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Some Comments on Dynamical Character of Cosmological Constant and Generalized Uncertainty Principle

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In this paper the results obtained by Minic and his colleagues on the uncertainty relation of the pair "cosmological constant - volume of space-time", where cosmological constant is a dynamical quantity, are reconsidered and generalized proceeding from a more natural viewpoint. It is demonstrated that on the basis of simple and natural assumptions this relation may be understood with the help of the uncertainty relation for the pair "energy - time". Since the latter is generalized at Planck's scales (Early Universe)- GUP, the first one may be generalized in a similar way. This means that we can suggest GUP for the pair "cosmological constant - space-time volume". Here the relation is derived in the explicit form, and also some implications are considered.

Keywords: Einstein's equations, cosmological constant,Generalized Uncertainty Principle, Early Universe

The Cosmological Constant Problem is basic for modern fundamental physics. There are three principal questions associated with this problem.
(a) Why the cosmological constant is nonzero?
(b) Why this constant is so small, being lower than the expected theoretical value by a factor of $\sim 10^{123}$?
(c)And why its actual value conforms well to the critical density of vacuum energy ρ_c?
Besides, the Cosmological Constant (Vacuum Energy Density) Problem is closely connected with the Dark Energy Problem that has become one of the key physical problems in basic research. Numerous works and review papers on this problem have been published in the last 10-15 years [1]. And a great number of approaches to this problem have been proposed: scalar field models (quintessence model, K-essence, tachyon field, phantom field, dilatonic, Chaplygin gas) [2], [3],[4],[5],[6], [7], braneworld models [8],dynamic approaches to the cosmological constant Λ [9], anthropic selection of Λ [10], etc.
At the same time, it should be noted that Cosmological Constant (Vacuum Energy) persists to be the main candidate to play a role of Dark Energy. But we still have no intelligible answers for the above questions. Because of this, any progress in this direction is of particular value. By author's opinion, most interesting in this respect are the works [11]–[14]. Specifically, of great interest is the Uncertainty Principle derived in these works for the pair of conjugate variables (Λ, V):

$$\Delta\Lambda\,\Delta V \sim \hbar, \tag{1}$$

where Λ is the vacuum energy density (cosmological constant). It is a dynamical value fluctuating around zero; V is the space-time volume. Here the volume of space-time V results from the Einstein-Hilbert action [12]:

$$S_{EH} \supset \Lambda \int d^4x\sqrt{-g} = \Lambda V \tag{2}$$

In this case "the notion of conjugation is well-defined, but approximate, as implied by the expansion about the static Fubini–Study metric" (Section 6.1 of [11]). Unfortunately, in the proof per se (1), relying on the procedure with a non-linear and non-local Wheeler–de-Witt-like equation of the background independent Matrix theory, some unconvincing arguments are used, making it insufficiently rigorous (Appendix 3 of [11]). But, without doubt, this proof has a significant result, though failing to clear up the situation.

Let us attempt to explain (1)(certainly at an heuristic level) using simpler and more natural terms involved with the other, more well-known, conjugate pair (E,t) - "energy - time". We use the designations of [11],[12]. In this way a four-dimensional volume will be denoted, as previously, by V.

Just from the start, the Generalized Uncertainty Principle (GUP) is used. Then a change over to the Heisenberg Uncertainty Principle at low energies will be only natural. As is known, the Uncertainty Principle of Heisenberg at Planck's scales (energies) may be extended to the Generalized Uncertainty Principle. To illustrate, for the conjugate pair "momentum-coordinate" (p,x) this has been noted in many works [15]–[19]:

$$\triangle x \geq \frac{\hbar}{\triangle p} + \alpha' l_p^2 \frac{\triangle p}{\hbar}. \tag{3}$$

In [20],[21] it is demonstrated that the corresponding Generalized Uncertainty Relation for the pair "energy - time" may be easily obtained from

$$\Delta t \geq \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar}, \tag{4}$$

where l_p and t_p - Planck length and time, respectively.

