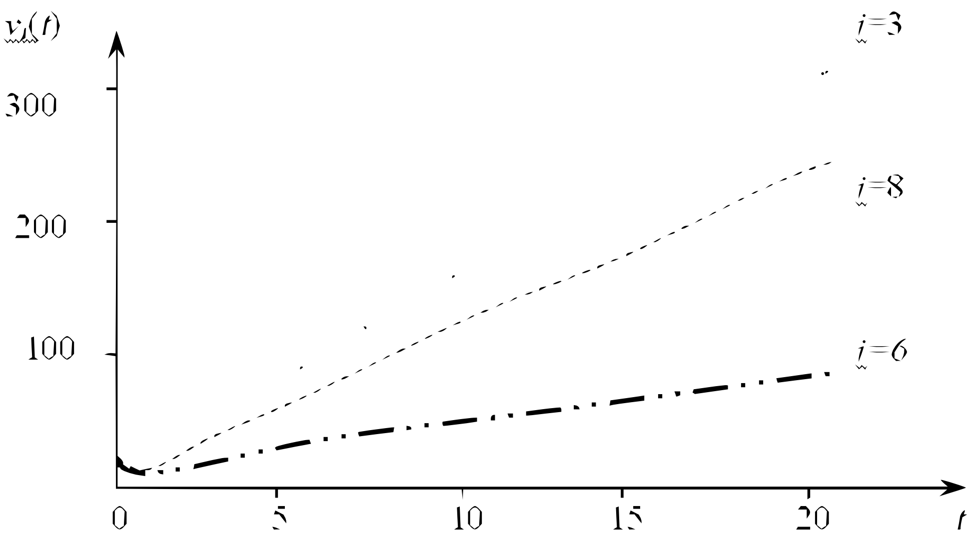

Fig. 2. Expected incomes of systems S_3, S_6, S_8

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