Theorem. *For the operator* $\Gamma_{\mu,\sigma}$ *to be bounded in the Hardy space* $H^2(\mathbb{D})$*, the condition*

$$\sup_{k\in\mathbb{Z}_+}\sum_{j=0}^{\infty}\left|\frac{\gamma_{k+j}}{\mu^j}\right|^2<\infty \tag{1}$$

is necessary. Under this condition, this operator is μ*-Hankel in* $H^2(\mathbb{D})$*, has the matrix* $(\gamma_{k+j}/\mu^j)_{k,j\in\mathbb{Z}_+}$ *with respect to the standard basis of this space, and the following statements are true.*

1) *Let* $|\mu|<1$*. Then the operator* $\Gamma_{\mu,\sigma}$ *is nuclear and*

$$\mathrm{tr}\Gamma_{\mu,\sigma}=\sum_{n=0}^{\infty}\frac{\gamma_{2n}}{\mu^n}=\mu\int_{\mathbb{D}}\frac{d\sigma(\zeta)}{\mu-\zeta^2}. \tag{2}$$

2) *Let* $|\mu|>1$*. Then the operator* $\Gamma_{\mu,\sigma}$ *is bounded if and only if* $(\gamma_n)\in\ell^2(\mathbb{Z}_+)$*. Moreover, it is nuclear, and its trace is expressed by the formula* (2).

3) *Let* $|\mu|=1$*. Operator* $\Gamma_{\mu,\sigma}$ *is bounded if and only if there is such function* $\psi\in L^\infty(\mathbb{T})$*, that* $\gamma_n=\widehat{\psi}(n)$ *for* $n\in\mathbb{Z}_+$*. Moreover*

$$\|\Gamma_{\mu,\sigma}\|=\inf\{\|\psi\|_{L^\infty}:\psi\in L^\infty(\mathbb{T}),\gamma_n=\widehat{\psi}(n)\forall n\in\mathbb{Z}_+\}.$$

For $|\mu|=1$, the sufficient boundedness condition gives the next

Corollary 1. *Let* $|\mu|=1$*. If the function*

$$\varphi_\sigma(\zeta):=\int_{\mathbb{D}}\frac{1-|z|^2}{|z-\zeta|^2}d\sigma(z)\quad(\zeta\in\mathbb{T})$$

belongs to $L^\infty(\mathbb{T})$*, the operator* $\Gamma_{\mu,\sigma}$ *is bounded in* $H^2(\mathbb{D})$ *and*

$$\|\Gamma_{\mu,\sigma}\|\le\|\varphi_\sigma\|_{L^\infty}.$$

Corollary 2. *Let the condition (1) be satisfied. The operator* $\Gamma_{\mu,\sigma}$ *has finite rank if and only if the function* $\Gamma_{\mu,\sigma}1$ *is rational. In this case* $\mathrm{rank}\Gamma_{\mu,\sigma}=\deg(z(\Gamma_{\mu,\sigma}1)(z))$.

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HAUSDORFF OPERATORS ON LEBESGUE SPACES AND HARDY SPACES

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The Hausdorff operator on the group $\mathbb{R}^n$ has the form

$$(\mathcal{H}f)(x)=\int_{\mathbb{R}^n}K(u)f(A(u)x)du,\ x\in\mathbb{R}^n,$$

where $K\in L^1_{loc}(\mathbb{R}^n)$, $A(u)\in\mathrm{GL}(n,\mathbb{R})$.

The one-dimensional Hausdorff operators were introduced by Garabedian and independently by Rogosinski as a continuous analog of Hausdorff means (see, e.g., [1, Chapter XI]). The n-dimensional definition is due to Brown-Moricz and Liflyand-Lerner (see, e.g., [2]).

The generalization of Hausdorff operators to general topological group G is as follows [3]:

$$(\mathcal{H}_{K,A}f)(x) = \int_{\Omega} K(u)f(A(u)(x))d\mu(u),\ x \in G,$$

where $K \in L^1_{loc}(\Omega, \mu)$, $A(u) \in \mathrm{Aut}(G)$, the group of automorphisms of G.

The following issues are expected to be discussed in the talk:

— the description of normal Hausdorff operator over $\mathbb{R}^n$ [4];

— the noncompactness of Hausdorff operators over $\mathbb{R}^n$ [5];

— the boundedness of Hausdorff operators on Hardy spaces H^1 over locally compact groups [3, 6, 7];

— the generalizations of Hausdorff operators to homogeneous spaces of locally compact groups [6, 7].

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ON PERIODIC GIBBS MEASURES FOR THE THREE-STATE POTTS-SOS MODEL

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The Cayley tree Γ^k (See [1]) of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles, from each vertex of which exactly $k+1$ edges issue. Let $\Gamma^k = (V, L, i)$, where V is the set of vertices of Γ^k, L is the set of edges of Γ^k and i is the incidence function associating each edge $l \in L$ with its endpoints $x, y \in V$. If $i(l) = \{x, y\}$, then x and y are called *nearest neighboring vertices*, and we write $l =< x, y >$.

Consider model where the spin takes values in the set $\Phi = \{0, 1, 2, ..., m\}, m \geq 1$. For $A \subseteq V$ a spin *configuration* σ_A on A is defined as a function $x \in A \to \sigma_A(x) \in \Phi$; the set of all configurations coincides with $\Omega_A = \Phi^A$. Denote $\Omega = \Omega_V$ and $\sigma = \sigma_V$.

Potts-SOS model is a generalization of the Potts and SOS (solid-on-solid) models. The Hamiltonian of the Potts-SOS model has the form

$$H(\sigma) = -J \sum_{<x,y>\in L} |\sigma(x) - \sigma(y)| - J_p \sum_{<x,y>\in L} \delta_{\sigma(x)\sigma(y)} \quad (1)$$

where $J, J_p \in R$ are nonzero coupling constants.