GUARANTEE OPTIMIZATION IN PROBLEMS WITH FUNCTIONALLY CONSTRAINED SET OF DISTURBANCES

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We deal with a problem of optimizing the guaranteed result for a dynamical system controlled under conditions of disturbances. A motion of the system is considered on a finite time interval and is described by an ordinary differential equation. Admissible controls are measurable functions with values in a given compact set. The goal of control is to minimize a cost functional, which evaluates system's motions.

In a classical formulation of such problems, admissible disturbances are usually assumed to satisfy two conditions similar to those imposed on admissible controls, i.e., instantaneous (or geometric) constraints coupled with the claim of measurability. In this case, the theory of zero-sum differential games can be involved to study and solve the problem. In particular, in order to obtain a constructive solution in the form of feedback (positional) controls, we can use, for example, the method of extremal aiming, developed within the positional approach (see, e.g., [1, 2]). At the same time, a direct application of many basic results of the differential games theory, including the method of extremal aiming [1, 2], requires (often implicitly) the following "gluing up" property: an admissible disturbance extended by any other one from any time composes a new admissible disturbance. On the other hand, there is a wide range of applied control problems in which standard two assumptions on the set of disturbances are supplemented by constraints of a functional nature. For example, they can be determined by such conditions as constancy, continuity, or Lipschitz continuity, as well as boundedness of a number of discontinuity points and compactness in some topology (see, e.g., [3]–[6]). Usually, by taking this additional information into account, we can significantly improve the value of optimal guaranteed result. However, under such functional constraints, the property of "gluing up" may no longer be satisfied. So, the presence of a functional constraint is the main feature of the considered guarantee optimization problem. Note that we do not fix a specific form of this constraint, and, in particular, the set of admissible disturbances may turn out to be hereditary.

Our aim is to provide a functional in the space of positions of the dynamical system that possesses suitable properties of u- and v-stability (see, e.g., [1, Sect. 4.2] and [2, Sect. 8]), which may allow to use this functional as a basis for obtaining optimal feedback controls via the extremal aiming technique. For these purposes we use the functional of the optimal guaranteed result with respect to non-anticipative control strategies. The construction is prompted by the classical result (see, e.g., [2, Sect. 9]) about the coincidence of the optimal guaranteed results in the classes of positional strategies and non-anticipative strategies under Isaacs' condition and similar results (see, e.g., [4, 7, 8]) under some other conditions imposed on the control problem. Note that dealing with the functional constraints may lead to a non-anticipative control strategy that differs from the standard ones. Namely, in considered case the appropriate class of strategies depends on the system's dynamics and the current history of the control process. Thus, the main result of the paper is the proof of the fact that the introduced functional satisfies the dynamic programming principle and, as a consequence, possesses the stability properties mentioned above.

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