$$C_2 = \frac{4(\sqrt{2}+1)(m-1)}{\sqrt[m-1]{C_1}}\left(\frac{C_3}{C_1}\operatorname{mes} I + 1\right), \quad C_3 = \max_{1\leqslant j,q\leqslant m} ||f_j^{(q)}||_{C(I;\mathbb{R})}.$$

Measure estimates for exceptional sets of smooth functions also play an important role in metric theory of Diophantine approximations (see [3–5]).

Aknowledgements. The work is partially supported by SFFR, grant F41.1/004.

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ON A SOLUTION OF A FUNCTIONAL EQUATION OF TYPE (4;2) ON QUASIGROUPS

H.V. Krainichuk

University "Ukraina", Vinnytsia Institute of Economics and Social Sciences, Vinnytsia, Ukraine
kraynichuk@ukr.net

We say that *a functional equation has a type* $(m;n)$ if it has two individual variables with appearances m and n. For example, V.D. Belousov investigated the minimal quasigroup functional equations and the minimal quasigroup identities. He proved [1] that such functional equations have the type $(3;2)$ and all of them are parastrophically equivalent to $A(x;B(x;C(x;y))) = y$ and he stated that there exist 7 identities up to parastrophic equivalency. In other words, V. D. Belousov classified functional equations and identities with 5 appearances of 2 individual variables. R.F.Koval' [2] considered functional equations with 6 appearances of 2 individual variables, i.e., functional equations of the types (4;2) and (3;3). She proved that there exist at most 13 functional equations up to parastrophic equivalency, but she did not prove that they are not parastrophically equivalent and did not find their solution sets.

Complete classification of functional equations of the type (4:2) is given by the author in the annotation [3]. There exist only six classes of such equations. The representative of each class is solved. Full solution of one of this type functional equations is given here.

A binary function f is called *invertible* (i.e., *quasigroup*) if every of the equations $f(x;a) = b$, $f(a;y) = b$ has a unique solution for all a, b. Two functional equations are known to be:

- *equivalent* if their solution sets on every carrier set coincide;
- *parastrophically equivalent* [4] if one can be obtained from the other in a finite number of the following steps:

1) application of: $F({}^{\ell}F(x,y),y) = x$, $F(x,{}^{r}F(x,y)) = y$, ${}^{\tau}({}^{\sigma}F) = {}^{\sigma\tau}F$;
2) replacing $\omega = \upsilon$ by $\upsilon = \omega$;
3) renaming the individual variables;
4) renaming the functional variables.

Let f be an invertible function. A parastrophe ${}^{\sigma}\!f$ of f is defined by

$$ {}^{\sigma}\!f(x_{1\sigma}; x_{2\sigma}) = x_{3\sigma} \iff f(x_1; x_2) = x_3 $$

for any $\sigma \in S_3$, where $S_3 := \{\varepsilon, \ell, r, s, \ell s, rs\}$ and $s := (12)$, $\ell := (13)$, $r := (23)$.

Theorem. *Let f_1, f_2, f_3, f_4 be binary invertible functions defined on a set Q. Then a quadruple $(f_1; \ldots; f_4)$ is a solution of*

$$ F_1(y; y) = F_2(x; F_3(x; F_4(x; x))) $$

iff there exists a substitution α and an element a of Q such that

$$ f_1(y; y) = a, \quad f_2(x; \alpha x) = a, \quad f_4(x; x) = {}^{r}\!f_3(x; \alpha x). $$

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SYSTEMS OF LINEAR CONGRUENCES, BALANCED MODULAR LABELLINGS OF GRAPHS AND CHROMATIC TOTIENTS

V.A. Liskovets

Institute of Mathematics, National Academy of Sciences of Belarus
220072, Minsk, Belarus
liskov@im.bas-net.by

The present research is devoted to some number-theoretic consequences of certain notions and results of algebraic graph theory. Given a finite simple connected graph $G = (V, E)$, an orientation of its edges and a natural number k, we consider edge k-labellings $f : E \to \mathbb{Z}_k^*$ satisfying Kirchhoff's circuit law, where $\mathbb{Z}_k^*$ is the set of invertible elements of the ring $\mathbb{Z}_k = \mathbb{Z}/k\mathbb{Z}$. In terms of variables $x_e = f(e)$, $e \in E$, we consider the system of homogeneous linear congruences modulo k which correspond to the (simple, independent) cycles of G, have coefficients 0 and ± 1 (moreover, their matrix is unimodular) and are subject to the 'side condition' that all values of their variables are coprime with k. The solutions with the latter property are called *invertible.* The choices of edge orientations and independent cycles do not influent the resulting system of congruences up to equivalence. Let $R(G, k)$ be the number of invertible solutions of such a system.

Theorem. *For any finite connected graph G, $R(G, k)$ is the multiplicative arithmetic function of k that is determined by the formula*

$$ R(G, p^a) = \chi(G, p)p^{(a-1)(n-1)-1} \tag{1} $$

for every prime p and integer $a \geqslant 1$, where $\chi(G, z)$ is the chromatic polynomial of G and $n = |V|$ is the number of vertices.

This basic equation shows that $R(G, k)$ is a kind of totient functions [1], which we call a *chromatic totient.* In particular, $R(K_2, k) = \phi(k)$, Euler's totient function, where $K_2 =$ •—•