group of $K/F$. We study the realization fields $K = F(G)$ obtained via adjoining to $F$ all matrix coefficients of all matrices $g \in G$ for some finite subgroup $G \subset GL_n(O_K)$.

**Definition.** Consider a finite Galois extension $K$ of $Q$ and a free $Z$-module $M$ of rank $n$ with basis $m_1, \ldots, m_n$. The group $GL_n(O_K)$ acts in a natural way on $O_K \otimes M \cong \bigoplus_{i=1}^n O_K m_i$. The finite group $G \subset GL_n(O_K)$ is said to be of $A$-type, if there exists a decomposition $M = \bigoplus_{i=1}^k M_i$ such that for every $g \in G$ there exists a permutation $\Pi(g)$ of $\{1, 2, \ldots, k\}$ and roots of unity $\epsilon_i(g)$ such that $\epsilon_i(g)gM_i = M_{\Pi(i)}$ for $1 \leq i \leq k$.

**Theorem 1.** Let $K/Q$ be a normal extension with Galois group $\Gamma$, and let $G \subset GL_n(O_K)$ be a finite $\Gamma$-stable subgroup. Then $G$ is a group of $A$-type.

The following theorem was proven using the results for finite flat group schemes over $Z$ annihilated by a prime $p$, obtained by V.A. Abrashkin and J.-M. Fontaine:

**Theorem 2.** Let $K/Q$ be a normal extension with Galois group $\Gamma = Gal(K/Q)$, and let $G \subset GL_n(O_K)$ be a finite $\Gamma$-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where $K_{ab}$ is the maximal abelian over $Q$ subfield of $K$. Moreover, $Q(G) = Q(\xi)$ for some root $\xi$ of $1$.

This theorem can be specified using a generalization of the concept of permutation lattices and permutation modules implemented by A. Weiss and K.W. Roggenkamp to study Zassenhaus Conjecture for group rings, and the structure of representations of the subgroups $G \subset GL_n(O_{K_{ab}})$ in the theorem above can be given more explicitly.

Similar results for totally real extensions and CM-fields $K/Q$ are interesting for classification problems of quadratic and Hermitian lattices, and also for Galois cohomology.

For instance, the theorem above implies that for definite arithmetic groups $G \subset GL_n(O_K)$ the kernel of the natural cohomology map

$$H^1(Gal(K/Q), G) \rightarrow \prod_v H^1(Gal(K_v/Q_v), G_v)$$

is trivial. This is a generalization of the Hasse principle for definite algebraic groups over rings of integers.

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**ON R-CONJUGATE-PERMUTABLE SUBGROUPS OF FINITE GROUPS**

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All groups considered are finite.

In 1997 T. Foguel [1] introduced the concept of conjugate-permutable subgroup. A subgroup $H$ of a group $G$ is called conjugate-permutable if $HH^x = H^xH$ for all $x \in G$.

In [2] we introduced a generalization of it.

**Definition [2].** Let $R$ be a subset of a group $G$. A subgroup $H$ of $G$ is called $R$-conjugate-permutable if $HH^x = H^xH$ for all $x \in R$.

Every conjugate-permutable subgroup is subnormal [1]. Let $F(G)$ be the Fitting subgroup of a group $G$. In the general case $F(G)$-conjugate-permutable subgroup need not to be subnormal (for example Sylow 2-subgroup in the symmetric group $S_4$).

In this paper we study the properties of $R$-conjugate-permutable subgroups. We found sufficient conditions on the subgroup $R$ such that $R$-conjugate-permutability implies subnormality.

We studied the influence of various systems of the $R$-conjugate-permutable subgroups on the structure groups, when $R$ is the Fitting subgroup $F(G)$, its generalizations ($F^s(G)$ and $F^s(G)$), nilpotent residual $\gamma_\infty(G)$ and others.
Theorem 1. The following statements for a group $G$ are equivalent.
1. $G$ is nilpotent.
2. All Sylow subgroups of $G$ are $\gamma_\infty(G)$-conjugate-permutable.
3. All Sylow subgroups of $G$ are $F^*(G)$-conjugate-permutable.
4. All Sylow subgroups of $G$ are $M$-conjugate-permutable where $M$ is an abnormal subgroup of $G$.
5. Every maximal subgroup of $G$ is $\hat{F}(G)$-conjugate-permutable.

Theorem 2. The following statements for a group $G$ are equivalent.
1. $G$ is supersolvable.
2. $G = AB$ where $A$ and $B$ are supersolvable $F(G)$-conjugate-permutable subgroups of $G$ and $G'$ is nilpotent.
3. $G$ is metanilpotent and contains two supersolvable $F(G)$-conjugate-permutable subgroups with coprime indexes in $G$.
4. $G = AB$ where $A$ and $B$ are supersolvable $F(G)$-conjugate-permutable subgroups of $G$ and $G' = A'B'$.

References


THE RESTRICTIONS OF REPRESENTATIONS OF THE SPECIAL LINEAR GROUP TO SUBGROUPS OF RANK 2

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Let $K$ be an algebraically closed field of characteristic $p > 0$, $G$ be a simply connected algebraic group of type $A_r$ over $K$ with $r \geq 3$, $\omega_1, \ldots, \omega_r$ be the fundamental weights of $G$ labeled in the standard way, and let $\phi$ be an irreducible rational representation of $G$ with highest weight $\omega = a_1\omega_1 + \ldots + a_r\omega_r$. A subsystem subgroup in $G$ is a subgroup generated by all the root subsystems of $G$ associated with a certain subsystem of roots. Let $H_1 \subset G$ be a subsystem subgroup of type $A_2$ and $H_2 \subset G$ be a subsystem subgroup of type $A_1 \times A_1$. The sets of all dominant weights of $H_1$ and $H_2$ can be identified with the set $\mathbb{N}^2$ of pairs of nonnegative integers with the help of the map $x_1\omega_1 + x_2\omega_2 \mapsto (x_1, x_2)$ in the first case and $(x_1\omega_1, x_2\omega_1) \mapsto (x_1, x_2)$ in the second case. Denote by $\text{Irr}(\phi|S)$ the set of highest weights of composition factors for the restriction of $\phi$ to a subgroup $S$. Taking into account the above maps, one can write $\text{Irr}(\phi|H_1) \subset \mathbb{N}^2$. Put

$$a = a_1 + \ldots + a_r, \quad A = a_1 + 2a_2 + \ldots + 2a_{r-1} + a_r,$$

$$S_1 = \{(x_1, x_2) \in \mathbb{N}^2 | x_1 \leq a - a_r, x_2 \leq a - a_1, x_1 + x_2 \leq a\},$$

$$S_2 = \{(x_1, x_2) \in \mathbb{N}^2 | x_1 \leq a, x_2 \leq a, x_1 + x_2 \leq A\}.$$

For representations $\phi$ with locally small highest weights the composition factors of the restriction of $\phi$ to $H_1$ are determined.

Theorem 1. Let $r > 5$ and $a_i + a_{i+1} + 1 < p$ for all $1 \leq i \leq r - 1$. Then $\text{Irr}(\phi|H_1) = S_1$.

Theorem 2. Let $r > 6$ and $a_i < p$ for all $1 \leq i \leq r$. Then $\text{Irr}(\phi|H_2) = S_2$. 