Now we assume that in the space-time volume $\int d^4x\sqrt{-g} = V$ the temporal and spatial parts may be separated (factored out) in the explicit form:

$$V(t) \approx t\bar{V}(t), \tag{5}$$

where $\bar{V}(t)$ - spatial part V. For the expanding (inflation) Universe such an assumption is quite natural. Then it is obvious that

$$\Delta V(t) = \Delta t\bar{V}(t) + t\Delta\bar{V}(t) + \Delta t\Delta\bar{V}(t). \tag{6}$$

Now we recall that for the inflation Universe the scaling factor is $a(t) \sim e^{Ht}$. Consequently, $\Delta\bar{V}(t) \sim \Delta t^3 f(H)$, where $f(H)$ is a particular function of Hubble's constant. From (4) it follows that

$$\Delta t \geq t_{min} \sim t_p. \tag{7}$$

However, it is suggested that, even though Δt is satisfying (7), its value is sufficiently small in order that ΔV be contributed significantly by the terms containing Δt to the power higher than the first. In this case the main contribution on the right-hand side of (6) is made by the first term $\Delta t\bar{V}(t)$ only. Then, multiplying the left- and right-hand sides of (4) by $\bar{V}$, we have

$$\Delta V \geq \frac{\hbar\bar{V}}{\Delta E} + \alpha' t_p^2 \frac{\Delta E\bar{V}}{\hbar} = \frac{\hbar}{\Delta\Lambda} + \alpha' t_p^2 \bar{V}^2 \frac{\Delta\Lambda}{\hbar}. \tag{8}$$

It is not surprising that a solution of the quadratic inequality (8) leads to a minimal volume of the space-time $V_{min} \sim V_p = l_p^3 t_p$ since (3) and (4) result in minimal length $l_{min} \sim l_p$ and minimal time $t_{min} \sim t_p$, respectively.

(8) is of interest from the viewpoint of two limits:

1)IR - limit: $t \to \infty$
2)UV - limit: $t \to t_{min}$.
In the case of IR-limit we have large volumes $\bar{V}$ and V at low $\Delta\Lambda$. Because of this, the main contribution in the right-hand side of (8) is made by the first term as great $\bar{V}$ in the second term is damped by small t_p and $\Delta\Lambda$. Thus, we derive at

$$\lim_{t\to\infty} \Delta V \approx \frac{\hbar}{\Delta\Lambda} \tag{9}$$

in accordance with (1) [11]. Here, similar to [11] , Λ is a dynamical value fluctuating around zero.
And for the case (2) $\Delta\Lambda$ becomes significant

$$\lim_{t\to t_{min}} \bar{V} = \bar{V}_{min} \sim \bar{V}_p = l_p^3; \lim_{t\to t_{min}} V = V_{min} \sim V_p = l_p^3 t_p. \tag{10}$$

As a result, we have

$$\lim_{t\to t_{min}} \Delta V = \frac{\hbar}{\Delta\Lambda} + \alpha_\Lambda V_p^2 \frac{\Delta\Lambda}{\hbar}, \tag{11}$$

where the parameter α_Λ absorbs all the above-mentioned proportionality coefficients.
For(11) $\Delta\Lambda \sim \Lambda_p \equiv \hbar/V_p = E_p/\bar{V}_p$.
It is easily seen that in this case $\Lambda \sim M_p^4$, in agreement with the value obtained using a naive (i.e. without super-symmetry and the like) quantum field theory [22],[23]. Despite the fact that Λ at Planck's scales (referred to as $\Lambda(UV)$) (11) is also a dynamical quantity, it is not directly related to well-known Λ (1),(9) (called $\Lambda(IR)$) because the latter, as opposed to the first one, is derived from Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N \left(-\Lambda g_{\mu\nu} + T_{\mu\nu}\right). \tag{12}$$

However, Einstein's equations (12) are not valid at the Planck scales and hence $\Lambda(UV)$ may be considered as some high-energy generalization of the conventional cosmological constant, leading to $\Lambda(IR)$ in the low-energy limit.
In conclusion, it should be noted that the right-hand side of (3),(4) in fact is a series. Of course, a similar statement is true for (11) as well.
Then, we obtain a system of the Generalized Uncertainty Relations for the Early Universe (Planck scales) in the symmetric form as follows:

$$\left\{ \begin{array}{lcl} \Delta x & \geq & \frac{\hbar}{\Delta p} + \alpha' \left(\frac{\Delta p}{p_{pl}}\right) \frac{\hbar}{p_{pl}} + ... \\ \\ \Delta t & \geq & \frac{\hbar}{\Delta E} + \alpha' \left(\frac{\Delta E}{E_p}\right) \frac{\hbar}{E_p} + ... \\ \\ \Delta V & \geq & \frac{\hbar}{\Delta\Lambda} + \alpha_\Lambda \left(\frac{\Delta\Lambda}{\Lambda_p}\right) \frac{\hbar}{\Lambda_p} + ... \end{array} \right. \tag{13}$$

The last of the relations (13) may be important when finding the general form for $\Lambda(UV)$, low-energy limit $\Lambda(IR)$, and also may be a step in the process of constructing future quantum-gravity equations, the low-energy limit of which is represented by Einstein's equations (12).

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