SUBDIFFERENTIABILITY OF FUNCTIONS THAT ARE ABSTRACT CONVEX WITH RESPECT TO THE SET OF LIPSCHITZ CONCAVE FUNCTIONS

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Abstract. For the functions defined on normed vector spaces we introduce the notion of the $\mathcal{L}\widehat{C}$-convexity that generalizes the classical notion of convex functions. A function f is called $\mathcal{L}\widehat{C}$-convex if it can be represented as the upper envelope of some subset of Lipschitz concave functions. In the terminology of abstract convexity [1] it means that f is abstract convex with respect to the set $\mathcal{L}\widehat{C}$ of Lipschitz concave functions. We prove [2, 3] that a function is $\mathcal{L}\widehat{C}$-convex if and only if it is lower semicontinuous and, in addition, it is bounded from below by a Lipschitz continuous function. For a function f and a point $x \in \operatorname{dom} f$ we introduce the notion of the $\mathcal{L}\widehat{C}$-subgradient as well as the notions of the $\mathcal{L}\widehat{C}$-presubdifferential and the $\mathcal{L}\widehat{C}$-subdifferential of f at x. We prove that for a $\mathcal{L}\widehat{C}$-convex function f the $\mathcal{L}\widehat{C}$-presubdifferential and the $\mathcal{L}\widehat{C}$-subdifferential of the function f are nonempty at any point of the dense subset of $\operatorname{dom} f$. This result extends the well-known Brøndsted-Rockafellar theorem on the existence of the Fenchel subdifferential of a convential convex function to the wider class of lower semicontinuous functions. As an application we derive the $\mathcal{L}\widehat{C}$-subdifferential criterium of global minimum and the $\mathcal{L}\widehat{C}$-subdifferential necessary condition of global maximum for a nonsmooth function.

References

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